

Eastern  
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Edition

# Engineering Economics

R. Panneerselvam



# Engineering Economics

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**ENGINEERING ECONOMICS**

by R. Panneerselvam

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To  
*My Great-Grand-father*

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## PREFACE

Efficient functioning of any business organization would enable it to provide goods/services at a lower price. In the process of managing organizations, the managers at different levels should take appropriate economic decisions which will help in minimizing investment, operating and maintenance expenditures besides increasing the revenue, savings and such other gains of the organization. These can be achieved through Engineering Economics which deals with the methods that enable one to make economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

This book on Engineering Economics is the outgrowth of my several years of teaching postgraduate courses in industrial engineering and production engineering and a year of teaching water resources management (all at Anna University, Chennai). It is intended as a text for these disciplines. It can also be used as a text for the undergraduate engineering courses and as a reference for management (project management) and commerce (financial management) courses. Besides, professional engineers and project consultants undertaking economic decision analysis would find the book useful.

I have tried not only to give a comprehensive coverage of the various aspects of engineering economic analysis but provided an exhaustive appendix on Interest Tables for a wide range of interest rates (0.25–50%) and a period ranging from one year to 100 years. These tables, along with the topics discussed, will, I believe, help both students and teachers in carrying out economic analysis and solving problems.

The book contains about 100 well-structured worked-out examples to illustrate the concepts explained. Each chapter also has a set of questions and problems to test the student's power of comprehending the topics.

I wish to thank Prof. Dr. K.N. Balasubramanian, formerly Head of the Department of Industrial Engineering Division of Anna University, who gave me the opportunity of teaching the subject to M.E. Industrial Engineering class continuously for seven years during my stay there as a faculty. This enabled me to enrich my knowledge and expertise in the subject. Many of my colleagues and academic friends helped me by giving valuable suggestions on the structure and content of this text and these were instrumental in improving the quality and presentation of this book. I wish to express my profound gratitude and appreciation to all of them.

Any suggestions for improving the contents would be warmly appreciated.

**R. Panneerselvam**

# 1

## INTRODUCTION

This chapter discusses the elements of economics and the interaction between its various components. This is followed by an analysis of the need and scope of engineering economics. Later, elements of cost and break-even analysis are presented.

### 1.1 ECONOMICS

Economics is the science that deals with the production and consumption of goods and services and the distribution and rendering of these for human welfare.

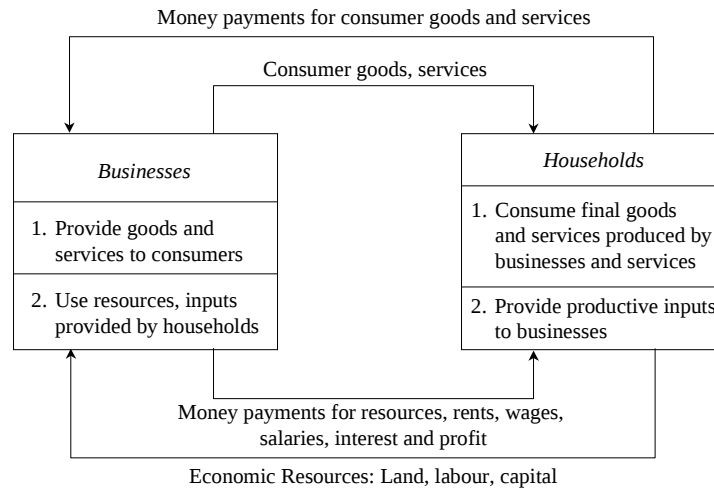
The following are the economic goals.

- A high level of employment
- Price stability
- Efficiency
- An equitable distribution of income
- Growth

Some of the above goals are interdependent. The economic goals are not always complementary; in many cases they are in conflict. For example, any move to have a significant reduction in unemployment will lead to an increase in inflation.

#### 1.1.1 Flow in an Economy

The flow of goods, services, resources and money payments in a simple economy are shown in Fig. 1.1. Households and businesses are the two major entities in a simple economy. Business organizations use various economic resources like land, labour and capital which are provided by households to produce consumer goods and services which will be used by them. Business organizations make payment of money to the households for receiving various resources. The households in turn make payment of money to business organizations for receiving consumer goods and services. This cycle shows the interdependence between the two major entities in a simple economy.



**Fig. 1.1** Flow of goods, services, resources and money payments in a simple economy.

### 1.1.2 Law of Supply and Demand

An interesting aspect of the economy is that the demand and supply of a product are interdependent and they are sensitive with respect to the price of that product. The interrelationships between them are shown in Fig. 1.2.

From Fig. 1.2 it is clear that when there is a decrease in the price of a product, the demand for the product increases and its supply decreases. Also, the product is more in demand and hence the demand of the product increases. At the same time, lowering of the price of the product makes the producers restrain from releasing more quantities of the product in the market. Hence, the supply of the product is decreased. The point of intersection of the supply curve and the demand curve is known as the *equilibrium point*. At the price corresponding to this point, the quantity of supply is equal to the quantity of demand. Hence, this point is called the *equilibrium point*.

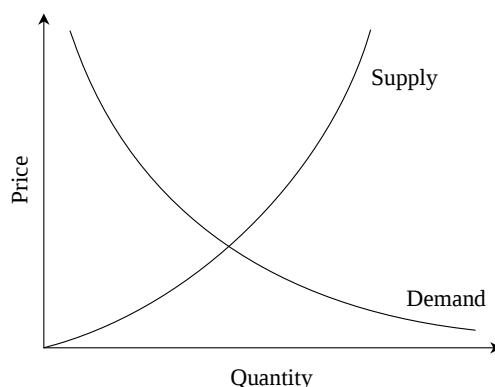
#### Factors influencing demand

The shape of the demand curve is influenced by the following factors:

- Income of the people
- Prices of related goods
- Tastes of consumers

If the income level of the people increases significantly, then their purchasing power will naturally improve. This would definitely shift the demand curve to the north-east direction of Fig. 1.2. A converse situation will shift the demand curve to the south-west direction.

If, for instance, the price of television sets is lowered drastically its demand would naturally go up. As a result, the demand for its associated product,



**Fig. 1.2** Demand and supply curve.

namely VCDs would also increase. Hence, the prices of related goods influences the demand of a product.

Over a period of time, the preference of the people for a particular product may increase, which in turn, will affect its demand. For instance, diabetic people prefer to have sugar-free products. If the incidence of diabetes rises naturally there will be increased demand for sugar-free products.

### **Factors influencing supply**

The shape of the supply curve is affected by the following factors:

- Cost of the inputs
- Technology
- Weather
- Prices of related goods

If the cost of inputs increases, then naturally, the cost of the product will go up. In such a situation, at the prevailing price of the product the profit margin per unit will be less. The producers will then reduce the production quantity, which in turn will affect the supply of the product. For instance, if the prices of fertilizers and cost of labour are increased significantly, in agriculture, the profit margin per bag of paddy will be reduced. So, the farmers will reduce the area of cultivation, and hence the quantity of supply of paddy will be reduced at the prevailing prices of the paddy.

If there is an advancement in technology used in the manufacture of the product in the long run, there will be a reduction in the production cost per unit. This will enable the manufacturer to have a greater profit margin per unit at the prevailing price of the product. Hence, the producer will be tempted to supply more quantity to the market.

Weather also has a direct bearing on the supply of products. For example, demand for woollen products will increase during winter. This means the prices of woollen goods will be increased in winter. So, naturally, manufacturers will supply more volume of woollen goods during winter.

Again, take the case of television sets. If the price of TV sets is lowered significantly, then its demand would naturally go up. As a result, the demand for associated products like VCDs would also go up. Over a period of time, this will lead to an increase in the price of VCDs, which would result in more supply of VCDs.

## 1.2 CONCEPT OF ENGINEERING ECONOMICS

Science is a field of study where the basic principles of different physical systems are formulated and tested. Engineering is the application of science. It establishes varied application systems based on different scientific principles.

From the discussions in the previous section, it is clear that price has a major role in deciding the demand and supply of a product. Hence, from the organization's point of view, efficient and effective functioning of the organization would certainly help it to provide goods/services at a lower cost which in turn will enable it to fix a lower price for its goods or services.

The following section discusses the different types of efficiency and their impact on the operation of businesses and the definition and scope of engineering economics.

### 1.2.1 Types of Efficiency

Efficiency of a system is generally defined as the ratio of its output to input. The efficiency can be classified into *technical efficiency* and *economic efficiency*.

#### Technical efficiency

It is the ratio of the output to input of a physical system. The physical system may be a diesel engine, a machine working in a shop floor, a furnace, etc.

$$\text{Technical efficiency (\%)} = \frac{\text{Output}}{\text{Input}} \times 100$$

The technical efficiency of a diesel engine is as follows:

$$\text{Technical efficiency (\%)} = \frac{\text{Heat equivalent of mechanical energy produced}}{\text{Heat equivalent of fuel used}} \times 100$$

In practice, technical efficiency can never be more than 100%. This is mainly due to frictional loss and incomplete combustion of fuel, which are considered to be unavoidable phenomena in the working of a diesel engine.

#### Economic efficiency

Economic efficiency is the ratio of output to input of a business system.

$$\text{Economic efficiency (\%)} = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{\text{Worth}}{\text{Cost}} \times 100$$

‘Worth’ is the annual revenue generated by way of operating the business and ‘cost’ is the total annual expenses incurred in carrying out the business. For the survival and growth of any business, the economic efficiency should be more than 100%.

Economic efficiency is also called ‘productivity’. There are several ways of improving productivity.

- Increased output for the same input
- Decreased input for the same output
- By a proportionate increase in the output which is more than the proportionate increase in the input
- By a proportionate decrease in the input which is more than the proportionate decrease in the output
- Through simultaneous increase in the output with decrease in the input.

*Increased output for the same input.* In this strategy, the output is increased while keeping the input constant. Let us assume that in a steel plant, the layout of the existing facilities is not proper. By slightly altering the location of the billet-making section, and bringing it closer to the furnace which produces hot metal, the scale formation at the top of ladles will be considerably reduced. The molten metal is usually carried in ladles to the billet-making section. In the long run, this would give more yield in terms of tonnes of billet produced. In this exercise, there is no extra cost involved. The only task is the relocation of the billet-making facility which involves an insignificant cost.

*Decreased input for the same output.* In this strategy, the input is decreased to produce the same output. Let us assume that there exists a substitute raw material to manufacture a product and it is available at a lower price. If we can identify such a material and use it for manufacturing the product, then certainly it will reduce the input. In this exercise, the job of the purchase department is to identify an alternate substitute material. The process of identification does not involve any extra cost. So, the productivity ratio will increase because of the decreased input by way of using cheaper raw materials to produce the same output.

*Less proportionate increase in output is more than that of the input.* Consider the example of introducing a new product into the existing product mix of an organization. Let us assume that the existing facilities are not fully utilized and the R&D wing of the company has identified a new product which has a very good market and which can be manufactured with the surplus facilities of the organization. If the new product is taken up for production, it will lead to—

- an increase in the revenue of the organization by way of selling the new product in addition to the existing product mix and

- an increase in the material cost and operation and maintenance cost of machineries because of producing the new product.

If we examine these two increases closely, the proportionate increase in the revenue will be more than the proportionate increase in the input cost. Hence, there will be a net increase in the productivity ratio.

*When proportionate decrease in input is more than that of the output.* Let us consider the converse of the previous example, i.e. dropping an uneconomical product from the existing product mix. This will result in the following:

- A decrease in the revenue of the organization
- A decrease in the material cost, and operation and maintenance cost of machinery

If we closely examine these two decreases, we will see that the proportionate decrease in the input cost will be more than the proportionate decrease in the revenue. Hence, there will be a net increase in the productivity ratio.

*Simultaneous increase in output and decrease in input.* Let us assume that there are advanced automated technologies like robots and automated guided vehicle system (AGVS), available in the market which can be employed in the organization we are interested in. If we employ these modern tools, then:

- There will be a drastic reduction in the operation cost. Initially, the cost on equipment would be very high. But, in the long run, the reduction in the operation cost would break-even the high initial investment and offer more savings on the input.
- These advanced facilities would help in producing more products because they do not experience fatigue. The increased production will yield more revenue.

In this example, in the long run, there is an increase in the revenue and a decrease in the input. Hence, the productivity ratio will increase at a faster rate.

### 1.2.2 Definition and Scope of Engineering Economics

As stated earlier, efficient functioning of any business organization would enable it to provide goods/services at a lower price. In the process of managing organizations, the managers at different levels should take appropriate economic decisions which will help in minimizing investment, operating and maintenance expenditures besides increasing the revenue, savings and other related gains of the organization.

#### Definition

Engineering economics deals with the methods that enable one to take economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

## Scope

The issues that are covered in this book are elementary economic analysis, interest formulae, bases for comparing alternatives, present worth method, future worth method, annual equivalent method, rate of return method, replacement analysis, depreciation, evaluation of public alternatives, inflation adjusted investment decisions, make or buy decisions, inventory control, project management, value engineering, and linear programming.

## 1.3 ELEMENTS OF COSTS

Cost can be broadly classified into *variable cost* and *overhead cost*. Variable cost varies with the volume of production while overhead cost is fixed, irrespective of the production volume.

Variable cost can be further classified into direct material cost, direct labour cost, and direct expenses. The overhead cost can be classified into factory overhead, administration overhead, selling overhead, and distribution overhead.

Direct material costs are those costs of materials that are used to produce the product. Direct labour cost is the amount of wages paid to the direct labour involved in the production activities. Direct expenses are those expenses that vary in relation to the production volume, other than the direct material costs and direct labour costs.

Overhead cost is the aggregate of indirect material costs, indirect labour costs and indirect expenses. Administration overhead includes all the costs that are incurred in administering the business. Selling overhead is the total expense that is incurred in the promotional activities and the expenses relating to sales force. Distribution overhead is the total cost of shipping the items from the factory site to the customer sites.

The selling price of a product is derived as shown below:

- (a)  $\text{Direct material costs} + \text{Direct labour costs} + \text{Direct expenses} = \text{Prime cost}$
- (b)  $\text{Prime cost} + \text{Factory overhead} = \text{Factory cost}$
- (c)  $\text{Factory cost} + \text{Office and administrative overhead} = \text{Costs of production}$
- (d)  $\text{Cost of production} + \text{Opening finished stock} - \text{Closing finished stock} = \text{Cost of goods sold}$
- (e)  $\text{Cost of goods sold} + \text{Selling and distribution overhead} = \text{Cost of sales}$
- (f)  $\text{Cost of sales} + \text{Profit} = \text{Sales}$
- (g)  $\text{Sales/Quantity sold} = \text{Selling price per unit}$

In the above calculations, if the opening finished stock is equal to the closing finished stock, then the cost of production is equal to the cost of goods sold.

## 1.4 OTHER COSTS/REVENUES

The following are the costs/revenues other than the costs which are presented in the previous section:

- Marginal cost
- Marginal revenue
- Sunk cost
- Opportunity cost

### 1.4.1 Marginal Cost

Marginal cost of a product is the cost of producing an additional unit of that product. Let the cost of producing 20 units of a product be Rs. 10,000, and the cost of producing 21 units of the same product be Rs. 10,045. Then the marginal cost of producing the 21st unit is Rs. 45.

### 1.4.2 Marginal Revenue

Marginal revenue of a product is the incremental revenue of selling an additional unit of that product. Let, the revenue of selling 20 units of a product be Rs. 15,000 and the revenue of selling 21 units of the same product be Rs. 15,085. Then, the marginal revenue of selling the 21st unit is Rs. 85.

### 1.4.3 Sunk Cost

This is known as the past cost of an equipment/asset. Let us assume that an equipment has been purchased for Rs. 1,00,000 about three years back. If it is considered for replacement, then its present value is not Rs. 1,00,000. Instead, its present market value should be taken as the present value of the equipment for further analysis. So, the purchase value of the equipment in the past is known as its sunk cost. The sunk cost should not be considered for any analysis done from now onwards.

### 1.4.4 Opportunity Cost

In practice, if an alternative ( $X$ ) is selected from a set of competing alternatives ( $X, Y$ ), then the corresponding investment in the selected alternative is not available for any other purpose. If the same money is invested in some other alternative ( $Y$ ), it may fetch some return. Since the money is invested in the selected alternative ( $X$ ), one has to forego the return from the other alternative ( $Y$ ). The amount that is foregone by not investing in the other alternative ( $Y$ ) is known as the opportunity cost of the selected alternative ( $X$ ). So the opportunity cost of an alternative is the return that will be foregone by not investing the same money in another alternative.

Consider that a person has invested a sum of Rs. 50,000 in shares. Let the expected annual return by this alternative be Rs. 7,500. If the same amount is

invested in a fixed deposit, a bank will pay a return of 18%. Then, the corresponding total return per year for the investment in the bank is Rs. 9,000. This return is greater than the return from shares. The foregone excess return of Rs. 1,500 by way of not investing in the bank is the opportunity cost of investing in shares.

## 1.5 BREAK-EVEN ANALYSIS

The main objective of break-even analysis is to find the cut-off production volume from where a firm will make profit. Let

$s$  = selling price per unit

$v$  = variable cost per unit

$FC$  = fixed cost per period

$Q$  = volume of production

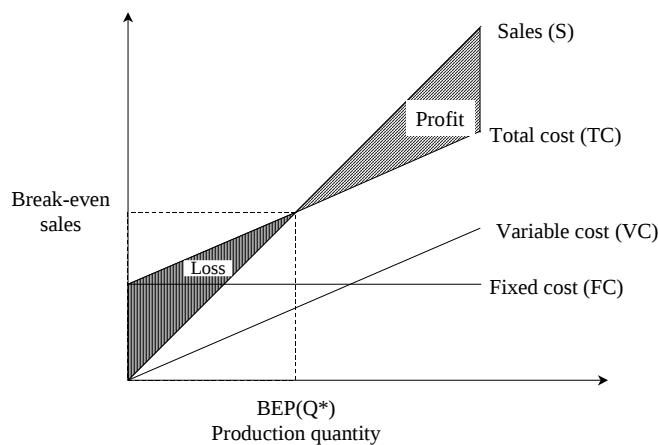
The total sales revenue ( $S$ ) of the firm is given by the following formula:

$$S = s \times Q$$

The total cost of the firm for a given production volume is given as

$$\begin{aligned} TC &= \text{Total variable cost} + \text{Fixed cost} \\ &= v \times Q + FC \end{aligned}$$

The linear plots of the above two equations are shown in Fig. 1.3. The intersection point of the total sales revenue line and the total cost line is called



**Fig. 1.3** Break-even chart.

the break-even point. The corresponding volume of production on the  $X$ -axis is known as the break-even sales quantity. At the intersection point, the total cost is equal to the total revenue. This point is also called the no-loss or no-gain situation. For any production quantity which is less than the break-even quantity, the total cost is more than the total revenue. Hence, the firm will be making loss.

For any production quantity which is more than the break-even quantity, the total revenue will be more than the total cost. Hence, the firm will be making profit.

$$\begin{aligned}\text{Profit} &= \text{Sales} - (\text{Fixed cost} + \text{Variable costs}) \\ &= s \times Q - (FC + v \times Q)\end{aligned}$$

The formulae to find the break-even quantity and break-even sales quantity

$$\begin{aligned}\text{Break-even quantity} &= \frac{\text{Fixed cost}}{\text{Selling price/unit} - \text{Variable cost/unit}} \\ &= \frac{FC}{s - v} \text{ (in units)}\end{aligned}$$

$$\begin{aligned}\text{Break-even sales} &= \frac{\text{Fixed cost}}{\text{Selling price/unit} - \text{Variable cost/unit}} \times \text{Selling price/unit} \\ &= \frac{FC}{s - v} \times s \text{ (Rs.)}\end{aligned}$$

The contribution is the difference between the sales and the variable costs. The margin of safety (M.S.) is the sales over and above the break-even sales. The formulae to compute these values are

$$\begin{aligned}\text{Contribution} &= \text{Sales} - \text{Variable costs} \\ \text{Contribution/unit} &= \text{Selling price/unit} - \text{Variable cost/unit} \\ \text{M.S.} &= \text{Actual sales} - \text{Break-even sales} \\ &= \frac{\text{Profit}}{\text{Contribution}} \times \text{sales}\end{aligned}$$

$$\text{M.S. as a per cent of sales} = (\text{M.S./Sales}) \times 100$$

**EXAMPLE 1.1** Alpha Associates has the following details:

Fixed cost = Rs. 20,00,000  
Variable cost per unit = Rs. 100  
Selling price per unit = Rs. 200

Find

- (a) The break-even sales quantity,
- (b) The break-even sales
- (c) If the actual production quantity is 60,000, find (i) contribution; and (ii) margin of safety by all methods.

**Solution**

Fixed cost ( $FC$ ) = Rs. 20,00,000  
Variable cost per unit ( $v$ ) = Rs. 100  
Selling price per unit ( $s$ ) = Rs. 200

$$\begin{aligned} \text{(a) Break-even quantity} &= \frac{FC}{s - v} = \frac{20,00,000}{200 - 100} \\ &= 20,00,000/100 = 20,000 \text{ units} \end{aligned}$$

$$\begin{aligned} \text{(b) Break-even sales} &= \frac{FC}{s - v} \times s \text{ (Rs.)} \\ &= \frac{20,00,000}{200 - 100} \times 200 \\ &= \frac{20,00,000}{100} \times 200 = \text{Rs. } 40,00,000 \end{aligned}$$

$$\begin{aligned} \text{(c) (i) Contribution} &= \text{Sales} - \text{Variable cost} \\ &= s \times Q - v \times Q \\ &= 200 \times 60,000 - 100 \times 60,000 \\ &= 1,20,00,000 - 60,00,000 \\ &= \text{Rs. } 60,00,000 \end{aligned}$$

(ii) Margin of safety

#### METHOD I

$$\begin{aligned} \text{M.S.} &= \text{Sales} - \text{Break-even sales} \\ &= 60,000 \times 200 - 40,00,000 \\ &= 1,20,00,000 - 40,00,000 = \text{Rs. } 80,00,000 \end{aligned}$$

#### METHOD II

$$\text{M.S.} = \frac{\text{Profit}}{\text{Contribution}} \times \text{Sales}$$

$$\begin{aligned} \text{Profit} &= \text{Sales} - (FC + v \times Q) \\ &= 60,000 \times 200 - (20,00,000 + 100 \times 60,000) \\ &= 1,20,00,000 - 80,00,000 \\ &= \text{Rs. } 40,00,000 \end{aligned}$$

$$\text{M.S.} = \frac{40,00,000}{60,00,000} \times 1,20,00,000 = \text{Rs. } 80,00,000$$

$$\text{M.S. as a per cent of sales} = \frac{80,00,000}{1,20,00,000} \times 100 = 67\%$$

### 1.6 PROFIT/VOLUME RATIO (P/V RATIO)

*P/V* ratio is a valid ratio which is useful for further analysis. The different formulae for the *P/V* ratio are as follows:

$$P/V \text{ ratio} = \frac{\text{Contribution}}{\text{Sales}} = \frac{\text{Sales} - \text{Variable costs}}{\text{Sales}}$$

The relationship between BEP and *P/V* ratio is as follows:

$$\text{BEP} = \frac{\text{Fixed cost}}{P/V \text{ ratio}}$$

The following formula helps us find the M.S. using the *P/V* ratio:

$$\text{M.S.} = \frac{\text{Profit}}{P/V \text{ ratio}}$$

**EXAMPLE 1.2** Consider the following data of a company for the year 1997:

Sales = Rs. 1,20,000

Fixed cost = Rs. 25,000

Variable cost = Rs. 45,000

Find the following:

- (a) Contribution
- (b) Profit
- (c) BEP
- (d) M.S.

**Solution**

$$\begin{aligned} \text{(a) Contribution} &= \text{Sales} - \text{Variable costs} \\ &= \text{Rs. } 1,20,000 - \text{Rs. } 45,000 \\ &= \text{Rs. } 75,000 \end{aligned}$$

$$\begin{aligned} \text{(b) Profit} &= \text{Contribution} - \text{Fixed cost} \\ &= \text{Rs. } 75,000 - \text{Rs. } 25,000 \\ &= \text{Rs. } 50,000 \end{aligned}$$

(c) BEP

$$\begin{aligned} P/V \text{ ratio} &= \frac{\text{Contribution}}{\text{Sales}} \\ &= \frac{75,000}{1,20,000} \times 100 = 62.50\% \end{aligned}$$

$$\text{BEP} = \frac{\text{Fixed cost}}{P/V \text{ ratio}} = \frac{25,000}{62.50} \times 100 = \text{Rs. } 40,000$$

$$\text{M.S.} = \frac{\text{Profit}}{P/V \text{ ratio}} = \frac{50,000}{62.50} \times 100 = \text{Rs. } 80,000$$

**EXAMPLE 1.3** Consider the following data of a company for the year 1998:

Sales = Rs. 80,000

Fixed cost = Rs. 15,000

Variable cost = 35,000

Find the following:

- (a) Contribution
- (b) Profit
- (c) BEP
- (d) M.S.

**Solution**

$$\begin{aligned} \text{(a) Contribution} &= \text{Sales} - \text{Variable costs} \\ &= \text{Rs. } 80,000 - \text{Rs. } 35,000 \\ &= \text{Rs. } 45,000 \end{aligned}$$

$$\begin{aligned} \text{(b) Profit} &= \text{Contribution} - \text{Fixed cost} \\ &= \text{Rs. } 45,000 - \text{Rs. } 15,000 \\ &= \text{Rs. } 30,000 \end{aligned}$$

(c) BEP

$$P/V \text{ ratio} = \frac{\text{Contribution}}{\text{Sales}} = \frac{45,000}{80,000} \times 100 = 56.25\%$$

$$\text{BEP} = \frac{\text{Fixed cost}}{P/V \text{ ratio}} = \frac{15,000}{56.25} \times 100 = \text{Rs. } 26,667$$

$$\text{(d) M.S.} = \frac{\text{Profit}}{P/V \text{ ratio}} = \frac{30,000}{56.25} \times 100 = \text{Rs. } 53,333.33$$

## QUESTIONS

1. Define economics. Also discuss the flow of goods, services, resources and money payments in a simple economy with the help of a suitable diagram.
2. Illustrate the effect of price on demand and supply; illustrate with the help of a diagram.
3. Discuss the factors which influence demand and supply.
4. Distinguish between technical efficiency and economic efficiency by giving examples.

5. What are the ways by which the economic efficiency can be improved?
6. Give the definition and scope of engineering economics.
7. Clearly explain the method of deriving the selling price of a product.
8. Define the following costs with examples:
  - (a) Sunk cost
  - (b) Opportunity cost
  - (c) Marginal cost
  - (d) Marginal revenue.
9. Define break-even point. Draw a break-even chart and explain its components.

10. Krishna Company Ltd. has the following details:

Fixed cost = Rs. 40,00,000  
Variable cost per unit = Rs. 300  
Selling price per unit = Rs. 500

Find

- (a) The break-even sales quantity
  - (b) The break-even sales
  - (c) If the actual production quantity is 1,20,000, find the following:
    - (i) Contribution
    - (ii) Margin of safety by all methods
11. Consider the following data of a company for the year 1998.

Sales = Rs. 2,40,000  
Fixed cost = Rs. 50,000  
Variable cost = Rs. 75,000

Find the following:

- (a) Contribution
- (b) Profit
- (c) BEP
- (d) Margin of safety

# 2

## ELEMENTARY ECONOMIC ANALYSIS

### 2.1 INTRODUCTION

Whether it is a business situation or a day-to-day event in somebody's personal life, there are a large number of economic decision making involved. One can manage many of these decision problems by using simple economic analysis. For example, an industry can source its raw materials from a nearby place or from a far-off place. In this problem, the following factors will affect the decision:

- Price of the raw material
- Transportation cost of the raw material
- Availability of the raw material
- Quality of the raw material

Consider the alternative of sourcing raw materials from a nearby place with the following characteristics:

- The raw material is more costly in the nearby area.
- The availability of the raw material is not sufficient enough to support the operation of the industry throughout the year.
- The raw material requires pre-processing before it is used in the production process. This would certainly add cost to the product.
- The cost of transportation is minimal under this alternative.

On the other hand, consider another alternative of sourcing the raw materials from a far-off place with the following characteristics:

- The raw material is less costly at the far off place.
- The cost of transportation is very high.
- The availability of the raw material at this site is abundant and it can support the plant throughout the year.
- The raw material from this site does not require any pre-processing before using it for production.

Under such a situation, the procurement of the raw material should be decided in such a way that the overall cost is minimized.

The above example clearly highlights the various components of cost that are involved in each of the alternatives of the decision-making process as well as a method of taking a suitable decision.

## 2.2 EXAMPLES FOR SIMPLE ECONOMIC ANALYSIS

In this section, the concept of simple economic analysis is illustrated using suitable examples in the following areas:

- Material selection for a product
- Design selection for a product
- Design selection for a process industry
- Building material selection for construction activities
- Process planning/Process modification

### 2.2.1 Material Selection for a Product/Substitution of Raw Material

The cost of a product can be reduced greatly by substitution of the raw materials. Among various elements of cost, raw material cost is most significant and it forms a major portion of the total cost of any product. So, any attempt to find a suitable raw material will bring a reduction in the total cost in any one or combinations of the following ways:

- Cheaper raw material price
- Reduced machining/process time
- Enhanced durability of the product

Therefore, the process of raw material selection/substitution will result in finding an alternate raw material which will provide the necessary functions that are provided by the raw material that is presently used. In this process, if the new raw material provides any additional benefit, then it should be treated as its welcoming feature. This concept is demonstrated with two numerical problems.

**EXAMPLE 2.1** In the design of a jet engine part, the designer has a choice of specifying either an aluminium alloy casting or a steel casting. Either material will provide equal service, but the aluminium casting will weigh 1.2 kg as compared with 1.35 kg for the steel casting.

The aluminium can be cast for Rs. 80.00 per kg. and the steel one for Rs. 35.00 per kg. The cost of machining per unit is Rs. 150.00 for aluminium and Rs. 170.00 for steel. Every kilogram of excess weight is associated with a penalty of Rs. 1,300 due to increased fuel consumption. Which material should be specified and what is the economic advantage of the selection per unit?

**Solution** (a) *Cost of using aluminium metal for the jet engine part:*

Weight of aluminium casting/unit = 1.2 kg  
 Cost of making aluminium casting = Rs. 80.00 per kg  
 Cost of machining aluminium casting per unit = Rs. 150.00  
 Total cost of jet engine part made of aluminium/unit  
 = Cost of making aluminium casting/unit  
 + Cost of machining aluminium casting/unit  
 =  $80 \times 1.2 + 150 = 96 + 150$   
 = Rs. 246

(b) *Cost of jet engine part made of steel/unit:*

Weight of steel casting/unit = 1.35 kg  
 Cost of making steel casting = Rs. 35.00 per kg  
 Cost of machining steel casting per unit = Rs. 170.00  
 Penalty of excess weight of steel casting = Rs. 1,300 per kg  
 Total cost of jet engine part made of steel/unit  
 = Cost of making steel casting/unit  
 + Cost of machining steel casting/unit  
 + Penalty for excess weight of steel casting  
 =  $35 \times 1.35 + 170 + 1,300(1.35 - 1.2)$   
 = Rs. 412.25

**DECISION** The total cost/unit of a jet engine part made of aluminium is less than that for an engine made of steel. Hence, aluminium is suggested for making the jet engine part. The economic advantage of using aluminium over steel/unit is Rs. 412.25 – Rs. 246 = Rs. 166.25

**EXAMPLE 2.2** A company manufactures dining tables which mainly consist of a wooden frame and a table top. The different materials used to manufacture the tables and their costs are given in Table 2.1.

**Table 2.1** Data for Example 2.2

| <i>Description of item</i>    | <i>Quantity</i>    | <i>Cost</i>               |
|-------------------------------|--------------------|---------------------------|
| Wood for frame and legs       | 0.1 m <sup>3</sup> | Rs. 12,000/m <sup>3</sup> |
| Table top with sunmica finish | 1                  | Rs. 3,000                 |
| Leg bushes                    | 4                  | Rs. 10/bush               |
| Nails                         | 100 g              | Rs. 300/kg                |
| Total labour                  | 15 hr              | Rs. 50/hr                 |

In view of the growing awareness towards deforestation and environmental conservation, the company feels that the use of wood should be minimal. The wooden top therefore could be replaced with a granite top. This would require additional wood for the frame and legs to take the extra weight of the granite top. The materials and labour requirements along with cost details to manufacture a table with granite top are given in Table 2.2.

**Table 2.2** Data for Example 2.2

| <i>Description of item</i> | <i>Quantity</i>     | <i>Cost</i>               |
|----------------------------|---------------------|---------------------------|
| Wood for frame and legs    | 0.15 m <sup>3</sup> | Rs. 12,000/m <sup>3</sup> |
| Granite table top          | 1.62 m <sup>2</sup> | Rs. 800/m <sup>2</sup>    |
| Leg bushes                 | 4                   | Rs. 25/bush               |
| Nails                      | 50 g                | Rs. 300/kg                |
| Total labour               | 8 hr                | Rs. 50/hr                 |

If the cost of the dining table with a granite top works out to be lesser than that of the table with wooden top, the company is willing to manufacture dining tables with granite tops. Compute the cost of manufacture of the table under each of the alternatives described above and suggest the best alternative. Also, find the economic advantage of the best alternative.

**Solution** (a) *Cost of table with wooden top*

|                                 |                     |             |
|---------------------------------|---------------------|-------------|
| Cost of wood for frame and legs | = 12,000 × 0.1      | = Rs. 1,200 |
| Cost of wooden top              |                     | = Rs. 3,000 |
| Cost of bushes                  | = 10 × 4            | = Rs. 40    |
| Cost of nails                   | = 300 × (100/1,000) | = Rs. 30    |
| Cost of labour                  | = 50 × 15           | = Rs. 750   |
| Total                           |                     | = Rs. 5,020 |

(b) *Cost of table with granite top*

|                                 |                    |             |
|---------------------------------|--------------------|-------------|
| Cost of wood for frame and legs | = 12,000 × 0.15    | = Rs. 1,800 |
| Cost of granite top             | = 800 × 1.62       | = Rs. 1,296 |
| Cost of bushes                  | = 25 × 4           | = Rs. 100   |
| Cost of nails                   | = 300 × (50/1,000) | = Rs. 15    |
| Cost of labour                  | = 50 × 8           | = Rs. 400   |
| Total                           |                    | = Rs. 3,611 |

The cost of a table with granite top works out to be less than that of a table with a wooden top. Hence, the table with granite top should be selected by the manufacturer.

(c) *Economic advantage*

|                                              |                    |
|----------------------------------------------|--------------------|
| Cost of a table with wooden top              | = Rs. 5,020        |
| Cost of a table with granite top             | = Rs. 3,611        |
| Economic advantage of table with granite top | <u>= Rs. 1,409</u> |

## 2.2.2 Design Selection for a Product

The design modification of a product may result in reduced raw material

requirements, increased machinability of the materials and reduced labour. Design is an important factor which decides the cost of the product for a specified level of performance of that product.

The elementary economic analysis applied to the selection of design for a product is illustrated with two example problems.

**EXAMPLE 2.3** Two alternatives are under consideration for a tapered fastening pin. Either design will serve the purpose and will involve the same material and manufacturing cost except for the lathe and grinder operations.

Design A will require 16 hours of lathe time and 4.5 hours of grinder time per 1,000 units. Design B will require 7 hours of lathe time and 12 hours of grinder time per 1,000 units. The operating cost of the lathe including labour is Rs. 200 per hour. The operating cost of the grinder including labour is Rs. 150 per hour. Which design should be adopted if 1,00,000 units are required per year and what is the economic advantage of the best alternative?

**Solution**

Operating cost of lathe including labour = Rs. 200 per hr  
Operating cost of grinder including labour = Rs. 150 per hr

*(a) Cost of design A*

No. of hours of lathe time per 1,000 units = 16 hr  
No. of hours of grinder time per 1,000 units = 4.5 hr

Total cost of design A/1,000 units

$$\begin{aligned} &= \text{Cost of lathe operation per 1,000 units} \\ &\quad + \text{Cost of grinder operation per 1,000 units} \\ &= 16 \times 200 + 4.5 \times 150 \\ &= \text{Rs. 3,875} \end{aligned}$$

$$\begin{aligned} \text{Total cost of design A/1,00,000 units} &= 3,875 \times 1,00,000/1,000 \\ &= \text{Rs. 3,87,500} \end{aligned}$$

*(b) Cost of design B*

No. of hours of lathe time per 1,000 units = 7 hr  
No. of hours of grinder time per 1,000 units = 12 hr

Total cost of design B/1,000 units

$$\begin{aligned} &= \text{Cost of lathe operation/1,000 units} \\ &\quad + \text{Cost of grinder operation/1,000 units} \\ &= 7 \times 200 + 12 \times 150 \\ &= \text{Rs. 3,200} \end{aligned}$$

$$\begin{aligned} \text{Total cost of design B/1,00,000 units} &= 3,200 \times 1,00,000/1,000 \\ &= \text{Rs. 3,20,000} \end{aligned}$$

DECISION The total cost/1,00,000 units of design B is less than that of design A. Hence, design B is recommended for making the tapered fastening pin.

$$\begin{aligned}\text{Economic advantage of the design B over design A per 1,00,000 units} \\ &= \text{Rs. } 3,87,500 - \text{Rs. } 3,20,000 \\ &= \text{Rs. } 67,500.\end{aligned}$$

**EXAMPLE 2.4 (Design selection for a process industry).** The chief engineer of refinery operations is not satisfied with the preliminary design for storage tanks to be used as part of a plant expansion programme. The engineer who submitted the design was called in and asked to reconsider the overall dimensions in the light of an article in the *Chemical Engineer*, entitled “How to size future process vessels?”

The original design submitted called for 4 tanks 5.2 m in diameter and 7 m in height. From a graph of the article, the engineer found that the present ratio of height to diameter of 1.35 is 111% of the minimum cost and that the minimum cost for a tank was when the ratio of height to diameter was 4 : 1. The cost for the tank design as originally submitted was estimated to be Rs. 9,00,000. What are the optimum tank dimensions if the volume remains the same as for the original design? What total savings may be expected through the redesign?

**Solution** (a) *Original design*

$$\begin{aligned}\text{Number of tanks} &= 4 \\ \text{Diameter of the tank} &= 5.2 \text{ m} \\ \text{Radius of the tank} &= 2.6 \text{ m} \\ \text{Height of the tank} &= 7 \text{ m} \\ \text{Ratio of height to diameter} &= 7/5.2 = 1.35 \\ \text{Volume/tank} &= (22/7)r^2h = (22/7)(2.6)^2 \times 7 \\ &= 148.72 \text{ m}^3\end{aligned}$$

(b) *New design*

Cost of the old design = 111% of the cost of the new design (optimal design)

Optimal ratio of the height to diameter = 4:1

$$h : d = 4 : 1$$

$$4d = h$$

$$d = h/4$$

$$r = h/8$$

$$\begin{aligned}\text{Volume} &= (22/7)r^2h = 148.72 \text{ (since, the volume remains the same)} \\ (22/7)(h/8)^2h &= 148.72\end{aligned}$$

$$h^3 = \frac{148.72}{(22/7)} \times 64 = 3,028.48$$

$$h = 14.47 \text{ m}$$

$$r = h/8 = 14.47/8 = 1.81 \text{ m}$$

Therefore,

$$\begin{aligned}\text{Diameter of the new design} &= 1.81 \times 2 \\ &= 3.62 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Cost of the new design} &= 9,00,000 \times (100/111) \\ &= \text{Rs. } 8,10,810.81\end{aligned}$$

$$\begin{aligned}\text{Expected savings by the redesign} &= \text{Rs. } 9,00,000 - \text{Rs. } 8,10,810.81 \\ &= \text{Rs. } 89,189.19\end{aligned}$$

### 2.2.3 Building Material Selection

As discussed in the introduction to this chapter, the sourcing of raw materials will have a significant effect on the cost of any product. Hence, it is assumed that the price of raw material is location dependent. While sourcing a raw material, the cost of transportation is to be considered in conjunction with the price of the raw material. This concept is demonstrated with a numerical example.

**EXAMPLE 2.5** In the design of buildings to be constructed in Alpha State, the designer is considering the type of window frame to specify. Either steel or aluminium window frames will satisfy the design criteria. Because of the remote location of the building site and lack of building materials in Alpha State, the window frames will be purchased in Beta State and transported for a distance of 2,500 km to the site. The price of window frames of the type required is Rs. 1,000 each for steel frames and Rs. 1,500 each for aluminium frames. The weight of steel window frames is 75 kg each and that of aluminium window frame is 28 kg each. The shipping rate is Re 1 per kg per 100 km. Which design should be specified and what is the economic advantage of the selection?

#### **Solution**

Distance between Alpha State and Beta State = 2,500 km  
Transportation cost = Re 1/kg/100 km

##### *(a) Steel window frame*

Price of steel window frame/unit = Rs 1,000  
Weight of steel window frame/unit = 75 kg

Total cost of steel window frame/unit

$$\begin{aligned}&= \text{Price of steel window frame/unit} \\ &\quad + \text{Transportation cost of steel window frame/unit} \\ &= 1,000 + (75 \times 2,500 \times 1)/100 \\ &= \text{Rs. } 2,875\end{aligned}$$

##### *(b) Aluminium window frame*

Price of aluminium window frame/unit = Rs. 1,500  
Weight of aluminium window frame/unit = 28 kg

Total cost of aluminium window frame/unit

$$\begin{aligned}
 &= \text{Price of aluminium window frame/unit} \\
 &\quad + \text{Transportation cost of aluminium window frame/unit} \\
 &= 1,500 + (28 \times 2,500 \times 1)/100 \\
 &= \text{Rs. } 2,200
 \end{aligned}$$

**DECISION** The total cost/unit of the aluminium window frame is less than that of steel window frame. Hence, aluminium window frame is recommended.

The economic advantage/unit of the aluminium window frame over the steel window frame = Rs. 2,875 – 2,200  
= Rs. 675

## 2.2.4 Process Planning/Process Modification

While planning for a new component, a feasible sequence of operations with the least cost of processing is to be considered. The process sequence of a component which has been planned in the past is not static. It is always subject to modification with a view to minimize the cost of manufacturing the component. So, the objective of process planning/process modification is to identify the most economical sequence of operations to produce a component.

The steps in process planning are as follows:

1. Analyze the part drawing to get an overall picture of what is required.
2. Make recommendations to or consult with product engineers on product design changes.
3. List the basic operations required to produce the part to the drawing or specifications.
4. Determine the most practical and economical manufacturing method and the form or tooling required for each operation.
5. Devise the best way to combine the operations and put them in sequence.
6. Specify the gauging required for the process.

Steps 3–5 aim to determine the most practical and economical sequence of operations to produce a component. This concept is demonstrated with a numerical problem.

**EXAMPLE 2.6** The process planning engineer of a firm listed the sequences of operations as shown in Table 2.3 to produce a component.

**Table 2.3** Data for Example 2.6

| <i>Sequence</i> | <i>Process sequence</i>                       |
|-----------------|-----------------------------------------------|
| 1               | Turning – Milling – Shaping – Drilling        |
| 2               | Turning – Milling – Drilling                  |
| 3               | All operations are performed with CNC machine |

The details of processing times of the component for various operations and their machine hour rates are summarized in Table 2.4.

**Table 2.4** Machine Hour Rates and Processing Times (minutes) for Example 2.6

| Operation      | Machine hour rate (Rs.) | Process sequence |    |   |
|----------------|-------------------------|------------------|----|---|
|                |                         | 1                | 2  | 3 |
| Turning        | 200                     | 5                | 5  | – |
| Milling        | 400                     | 8                | 14 | – |
| Shaping        | 350                     | 10               | –  | – |
| Drilling       | 300                     | 3                | 3  | – |
| CNC operations | 1,000                   | –                | –  | 8 |

Find the most economical sequence of operations to manufacture the component.

**Solution** (a) *Cost of component using process sequence 1.* The process sequence 1 of the component is as follows:

Turning – Milling – Shaping – Drilling

The calculations for the cost of the above process sequence are summarized in Table 2.5.

**Table 2.5** Workings for Process Sequence 1

| Operation No. | Operation | Time  |       | Machine hour rate (Rs.) | Cost (Rs.)    |
|---------------|-----------|-------|-------|-------------------------|---------------|
|               |           | (min) | (hr)  |                         |               |
| 1             | Turning   | 5     | 0.083 | 200                     | 16.60         |
| 2             | Milling   | 8     | 0.133 | 400                     | 53.20         |
| 3             | Shaping   | 10    | 0.167 | 350                     | 58.45         |
| 4             | Drilling  | 3     | 0.050 | 300                     | 15.00         |
|               |           |       |       |                         | Total: 143.25 |

(b) *Cost of component using process sequence 2.* The process sequence 2 of the component is as follows:

Turning – Milling – Drilling

The calculations for the cost of the above process sequence are given in Table 2.6.

**Table 2.6** Workings for Process Sequence 2

| Operation No. | Operation | Time  |       | Machine hour rate (Rs.) | Cost (Rs.) |
|---------------|-----------|-------|-------|-------------------------|------------|
|               |           | (min) | (hr)  |                         |            |
| 1             | Turning   | 5     | 0.083 | 200                     | 16.60      |
| 2             | Milling   | 14    | 0.233 | 400                     | 93.20      |
| 3             | Drilling  | 3     | 0.050 | 300                     | 15.00      |
| Total:        |           |       |       |                         | 124.80     |

(c) *Cost of component using process sequence 3.* The process sequence 3 of the component is as follows:

Only CNC operations

The calculations for the cost of the above process sequence are summarized in Table 2.7

**Table 2.7** Workings for Process Sequence 3

| Operation No. | Operation      | Time  |       | Machine hour rate (Rs.) | Cost (Rs.) |
|---------------|----------------|-------|-------|-------------------------|------------|
|               |                | (min) | (hr)  |                         |            |
| 1             | CNC operations | 8     | 0.133 | 1,000                   | 133        |

The process sequence 2 has the least cost. Therefore, it should be selected for manufacturing the component.

## QUESTIONS

1. List and explain the different situations deserving elementary economic analysis.
2. Explain the steps in the process planning.
3. In the design of an aircraft jet engine part, the designer has a choice of specifying either an aluminium alloy casting or a steel casting. Either material will provide equal service, but the aluminium casting will weigh 5 kg as compared with 7 kg for the steel casting.  
The aluminium part can be cast for Rs. 125 per kg and the steel part can be cast for Rs. 60 per kg. The cost of machining per unit is Rs. 200 for the aluminium part and Rs. 250 for the steel part. Every kilogram of excess weight is associated with a penalty of Rs. 2,500 due to increased fuel consumption. Which material should be specified and what is the economic advantage of the selection per unit?
4. Two alternatives are under consideration for a hexagonal bolt fastening pin. Either design will serve equally well and will involve the same material and manufacturing cost except for the lathe and grinder operations.

Design A will require 20 hours of lathe time and 8 hours of grinder time per 10,000 units. Design B will require 10 hours of lathe time and 22 hours of grinder time per 10,000 units. The operating cost of the lathe including labour is Rs. 400 per hour. The operating cost of the grinder including labour is Rs. 300 per hour. Which design should be adopted if 10,00,000 units are required per year and what is the economic advantage of the best alternative?

5. A building contractor can source door frames from either a nearby shop or a far-off forest area. The cost details are as summarized in the following table. The total requirement of wood for the construction work is 75 tons.

| <i>Items</i>                          | <i>Nearby shop</i> | <i>Far-off forest area</i> |
|---------------------------------------|--------------------|----------------------------|
| Distance to site                      | Negligible         | 900 km                     |
| Transportation cost<br>per ton per km | Negligible         | Rs. 100                    |
| Material cost/ton                     | Rs. 2,000          | Rs. 1,250                  |

Find the best alternative for buying the wooden frames. Also find the economic advantage of the best decision.

6. Consider Example 2.4. Rework this example if the ratio of the height to diameter corresponding to the minimum cost is 6:1 instead of 4:1.
7. The process planning engineer of a firm listed down the sequences of operations, as shown in the following table to produce a component:

| <i>Sequence</i> | <i>Process sequence</i>                       |
|-----------------|-----------------------------------------------|
| 1               | Turning – Milling – Shaping – Drilling        |
| 2               | Turning – Milling – Drilling                  |
| 3               | All operations are performed with CNC machine |

The details of process time for the components for various operations and their machine hour rates are tabulated now.

| <i>Operation</i> | <i>Machine hour<br/>rate<br/>(Rs.)</i> | <i>Process sequence</i> |    |    |
|------------------|----------------------------------------|-------------------------|----|----|
|                  |                                        | 1                       | 2  | 3  |
| Turning          | 300                                    | 8                       | 8  |    |
| Milling          | 350                                    | 10                      | 14 |    |
| Shaping          | 380                                    | 12                      |    |    |
| Drilling         | 350                                    | 5                       | 5  |    |
| CNC operation    | 1,200                                  |                         |    | 10 |

Find the most economical sequence of operations to manufacture the component.

# 3

## INTEREST FORMULAS AND THEIR APPLICATIONS

### 3.1 INTRODUCTION

Interest rate is the rental value of money. It represents the growth of capital per unit period. The period may be a month, a quarter, semiannual or a year. An interest rate 15% compounded annually means that for every hundred rupees invested now, an amount of Rs. 15 will be added to the account at the end of the first year. So, the total amount at the end of the first year will be Rs. 115. At the end of the second year, again 15% of Rs. 115, i.e. Rs. 17.25 will be added to the account. Hence the total amount at the end of the second year will be Rs. 132.25. The process will continue thus till the specified number of years.

### 3.2 TIME VALUE OF MONEY

If an investor invests a sum of Rs. 100 in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of every year will be as shown in Table 3.1.

**Table 3.1** Compound Amounts

(amount of deposit = Rs. 100.00)

| <i>Year end</i> | <i>Interest<br/>(Rs.)</i> | <i>Compound amount<br/>(Rs.)</i> |
|-----------------|---------------------------|----------------------------------|
| 0               |                           | 100.00                           |
| 1               | 15.00                     | 115.00                           |
| 2               | 17.25                     | 132.25                           |
| 3               | 19.84                     | 152.09                           |
| 4               | 22.81                     | 174.90                           |
| 5               | 26.24                     | 201.14                           |

The formula to find the future worth in the third column is

$$F = P \times (1 + i)^n$$

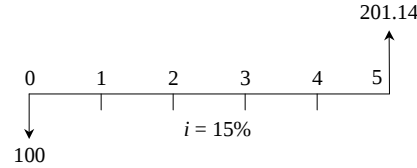
where

$P$  = principal amount invested at time 0,

$F$  = future amount,

$i$  = interest rate compounded annually,  
 $n$  = period of deposit.

The maturity value at the end of the fifth year is Rs. 201.14. This means that the amount Rs. 201.14 at the end of the fifth year is equivalent to Rs. 100.00 at time 0 (i.e. at present). This is diagrammatically shown in Fig. 3.1. This explanation assumes that the inflation is at zero percentage.



**Fig. 3.1** Time value of money.

Alternatively, the above concept may be discussed as follows: If we want Rs. 100.00 at the end of the  $n$ th year, what is the amount that we should deposit now at a given interest rate, say 15%? A detailed working is shown in Table 3.2.

**Table 3.2** Present Worth Amounts

(rate of interest = 15%)

| End of year<br>( $n$ ) | Present worth | Compound amount<br>after $n$ year(s) |
|------------------------|---------------|--------------------------------------|
| 0                      |               | 100                                  |
| 1                      | 86.96         | 100                                  |
| 2                      | 75.61         | 100                                  |
| 3                      | 65.75         | 100                                  |
| 4                      | 57.18         | 100                                  |
| 5                      | 49.72         | 100                                  |
| 6                      | 43.29         | 100                                  |
| 7                      | 37.59         | 100                                  |
| 8                      | 32.69         | 100                                  |
| 9                      | 28.43         | 100                                  |
| 10                     | 24.72         | 100                                  |

The formula to find the present worth in the second column is

$$P = \frac{F}{(1 + i)^n}$$

From Table 3.2, it is clear that if we want Rs. 100 at the end of the fifth year, we should now deposit an amount of Rs. 49.72. Similarly, if we want Rs. 100.00 at the end of the 10th year, we should now deposit an amount of Rs. 24.72.

Also, this concept can be stated as follows:

A person has received a prize from a finance company during the recent festival contest. But the prize will be given in either of the following two modes:

1. Spot payment of Rs. 24.72 or
2. Rs. 100 after 10 years from now (this is based on 15% interest rate compounded annually).

If the prize winner has no better choice that can yield more than 15% interest rate compounded annually, and if 15% compounded annually is the common interest rate paid in all the finance companies, then it makes no difference whether he receives Rs. 24.72 now or Rs. 100 after 10 years.

On the other hand, let us assume that the prize winner has his own business wherein he can get a yield of 24% interest rate (more than 15%) compounded annually, it is better for him to receive the prize money of Rs. 24.72 at present and utilize it in his business. If this option is followed, the equivalent amount for Rs. 24.72 at the end of the 10th year is Rs. 212.45. This example clearly demonstrates the time value of money.

### 3.3 INTEREST FORMULAS

While making investment decisions, computations will be done in many ways. To simplify all these computations, it is extremely important to know how to use interest formulas more effectively. Before discussing the effective application of the interest formulas for investment-decision making, the various interest formulas are presented first.

Interest rate can be classified into *simple interest rate* and *compound interest rate*.

In simple interest, the interest is calculated, based on the initial deposit for every interest period. In this case, calculation of interest on interest is not applicable. In compound interest, the interest for the current period is computed based on the amount (principal plus interest up to the end of the previous period) at the beginning of the current period.

The notations which are used in various interest formulae are as follows:

$P$  = principal amount

$n$  = No. of interest periods

$i$  = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

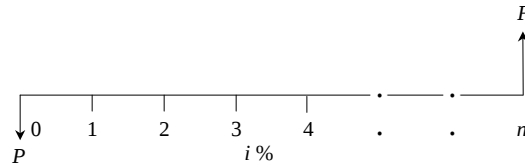
$F$  = future amount at the end of year  $n$

$A$  = equal amount deposited at the end of every interest period

$G$  = uniform amount which will be added/subtracted period after period to/from the amount of deposit  $A_1$  at the end of period 1

#### 3.3.1 Single-Payment Compound Amount

Here, the objective is to find the single future sum ( $F$ ) of the initial payment ( $P$ ) made at time 0 after  $n$  periods at an interest rate  $i$  compounded every period. The cash flow diagram of this situation is shown in Fig. 3.2.



**Fig. 3.2** Cash flow diagram of single-payment compound amount.

The formula to obtain the single-payment compound amount is

$$F = P(1 + i)^n = P(F/P, i, n)$$

where

$(F/P, i, n)$  is called as single-payment compound amount factor.

**EXAMPLE 3.1** A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

**Solution**

$$P = \text{Rs. } 20,000$$

$$i = 18\% \text{ compounded annually}$$

$$n = 10 \text{ years}$$

$$F = P(1 + i)^n = P(F/P, i, n)$$

$$= 20,000 (F/P, 18\%, 10)$$

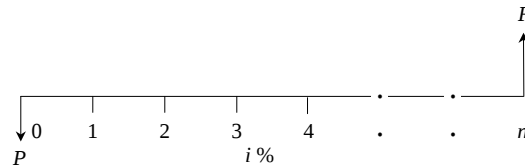
$$= 20,000 \times 5.234 = \text{Rs. } 1,04,680$$

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

### 3.3.2 Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount ( $P$ ) of a single future sum ( $F$ ) which will be received after  $n$  periods at an interest rate of  $i$  compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.3.



**Fig. 3.3** Cash flow diagram of single-payment present worth amount.

The formula to obtain the present worth is

$$P = \frac{F}{(1 + i)^n} = F(P/F, i, n)$$

where

$(P/F, i, n)$  is termed as *single-payment present worth factor*.

**EXAMPLE 3.2** A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

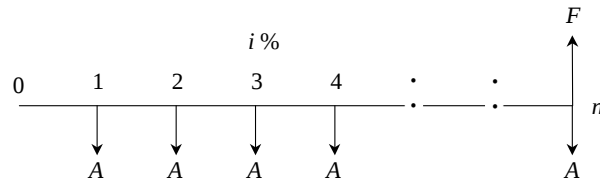
**Solution**

$$\begin{aligned}
 F &= \text{Rs. } 1,00,000 \\
 i &= 15\%, \text{ compounded annually} \\
 n &= 10 \text{ years} \\
 P &= F/(1 + i)^n = F(P/F, i, n) \\
 &= 1,00,000 (P/F, 15\%, 10) \\
 &= 1,00,000 \times 0.2472 \\
 &= \text{Rs. } 24,720
 \end{aligned}$$

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

### 3.3.3 Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of  $n$  equal payments which are made at the end of every interest period till the end of the  $n$ th interest period at an interest rate of  $i$  compounded at the end of each interest period. The corresponding cash flow diagram is shown in Fig. 3.4.



**Fig. 3.4** Cash flow diagram of equal-payment series compound amount.

In Fig. 3.4,

$A$  = equal amount deposited at the end of each interest period

$n$  = No. of interest periods

$i$  = rate of interest

$F$  = single future amount

The formula to get  $F$  is

$$F = A \frac{(1 + i)^n - 1}{i} = A(F/A, i, n)$$

where

$(F/A, i, n)$  is termed as *equal-payment series compound amount factor*.

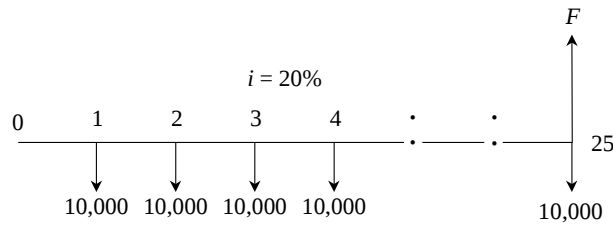
**EXAMPLE 3.3** A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for

the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

**Solution**

$$\begin{aligned} A &= \text{Rs. } 10,000 \\ n &= 25 \text{ years} \\ i &= 20\% \\ F &= ? \end{aligned}$$

The corresponding cash flow diagram is shown in Fig. 3.5.



**Fig. 3.5** Cash flow diagram of equal-payment series compound amount.

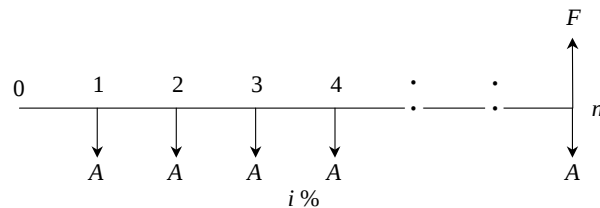
$$\begin{aligned} F &= A \frac{(1+i)^n - 1}{i} \\ &= A(F/A, i, n) \\ &= 10,000(F/A, 20\%, 25) \\ &= 10,000 \times 471.981 \\ &= \text{Rs. } 47,19,810 \end{aligned}$$

The future sum of the annual equal payments after 25 years is equal to Rs. 47,19,810.

### 3.3.4 Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount ( $A$ ) that should be deposited at the end of every interest period for  $n$  interest periods to realize a future sum ( $F$ ) at the end of the  $n$ th interest period at an interest rate of  $i$ .

The corresponding cash flow diagram is shown in Fig. 3.6.



**Fig. 3.6** Cash flow diagram of equal-payment series sinking fund.

In Fig. 3.6,

$A$  = equal amount to be deposited at the end of each interest period

$n$  = No. of interest periods

$i$  = rate of interest

$F$  = single future amount at the end of the  $n$ th period

The formula to get  $F$  is

$$A = F \frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

where

$(A/F, i, n)$  is called as *equal-payment series sinking fund factor*.

**EXAMPLE 3.4** A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

**Solution**

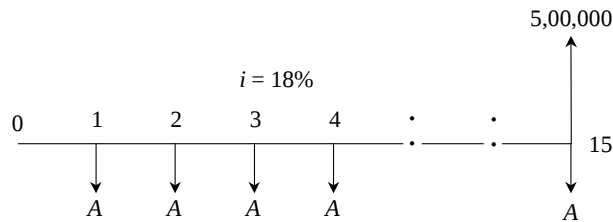
$F$  = Rs. 5,00,000

$n$  = 15 years

$i$  = 18%

$A$  = ?

The corresponding cash flow diagram is shown in Fig. 3.7.



**Fig. 3.7** Cash flow diagram of equal-payment series sinking fund.

$$\begin{aligned} A &= F \frac{i}{(1+i)^n - 1} = F(A/F, i, n) \\ &= 5,00,000(A/F, 18\%, 15) \\ &= 5,00,000 \times 0.0164 \\ &= \text{Rs. } 8,200 \end{aligned}$$

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

### 3.3.5 Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal

payment made at the end of every interest period for  $n$  interest periods at an interest rate of  $i$  compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.8. Here,

$P$  = present worth

$A$  = annual equivalent payment

$i$  = interest rate

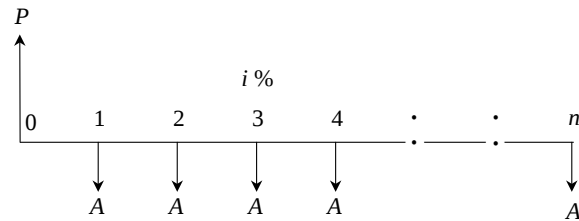
$n$  = No. of interest periods

The formula to compute  $P$  is

$$P = A \frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

where

$(P/A, i, n)$  is called *equal-payment series present worth factor*.



**Fig. 3.8** Cash flow diagram of equal-payment series present worth amount.

**EXAMPLE 3.5** A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

**Solution**

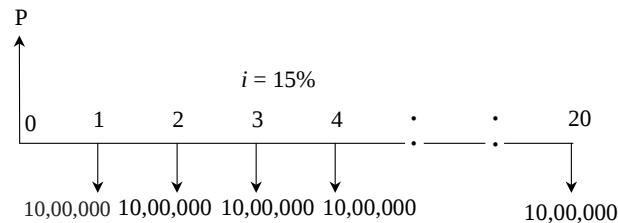
$A$  = Rs. 10,00,000

$i$  = 15%

$n$  = 20 years

$P$  = ?

The corresponding cash flow diagram is illustrated in Fig. 3.9.



**Fig. 3.9** Cash flow diagram of equal-payment series present worth amount.

$$\begin{aligned}
 P &= A \frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n) \\
 &= 10,00,000 \times (P/A, 15\%, 20) \\
 &= 10,00,000 \times 6.2593 \\
 &= \text{Rs. } 62,59,300
 \end{aligned}$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

### 3.3.6 Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount ( $A$ ) which is to be recovered at the end of every interest period for  $n$  interest periods for a loan ( $P$ ) which is sanctioned now at an interest rate of  $i$  compounded at the end of every interest period (see Fig. 3.10).

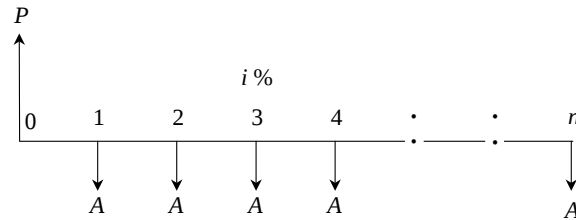


Fig. 3.10 Cash flow diagram of equal-payment series capital recovery amount.

In Fig. 3.10,

$P$  = present worth (loan amount)

$A$  = annual equivalent payment (recovery amount)

$i$  = interest rate

$n$  = No. of interest periods

The formula to compute  $P$  is as follows:

$$A = P \frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n)$$

where,

$(A/P, i, n)$  is called *equal-payment series capital recovery factor*.

**EXAMPLE 3.6** A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

#### Solution

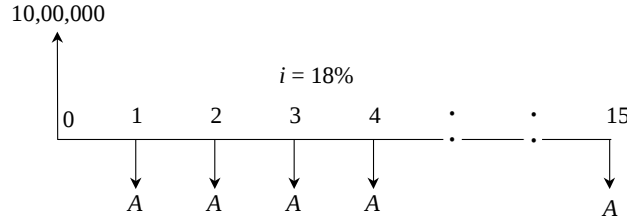
$P$  = Rs. 10,00,000

$i$  = 18%

$n$  = 15 years

$A$  = ?

The corresponding cash flow diagram is shown in Fig. 3.11.



**Fig. 3.11** Cash flow diagram of equal-payment series capital recovery amount.

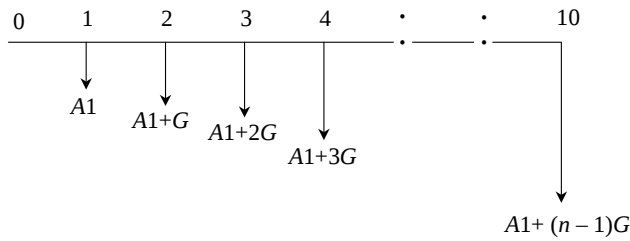
$$\begin{aligned}
 A &= P \frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n) \\
 &= 10,00,000 \times (A/P, 18\%, 15) \\
 &= 10,00,000 \times (0.1964) \\
 &= \text{Rs. } 1,96,400
 \end{aligned}$$

The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

### 3.3.7 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount  $A_1$  at the end of the first year and with an equal increment ( $G$ ) at the end of each of the following  $n - 1$  years with an interest rate  $i$  compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.12.



**Fig. 3.12** Cash flow diagram of uniform gradient series annual equivalent amount.

The formula to compute  $A$  under this situation is

$$\begin{aligned}
 A &= A_1 + G \frac{(1+i)^n - in - 1}{i(1+i)^n - i} \\
 &= A_1 + G (A/G, i, n)
 \end{aligned}$$

where

$(A/G, i, n)$  is called *uniform gradient series factor*.

**EXAMPLE 3.7** A person is planning for his retired life. He has 10 more years

of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

**Solution** Here,

$$A_1 = \text{Rs. } 4,000$$

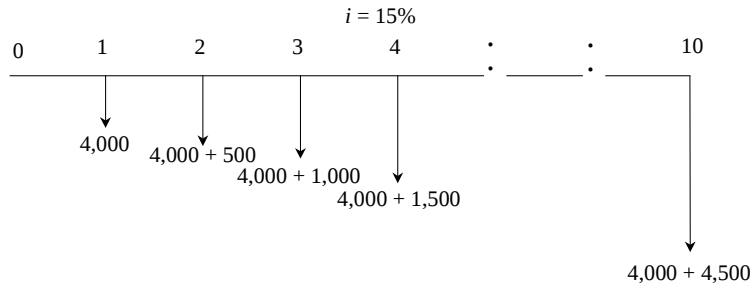
$$G = \text{Rs. } 500$$

$$i = 15\%$$

$$n = 10 \text{ years}$$

$$A = ? \text{ \& } F = ?$$

The cash flow diagram is shown in Fig. 3.13.



**Fig. 3.13** Cash flow diagram of uniform gradient series annual equivalent amount.

$$\begin{aligned} A &= A_1 + G \frac{(1+i)^n - in - 1}{i(1+i)^n - i} \\ &= A_1 + G(A/G, i, n) \\ &= 4,000 + 500(A/G, 15\%, 10) \\ &= 4,000 + 500 \times 3.3832 \\ &= \text{Rs. } 5,691.60 \end{aligned}$$

This is equivalent to paying an equivalent amount of Rs. 5,691.60 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$\begin{aligned} F &= A(F/A, i, n) \\ &= A(F/A, 15\%, 10) \\ &= 5,691.60(20.304) \\ &= \text{Rs. } 1,15,562.25 \end{aligned}$$

At the end of the 10th year, the compound amount of all his payments will be Rs. 1,15,562.25.

**EXAMPLE 3.8** A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 8,500 at the end of the first year and

thereafter he wishes to deposit the amount with an annual decrease of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

**Solution** Here,

$$A_1 = \text{Rs. } 8,500$$

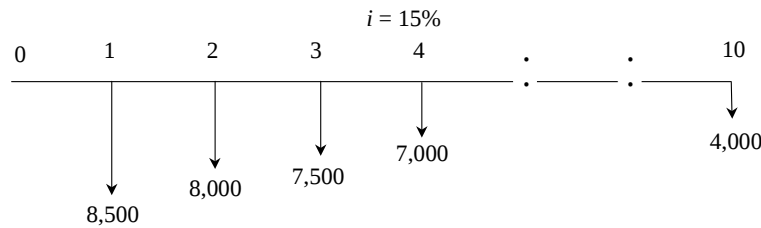
$$G = -\text{Rs. } 500$$

$$i = 15\%$$

$$n = 10 \text{ years}$$

$$A = ? \text{ \& } F = ?$$

The cash flow diagram is shown in Fig. 3.14.



**Fig. 3.14** Cash flow diagram of uniform gradient series annual equivalent amount.

$$\begin{aligned} A &= A_1 - G \frac{(1+i)^n - in - 1}{i(1+i)^n - i} \\ &= A_1 - G (A/G, i, n) \\ &= 8,500 - 500(A/G, 15\%, 10) \\ &= 8,500 - 500 \times 3.3832 \\ &= \text{Rs. } 6,808.40 \end{aligned}$$

This is equivalent to paying an equivalent amount of Rs. 6,808.40 at the end of every year for the next 10 years.

The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$\begin{aligned} F &= A(F/A, i, n) \\ &= A(F/A, 15\%, 10) \\ &= 6,808.40(20.304) \\ &= \text{Rs. } 1,38,237.75 \end{aligned}$$

At the end of the 10th year, the compound amount of all his payments is Rs. 1,38,237.75.

### 3.3.8 Effective Interest Rate

Let  $i$  be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year. For example, compounding may be monthly, quarterly, or semi-annually. Compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in

a year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is

$$\text{Effective interest rate, } R = \left(1 + \frac{i}{C}\right)^C - 1$$

where,

$i$  = the nominal interest rate

$C$  = the number of interest periods in a year.

**EXAMPLE 3.9** A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

**Solution**

$$P = \text{Rs. } 5,000$$

$$n = 10 \text{ years}$$

$$i = 12\% \text{ (Nominal interest rate)}$$

$$F = ?$$

**METHOD 1**

$$\text{No. of interest periods per year} = 4$$

$$\text{No. of interest periods in 10 years} = 10 \times 4 = 40$$

$$\text{Revised No. of periods (No. of quarters), } N = 40$$

$$\text{Interest rate per quarter, } r = 12\%/4$$

$$= 3\%, \text{ compounded quarterly.}$$

$$\begin{aligned} F &= P(1 + r)^N = 5,000(1 + 0.03)^{40} \\ &= \text{Rs. } 16,310.19 \end{aligned}$$

**METHOD 2**

$$\text{No. of interest periods per year, } C = 4$$

$$\text{Effective interest rate, } R = \left(1 + \frac{i}{C}\right)^C - 1$$

$$= \left(1 + \frac{12\%}{4}\right)^4 - 1$$

$$= 12.55\%, \text{ compounded annually.}$$

$$\begin{aligned} F &= P(1 + R)^n = 5,000(1 + 0.1255)^{10} \\ &= \text{Rs. } 16,308.91 \end{aligned}$$

### 3.4 BASES FOR COMPARISON OF ALTERNATIVES

In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives. Let us assume that an organization has a huge sum of money for potential investment and there are three different projects whose initial outlay and annual revenues during their lives are known. The executive has to select the best alternative among these three competing projects.

There are several bases for comparing the worthiness of the projects. These bases are:

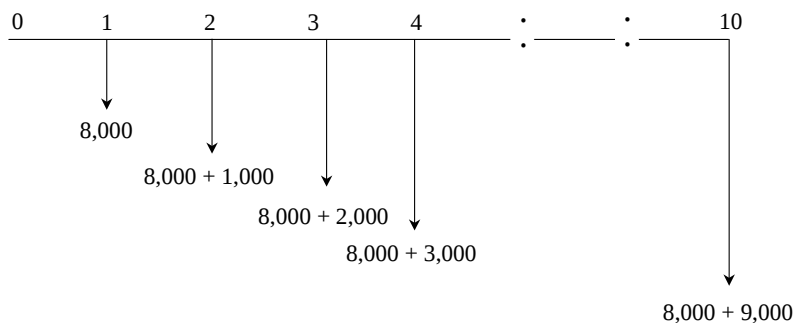
1. Present worth method
2. Future worth method
3. Annual equivalent method
4. Rate of return method

These methods are discussed in detail in Chapters 4–7.

### QUESTIONS

1. Explain the time value of money.
2. Give practical applications of various interest formulas.
3. A person deposits a sum of Rs. 1,00,000 in a bank for his son's education who will be admitted to a professional course after 6 years. The bank pays 15% interest rate, compounded annually. Find the future amount of the deposited money at the time of admitting his son in the professional course.
4. A person needs a sum of Rs. 2,00,000 for his daughter's marriage which will take place 15 years from now. Find the amount of money that he should deposit now in a bank if the bank gives 18% interest, compounded annually.
5. A person who is just 30 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 30 years starting from the end of next year. The bank gives 15% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.
6. A company is planning to expand its business after 5 years from now. The expected money required for the expansion programme is Rs. 5,00,00,000. The company can invest Rs. 50,00,000 at the end of every year for the next five years. If the assured rate of return of investment is 18% for the company, check whether the accumulated sum in the account would be sufficient to meet the fund for the expansion programme. If not, find the difference in amounts for which the company should make some other arrangement after 5 years.
7. A financial institution introduces a plan to pay a sum of Rs. 15,00,000 after 10 years at the rate of 18%, compounded annually. Find the annual equivalent amount that a person should invest at the end of every year for the next 10 years to receive Rs. 15,00,000 after 10 years from the institution.
8. A company is planning to expand its business after 5 years from now. The money required for the expansion programme is Rs. 4,00,00,000. What annual equivalent amount should the company deposit at the end of every year at an interest rate of 15% compounded annually to get Rs. 4,00,00,000 after 5 years from now?

9. A company wants to set-up a reserve which will help it to have an annual equivalent amount of Rs. 15,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made as the reserve amount now.
10. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually. Suggest the best alternative to the customers.
11. A company takes a loan of Rs. 20,00,000 to modernize its boiler section. The loan is to be repaid in 20 equal installments at 12% interest rate, compounded annually. Find the equal installment amount that should be paid for the next 20 years.
12. A bank gives loan to a company to purchase an equipment which is worth of Rs. 5,00,000, at an interest rate of 18% compounded annually. This amount should be repaid in 25 yearly equal installments. Find the installment amount that the company has to pay to the bank.
13. A working woman is planning for her retired life. She has 20 more years of service. She would like to deposit 10% of her salary which is Rs. 5,000 at the end of the first year and thereafter she wishes to deposit the same amount (Rs. 5,000) with an annual increase of Rs. 1,000 for the next 14 years with an interest rate of 18%. Find the total amount at the end of the 15th year of the above series.
14. Consider the following cash flow diagram. Find the total amount at the end of the 10th year at an interest rate of 12%, compounded annually.



15. A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 10,000, at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 10,000) with an annual increase of Rs. 2,000 for the next 9 years with an interest rate of 20%. Find the total amount at the end of the 10th year of the above series.

16. A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 30,000 at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 30,000) with an annual decrease of Rs. 2,000 for the next 9 years with an interest rate of 18%. Find the total amount at the end of the 10th year of the above series.
17. A person invests a sum of Rs. 50,000 in a bank at a nominal interest rate of 18% for 15 years. The compounding is monthly. Find the maturity amount of the deposit after 15 years.

# 4

## PRESENT WORTH METHOD OF COMPARISON

### 4.1 INTRODUCTION

In this method of comparison, the cash flows of each alternative will be reduced to time zero by assuming an interest rate  $i$ . Then, depending on the type of decision, the best alternative will be selected by comparing the present worth amounts of the alternatives.

The sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem.

In a cost dominated cash flow diagram, the costs (outflows) will be assigned with positive sign and the profit, revenue, salvage value (all inflows), etc. will be assigned with negative sign.

In a revenue/profit-dominated cash flow diagram, the profit, revenue, salvage value (all inflows to an organization) will be assigned with positive sign. The costs (outflows) will be assigned with negative sign.

In case the decision is to select the alternative with the minimum cost, then the alternative with the least present worth amount will be selected. On the other hand, if the decision is to select the alternative with the maximum profit, then the alternative with the maximum present worth will be selected.

### 4.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.1.

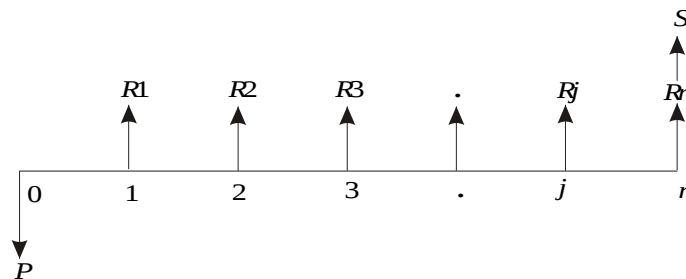


Fig. 4.1 Revenue-dominated cash flow diagram.

In Fig. 4.1,  $P$  represents an initial investment and  $R_j$  the net revenue at the end of the  $j$ th year. The interest rate is  $i$ , compounded annually.  $S$  is the salvage value at the end of the  $n$ th year.

To find the present worth of the above cash flow diagram for a given interest rate, the formula is

$$PW(i) = -P + R_1[1/(1+i)^1] + R_2[1/(1+i)^2] + \dots \\ + R_j[1/(1+i)^j] + R_n[1/(1+i)^n] + S[1/(1+i)^n]$$

In this formula, expenditure is assigned a negative sign and revenues are assigned a positive sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the alternative with the maximum present worth amount should be selected as the best alternative.

### 4.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.2.

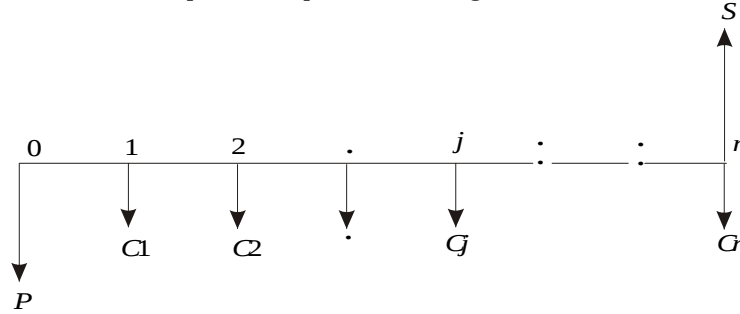


Fig. 4.2 Cost-dominated cash flow diagram.

In Fig. 4.2,  $P$  represents an initial investment,  $C_j$  the net cost of operation and maintenance at the end of the  $j$ th year, and  $S$  the salvage value at the end of the  $n$ th year.

To compute the present worth amount of the above cash flow diagram for a given interest rate  $i$ , we have the formula

$$PW(i) = P + C_1[1/(1+i)^1] + C_2[1/(1+i)^2] + \dots + C_j[1/(1+i)^j] \\ + C_n[1/(1+i)^n] - S[1/(1+i)^n]$$

In the above formula, the expenditure is assigned a positive sign and the revenue a negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the alternative with the minimum present worth amount should be selected as the best alternative.

#### 4.4 EXAMPLES

In this section, the concept of present worth method of comparison applied to the selection of the best alternative is demonstrated with several illustrations.

**EXAMPLE 4.1** Alpha Industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized in Table 4.1. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

**Table 4.1**

|              | Initial outlay<br>(Rs.) | Annual revenue<br>(Rs.) | Life<br>(years) |
|--------------|-------------------------|-------------------------|-----------------|
| Technology 1 | 12,00,000               | 4,00,000                | 10              |
| Technology 2 | 20,00,000               | 6,00,000                | 10              |
| Technology 3 | 18,00,000               | 5,00,000                | 10              |

**Solution** In all the technologies, the initial outlay is assigned a negative sign and the annual revenues are assigned a positive sign.

**TECHNOLOGY 1**

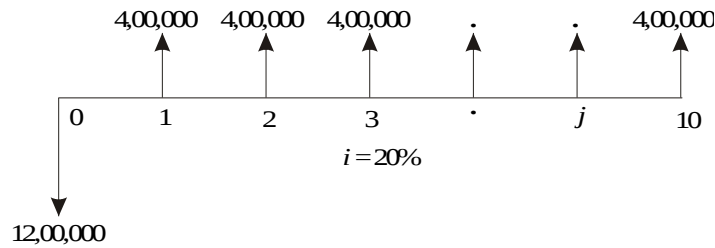
Initial outlay,  $P = \text{Rs. } 12,00,000$

Annual revenue,  $A = \text{Rs. } 4,00,000$

Interest rate,  $i = 20\%$ , compounded annually

Life of this technology,  $n = 10$  years

The cash flow diagram of this technology is as shown in Fig. 4.3.



**Fig. 4.3** Cash flow diagram for technology 1.

The present worth expression for this technology is

$$\begin{aligned}
 PW(20\%)_1 &= -12,00,000 + 4,00,000 \times (P/A, 20\%, 10) \\
 &= -12,00,000 + 4,00,000 \times (4.1925) \\
 &= -12,00,000 + 16,77,000 \\
 &= \text{Rs. } 4,77,000
 \end{aligned}$$

## TECHNOLOGY 2

Initial outlay,  $P = \text{Rs. } 20,00,000$

Annual revenue,  $A = \text{Rs. } 6,00,000$

Interest rate,  $i = 20\%$ , compounded annually

Life of this technology,  $n = 10$  years

The cash flow diagram of this technology is shown in Fig. 4.4.

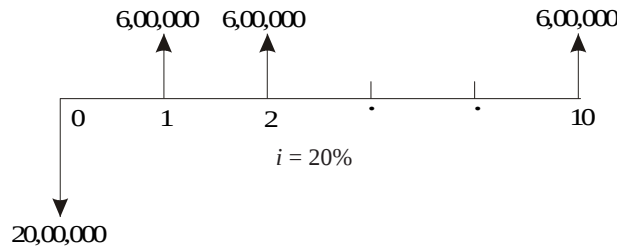


Fig. 4.4 Cash flow diagram for technology 2.

The present worth expression for this technology is

$$\begin{aligned}
 PW(20\%)_2 &= -20,00,000 + 6,00,000 \times (P/A, 20\%, 10) \\
 &= -20,00,000 + 6,00,000 \times (4.1925) \\
 &= -20,00,000 + 25,15,500 \\
 &= \text{Rs. } 5,15,500
 \end{aligned}$$

## TECHNOLOGY 3

Initial outlay,  $P = \text{Rs. } 18,00,000$

Annual revenue,  $A = \text{Rs. } 5,00,000$

Interest rate,  $i = 20\%$ , compounded annually

Life of this technology,  $n = 10$  years

The cash flow diagram of this technology is shown in Fig. 4.5.

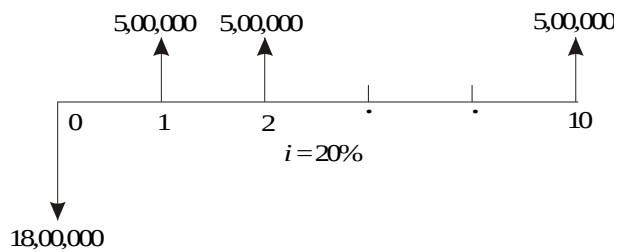


Fig. 4.5 Cash flow diagram for technology 3.

The present worth expression for this technology is

$$\begin{aligned}
 PW(20\%)_3 &= -18,00,000 + 5,00,000 \times (P/A, 20\%, 10) \\
 &= -18,00,000 + 5,00,000 \times (4.1925) \\
 &= -18,00,000 + 20,96,250 \\
 &= \text{Rs. } 2,96,250
 \end{aligned}$$

From the above calculations, it is clear that the present worth of technology 2 is the highest among all the technologies. Therefore, technology 2 is suggested for implementation to expand the production.

**EXAMPLE 4.2** An engineer has two bids for an elevator to be installed in a new building. The details of the bids for the elevators are as follows:

| Bid                 | Engineer's estimates |                      |                                            |
|---------------------|----------------------|----------------------|--------------------------------------------|
|                     | Initial cost         | Service life (years) | Annual operations & maintenance cost (Rs.) |
|                     | (Rs.)                |                      |                                            |
| Alpha Elevator Inc. | 4,50,000             | 15                   | 27,000                                     |
| Beta Elevator Inc.  | 5,40,000             | 15                   | 28,500                                     |

Determine which bid should be accepted, based on the present worth method of comparison assuming 15% interest rate, compounded annually.

**Solution**

**Bid 1: Alpha Elevator Inc.**

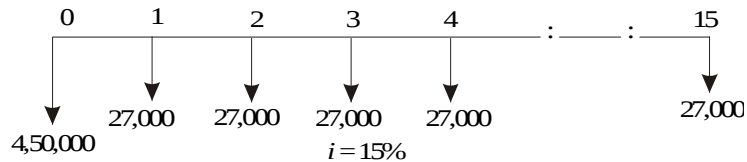
Initial cost,  $P = \text{Rs. } 4,50,000$

Annual operation and maintenance cost,  $A = \text{Rs. } 27,000$

Life = 15 years

Interest rate,  $i = 15\%$ , compounded annually.

The cash flow diagram of bid 1 is shown in Fig. 4.6.



**Fig. 4.6** Cash flow diagram for bid 1.

The present worth of the above cash flow diagram is computed as follows:

$$\begin{aligned}
 PW(15\%) &= 4,50,000 + 27,000(P/A, 15\%, 15) \\
 &= 4,50,000 + 27,000 \times 5.8474 \\
 &= 4,50,000 + 1,57,879.80 \\
 &= \text{Rs. } 6,07,879.80
 \end{aligned}$$

**Bid 2: Beta Elevator Inc.**

Initial cost,  $P = \text{Rs. } 5,40,000$

Annual operation and maintenance cost,  $A = \text{Rs. } 28,500$

Life = 15 years

Interest rate,  $i = 15\%$ , compounded annually.

The cash flow diagram of bid 2 is shown in Fig. 4.7.

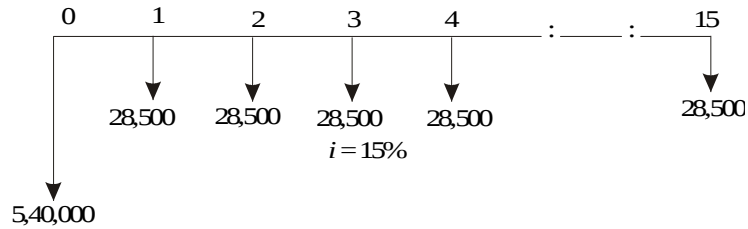


Fig. 4.7 Cash flow diagram for bid 2.

The present worth of the above cash flow diagram is computed as follows:

$$\begin{aligned}
 PW(15\%) &= 5,40,000 + 28,500(P/A, 15\%, 15) \\
 &= 5,40,000 + 28,500 \times 5.8474 \\
 &= 5,40,000 + 1,66,650.90 \\
 &= \text{Rs. } 7,06,650.90
 \end{aligned}$$

The total present worth cost of bid 1 is less than that of bid 2. Hence, bid 1 is to be selected for implementation. That is, the elevator from Alpha Elevator Inc. is to be purchased and installed in the new building.

**EXAMPLE 4.3** Investment proposals A and B have the net cash flows as follows:

| Proposal | End of years |       |       |       |       |
|----------|--------------|-------|-------|-------|-------|
|          | 0            | 1     | 2     | 3     | 4     |
| A (Rs.)  | -10,000      | 3,000 | 3,000 | 7,000 | 6,000 |
| B (Rs.)  | -10,000      | 6,000 | 6,000 | 3,000 | 3,000 |

Compare the present worth of A with that of B at  $i = 18\%$ . Which proposal should be selected?

#### Solution

**Present worth of A at  $i = 18\%$ .** The cash flow diagram of proposal A is shown in Fig. 4.8.

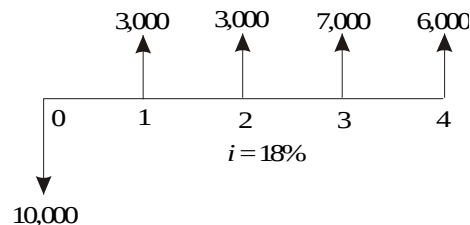
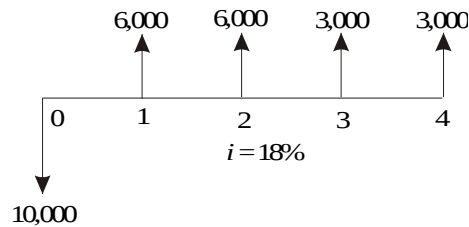


Fig. 4.8 Cash flow diagram for proposal A.

The present worth of the above cash flow diagram is computed as

$$\begin{aligned}
 PW_A(18\%) &= -10,000 + 3,000(P/F, 18\%, 1) + 3,000(P/F, 18\%, 2) \\
 &\quad + 7,000(P/F, 18\%, 3) + 6,000(P/F, 18\%, 4) \\
 &= -10,000 + 3,000(0.8475) + 3,000(0.7182) \\
 &\quad + 7,000(0.6086) + 6,000(0.5158) \\
 &= \text{Rs. } 2,052.10
 \end{aligned}$$

**Present worth of B at  $i = 18\%$ .** The cash flow diagram of the proposal B is shown in Fig. 4.9.



**Fig. 4.9** Cash flow diagram for proposal B.

The present worth of the above cash flow diagram is calculated as

$$\begin{aligned}
 PW_B(18\%) &= -10,000 + 6,000(P/F, 18\%, 1) + 6,000(P/F, 18\%, 2) \\
 &\quad + 3,000(P/F, 18\%, 3) + 3,000(P/F, 18\%, 4) \\
 &= -10,000 + 6,000(0.8475) + 6,000(0.7182) \\
 &\quad + 3,000(0.6086) + 3,000(0.5158) \\
 &= \text{Rs. } 2,767.40
 \end{aligned}$$

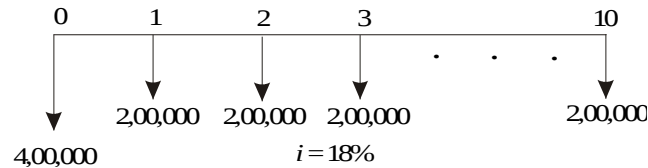
At  $i = 18\%$ , the present worth of proposal B is higher than that of proposal A. Therefore, select proposal B.

**EXAMPLE 4.4** A granite company is planning to buy a fully automated granite cutting machine. If it is purchased under down payment, the cost of the machine is Rs. 16,00,000. If it is purchased under installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of Rs. 2,00,000 each. Suggest the best alternative for the company using the present worth basis at  $i = 18\%$ , compounded annually.

**Solution** There are two alternatives available for the company:

1. Down payment of Rs. 16,00,000
2. Down payment of Rs. 4,00,000 and 10 annual equal installments of Rs. 2,00,000 each

**Present worth calculation of the second alternative.** The cash flow diagram of the second alternative is shown in Fig. 4.10.



**Fig. 4.10** Cash flow diagram for the second alternative.

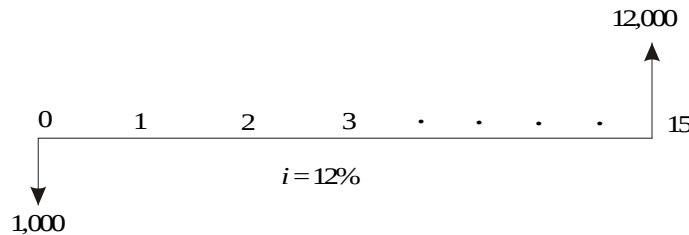
The present worth of the above cash flow diagram is computed as

$$\begin{aligned} PW(18\%) &= 4,00,000 + 2,00,000(P/A, 18\%, 10) \\ &= 4,00,000 + 2,00,000 \times 4.4941 \\ &= \text{Rs. } 12,98,820 \end{aligned}$$

The present worth of this option is Rs. 12,98,820, which is less than the first option of complete down payment of Rs. 16,00,000. Hence, the company should select the second alternative to buy the fully automated granite cutting machine.

**EXAMPLE 4.5** A finance company advertises two investment plans. In plan 1, the company pays Rs. 12,000 after 15 years for every Rs. 1,000 invested now. In plan 2, for every Rs. 1,000 invested, the company pays Rs. 4,000 at the end of the 10th year and Rs. 4,000 at the end of 15th year. Select the best investment plan from the investor's point of view at  $i = 12\%$ , compounded annually.

**Solution Plan 1.** The cash flow diagram for plan 1 is illustrated in Fig. 4.11.



**Fig. 4.11** Cash flow diagram for plan 1.

The present worth of the above cash flow diagram is calculated as

$$\begin{aligned} PW(12\%) &= -1,000 + 12,000(P/F, 12\%, 15) \\ &= -1,000 + 12,000(0.1827) \\ &= \text{Rs. } 1,192.40 \end{aligned}$$

**Plan 2.** The cash flow diagram for plan 2 is shown in Fig. 4.12.

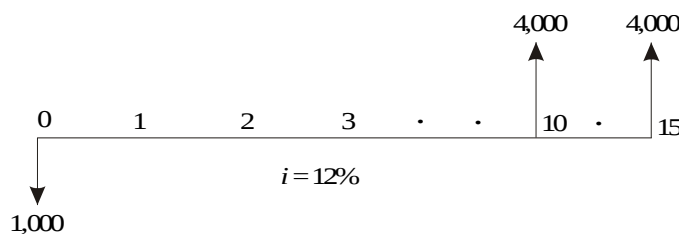


Fig. 4.12 Cash flow diagram for plan 2.

The present worth of the above cash flow diagram is computed as

$$\begin{aligned}
 PW(12\%) &= -1,000 + 4,000(P/F, 12\%, 10) + 4,000(P/F, 12\%, 15) \\
 &= -1,000 + 4,000(0.3220) + 4,000(0.1827) \\
 &= \text{Rs. } 1,018.80
 \end{aligned}$$

The present worth of plan 1 is more than that of plan 2. Therefore, plan 1 is the best plan from the investor's point of view.

**EXAMPLE 4.6** Novel Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 8,00,000 at the end of the 20th year. Innovative Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 15,00,000 at the end of the 25th year. Which is the best investment alternative? Use present worth base with  $i = 12\%$ .

**Solution** *Novel Investment Ltd's plan.* The cash flow diagram of Novel Investment Ltd's plan is shown in Fig. 4.13.

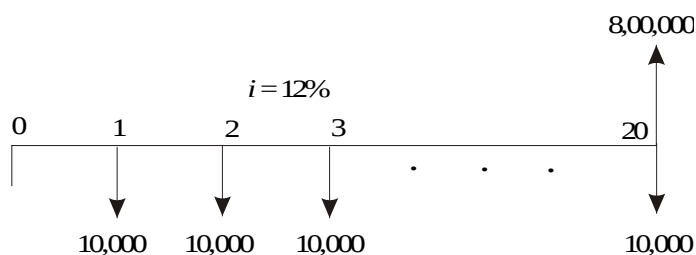
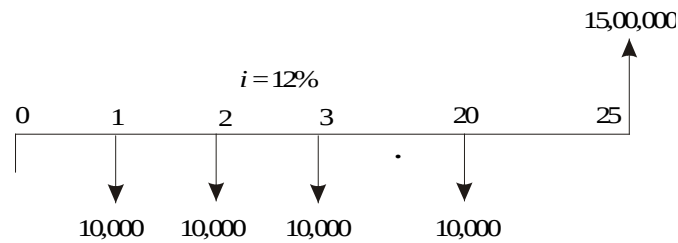


Fig. 4.13 Cash flow diagram for Novel Investment Ltd.

The present worth of the above cash flow diagram is computed as

$$\begin{aligned}
 PW(12\%) &= -10,000(P/A, 12\%, 20) + 8,00,000(P/F, 12\%, 20) \\
 &= -10,000(7.4694) + 8,00,000(0.1037) \\
 &= \text{Rs. } 8,266
 \end{aligned}$$

*Innovative Investment Ltd's plan.* The cash flow diagram of the Innovative Investment Ltd's plan is illustrated in Fig. 4.14.



**Fig. 4.14** Cash flow diagram for Innovative Investment Ltd.

The present worth of the above cash flow diagram is calculated as

$$\begin{aligned}
 PW(12\%) &= -10,000(P/A, 12\%, 20) + 15,00,000(P/F, 12\%, 25) \\
 &= -10,000(7.4694) + 15,00,000(0.0588) \\
 &= \text{Rs. } 13,506
 \end{aligned}$$

The present worth of Innovative Investment Ltd's plan is more than that of Novel Investment Ltd's plan. Therefore, Innovative Investment Ltd's plan is the best from investor's point of view.

**EXAMPLE 4.7** A small business with an initial outlay of Rs. 12,000 yields Rs. 10,000 during the first year of its operation and the yield increases by Rs. 1,000 from its second year of operation up to its 10th year of operation. At the end of the life of the business, the salvage value is zero. Find the present worth of the business by assuming an interest rate of 18%, compounded annually.

**Solution**

Initial investment,  $P = \text{Rs. } 12,000$

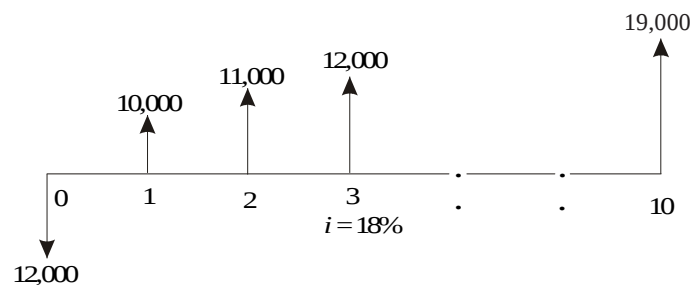
Income during the first year,  $A = \text{Rs. } 10,000$

Annual increase in income,  $G = \text{Rs. } 1,000$

$n = 10$  years

$i = 18\%$ , compounded annually

The cash flow diagram for the small business is depicted in Fig. 4.15.



**Fig. 4.15** Cash flow diagram for the small business.

The equation for the present worth is

$$\begin{aligned}
 PW(18\%) &= -12,000 + (10,000 + 1,000 \times (A/G, 18\%, 10)) \times (P/A, 18\%, 10) \\
 &= -12,000 + (10,000 + 1,000 \times 3.1936) \times 4.4941 \\
 &= -12,000 + 59,293.36 \\
 &= \text{Rs. } 47,293.36
 \end{aligned}$$

The present worth of the small business is Rs. 47,293.36.

### QUESTIONS

1. A project involves an initial outlay of Rs. 30,00,000 and with the following transactions for the next five years. The salvage value at the end of the life of the project after five years is Rs. 2,00,000. Draw a cash flow diagram of the project and find its present worth by assuming  $i = 15\%$ , compounded annually.

| <i>End of year</i> | <i>Maintenance and operating expense (Rs.)</i> | <i>Revenue (Rs.)</i> |
|--------------------|------------------------------------------------|----------------------|
| 1                  | 2,00,000                                       | 9,00,000             |
| 2                  | 2,50,000                                       | 10,00,000            |
| 3                  | 3,00,000                                       | 12,00,000            |
| 4                  | 3,00,000                                       | 13,00,000            |
| 5                  | 4,00,000                                       | 12,00,000            |

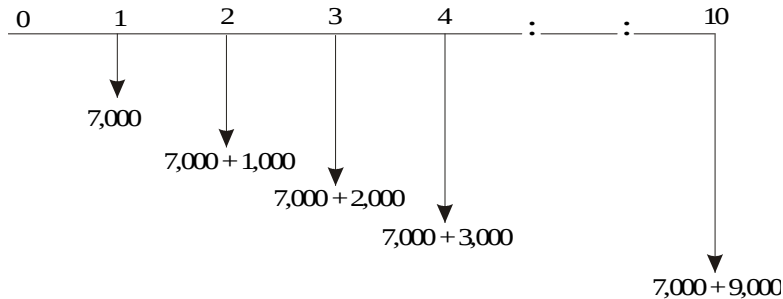
2. Find the present worth of the following cash flow series. Assume  $i = 15\%$ , compounded annually.

| <i>End of year</i>     | 0       | 1      | 2      | 3      | 4      | 5      |
|------------------------|---------|--------|--------|--------|--------|--------|
| <i>Cash flow (Rs.)</i> | -10,000 | 30,000 | 30,000 | 30,000 | 30,000 | 30,000 |

3. Consider the following cash flow series over a 20-year period. Assuming the interest rate as 18% compounded annually, compute the present worth of the series; give your comments.

| <i>End of year</i> | <i>Cash flow (Rs.)</i> |
|--------------------|------------------------|
| 0                  | -50,00,000             |
| 1                  | 6,00,000               |
| 2                  | 6,00,000               |
| ⋮                  | ⋮                      |
| ⋮                  | ⋮                      |
| 20                 | 6,00,000               |

4. The cost of erecting an oil well is Rs. 1,50,00,000. The annual equivalent yield from the oil well is Rs. 30,00,000. The salvage value after its useful life of 10 years is Rs. 2,00,000. Assuming an interest rate of 18%, compounded annually, find out whether the erection of the oil well is financially feasible, based on the present worth method.
5. The details of the feasibility report of a project are as shown below. Check the feasibility of the project based on present worth method, using  $i = 20\%$ .  
 Initial outlay = Rs. 50,00,000  
 Life of the project = 20 years.  
 Annual equivalent revenue = Rs. 15,00,000  
 Modernizing cost at the end of the 10th year = Rs. 20,00,000  
 Salvage value at the end of project life = Rs. 5,00,000.
6. Consider the following cash flow diagram. Find the present worth using an interest rate of 15%, compounded annually.



7. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually.  
 You are asked to advise the best alternative for the customers based on the present worth method of comparison.
8. The cash flows of two project proposals are as given below. Each of the project has an expected life of 10 years. Select the best project based on present worth method of comparison using an interest rate of 18%, compounded annually.

|           | <i>Initial<br/>outlay</i><br>(Rs.) | <i>Annual<br/>equivalent<br/>revenue</i><br>(Rs.) | <i>Salvage<br/>value after<br/>10 years</i><br>(Rs.) |
|-----------|------------------------------------|---------------------------------------------------|------------------------------------------------------|
| Project 1 | -7,50,000                          | 2,00,000                                          | 50,000                                               |
| Project 2 | -9,50,000                          | 2,25,000                                          | 1,00,000                                             |

9. A company has two alternatives for satisfying its daily travel requirements of its employees for the next five years:

*Alternative 1:* Renting a vehicle at a cost of Rs. 10,00,000 per year.

*Alternative 2:* Buying a vehicle for Rs. 5,00,000 with an operating and maintenance cost of Rs. 3,50,000 per year. The salvage value of the vehicle after five years is Rs. 1,00,000.

Select the best alternative based on the present worth method of comparison using the interest rate of 20%, compounded annually.

10. A working woman is planning for her retired life. She has 20 more years of service. She would like to have an annual equivalent amount of Rs. 3,00,000, starting from the end of the first year of her retirement. Find the single amount that should be deposited now so that she receives the above mentioned annual equivalent amount at the end of every year for 20 years after her retirement. Assume  $i = 15\%$ , compounded annually.

# 5

## FUTURE WORTH METHOD

### 5.1 INTRODUCTION

In the future worth method of comparison of alternatives, the future worth of various alternatives will be computed. Then, the alternative with the maximum future worth of net revenue or with the minimum future worth of net cost will be selected as the best alternative for implementation.

### 5.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the future worth method of comparison is presented in Fig. 5.1.

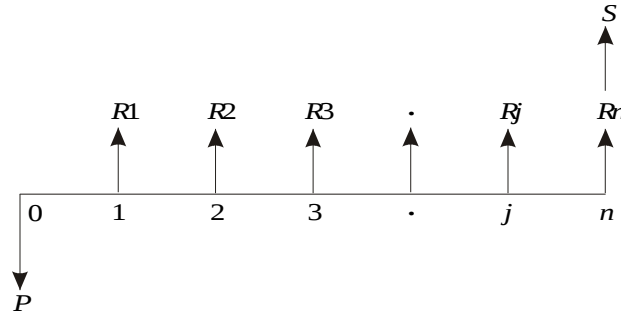


Fig. 5.1 Revenue-dominated cash flow diagram.

In Fig. 5.1,  $P$  represents an initial investment,  $R_j$  the net-revenue at the end of the  $j$ th year, and  $S$  the salvage value at the end of the  $n$ th year.

The formula for the future worth of the above cash flow diagram for a given interest rate,  $i$  is

$$FW(i) = -P(1 + i)^n + R1(1 + i)^{n-1} + R2(1 + i)^{n-2} + \dots + Rj(1 + i)^{n-j} + \dots + Rn + S$$

In the above formula, the expenditure is assigned with negative sign and the revenues are assigned with positive sign.

If we have some more alternatives which are to be compared with this

alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the maximum future worth amount should be selected as the best alternative.

### 5.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the future worth method of comparison is given in Fig. 5.2.

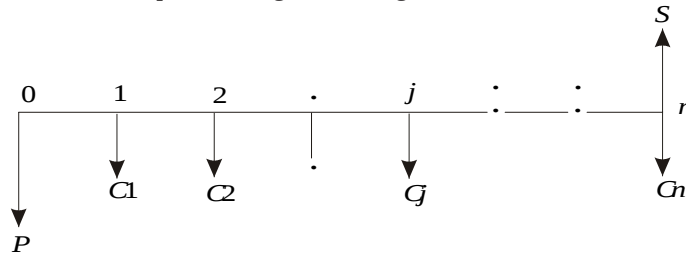


Fig. 5.2 Cost-dominated cash flow diagram.

In Fig. 5.2,  $P$  represents an initial investment,  $C_j$  the net cost of operation and maintenance at the end of the  $j$ th year, and  $S$  the salvage value at the end of the  $n$ th year.

The formula for the future worth of the above cash flow diagram for a given interest rate,  $i$  is

$$FW(i) = P(1 + i)^n + C_1(1 + i)^{n-1} + C_2(1 + i)^{n-2} + \dots + C_j(1 + i)^{n-j} + \dots + C_n - S$$

In this formula, the expenditures are assigned with positive sign and revenues with negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the minimum future worth amount should be selected as the best alternative.

### 5.4 EXAMPLES

In this section, several examples highlighting the applications of the future worth method of comparison are presented.

**EXAMPLE 5.1** Consider the following two mutually exclusive alternatives:

| Alternative | End of year |           |           |           |           |
|-------------|-------------|-----------|-----------|-----------|-----------|
|             | 0           | 1         | 2         | 3         | 4         |
| A (Rs.)     | -50,00,000  | 20,00,000 | 20,00,000 | 20,00,000 | 20,00,000 |
| B (Rs.)     | -45,00,000  | 18,00,000 | 18,00,000 | 18,00,000 | 18,00,000 |

At  $i = 18\%$ , select the best alternative based on future worth method of comparison.

**Solution Alternative A**

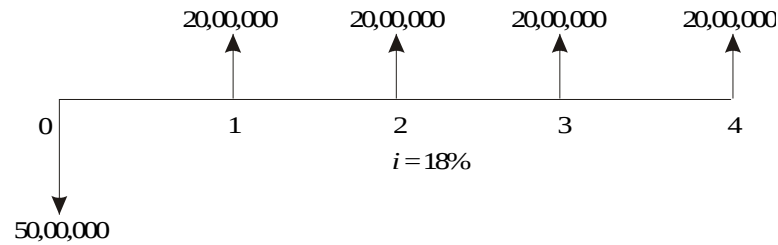
Initial investment,  $P = \text{Rs. } 50,00,000$

Annual equivalent revenue,  $A = \text{Rs. } 20,00,000$

Interest rate,  $i = 18\%$ , compounded annually

Life of alternative A = 4 years

The cash flow diagram of alternative A is shown in Fig. 5.3.



**Fig. 5.3** Cash flow diagram for alternative A.

The future worth amount of alternative B is computed as

$$\begin{aligned} FW_A(18\%) &= -50,00,000(F/P, 18\%, 4) + 20,00,000(F/A, 18\%, 4) \\ &= -50,00,000(1.939) + 20,00,000(5.215) \\ &= \text{Rs. } 7,35,000 \end{aligned}$$

**Alternative B**

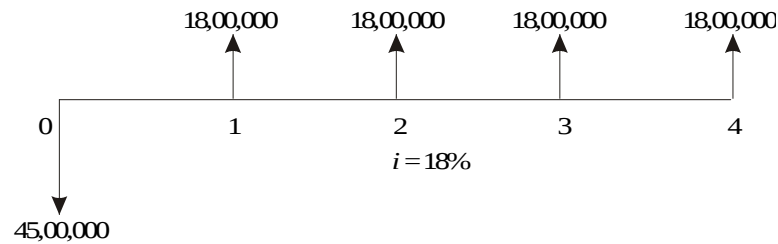
Initial investment,  $P = \text{Rs. } 45,00,000$

Annual equivalent revenue,  $A = \text{Rs. } 18,00,000$

Interest rate,  $i = 18\%$ , compounded annually

Life of alternative B = 4 years

The cash flow diagram of alternative B is illustrated in Fig. 5.4.



**Fig. 5.4** Cash flow diagram for alternative B.

The future worth amount of alternative B is computed as

$$\begin{aligned} FW_B(18\%) &= -45,00,000(F/P, 18\%, 4) + 18,00,000 (F/A, 18\%, 4) \\ &= -45,00,000(1.939) + 18,00,000(5.215) \\ &= \text{Rs. } 6,61,500 \end{aligned}$$

The future worth of alternative A is greater than that of alternative B. Thus, alternative A should be selected.

**EXAMPLE 5.2** A man owns a corner plot. He must decide which of the several alternatives to select in trying to obtain a desirable return on his investment. After much study and calculation, he decides that the two best alternatives are as given in the following table:

|                             | <i>Build<br/>gas station</i> | <i>Build soft<br/>ice-cream stand</i> |
|-----------------------------|------------------------------|---------------------------------------|
| First cost (Rs.)            | 20,00,000                    | 36,00,000                             |
| Annual property taxes (Rs.) | 80,000                       | 1,50,000                              |
| Annual income (Rs.)         | 8,00,000                     | 9,80,000                              |
| Life of building (years)    | 20                           | 20                                    |
| Salvage value (Rs.)         | 0                            | 0                                     |

Evaluate the alternatives based on the future worth method at  $i = 12\%$ .

**Alternative 1—Build gas station**

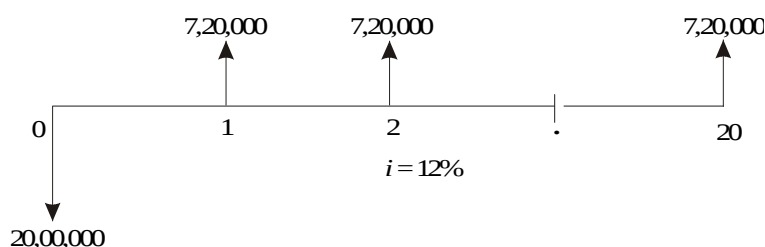
First cost = Rs. 20,00,000

Net annual income = Annual income – Annual property tax  
 = Rs. 8,00,000 – Rs. 80,000  
 = Rs. 7,20,000

Life = 20 years

Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is depicted in Fig. 5.5.



**Fig. 5.5** Cash flow diagram for alternative 1.

The future worth of alternative 1 is computed as

$$\begin{aligned}
 FW_1(12\%) &= -20,00,000(F/P, 12\%, 20) + 7,20,000(F/A, 12\%, 20) \\
 &= -20,00,000(9.646) + 7,20,000(72.052) \\
 &= \text{Rs. } 3,25,85,440
 \end{aligned}$$

**Alternative 2—Build soft ice-cream stand**

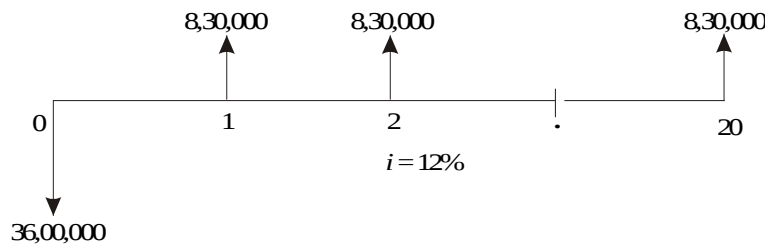
First cost = Rs. 36,00,000

Net annual income = Annual income – Annual property tax  
 = Rs. 9,80,000 – Rs. 1,50,000  
 = Rs. 8,30,000

Life = 20 years

Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is shown in Fig. 5.6.



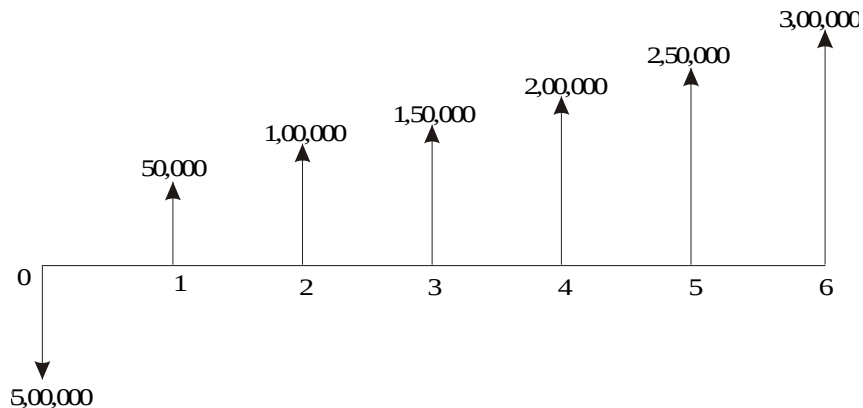
**Fig. 5.6** Cash flow diagram for alternative 2.

The future worth of alternative 2 is calculated as

$$\begin{aligned} FW_2(12\%) &= -36,00,000(F/P, 12\%, 20) + 8,30,000(F/A, 12\%, 20) \\ &= -36,00,000(9.646) + 8,30,000(72.052) \\ &= \text{Rs. } 2,50,77,560 \end{aligned}$$

The future worth of alternative 1 is greater than that of alternative 2. Thus, building the gas station is the best alternative.

**EXAMPLE 5.3** The cash flow diagram of two mutually exclusive alternatives are given in Figs. 5.7 and 5.8.



**Fig. 5.7** Cash flow diagram for alternative 1.

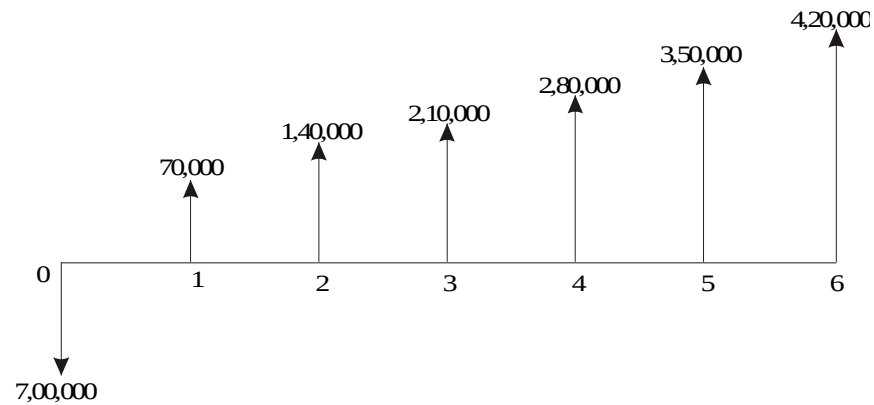


Fig. 5.8 Cash flow diagram for alternative 2.

- (a) Select the best alternative based on future worth method at  $i = 8\%$ .  
 (b) Rework part (a) with  $i = 9\%$  and  $20\%$

**(a) Evaluation at  $i = 8\%$**

**Alternative 1—This comes under equal payment gradient series.**

$$P = \text{Rs. } 5,00,000$$

$$A_1 = \text{Rs. } 50,000$$

$$G = \text{Rs. } 50,000$$

$$i = 8\%$$

$$n = 6 \text{ years}$$

The formula for the future worth of alternative 1 is

$$\begin{aligned} FW_1(8\%) &= -P(F/P, 8\%, 6) + [A_1 + G(A/G, 8\%, 6)] \times (F/A, 8\%, 6) \\ &= -5,00,000(1.587) + [50,000 + 50,000(2.2764)] \times 7.336 \\ &= -79,35,000 + 1,63,820 \times 7.336 \\ &= -79,35,000 + 12,01,784 \\ &= \text{Rs. } 4,08,283.52 \end{aligned}$$

**Alternative 2—This comes under equal payment gradient series.**

$$P = \text{Rs. } 7,00,000$$

$$A_1 = \text{Rs. } 70,000$$

$$G = \text{Rs. } 70,000$$

$$i = 8\%$$

$$n = 6 \text{ years}$$

The formula for the future worth of alternative 2 is

$$\begin{aligned}
 FW_2(8\%) &= -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] \times (F/A, 8\%, 6) \\
 FW_2(8\%) &= -7,00,000 \times 1.587 + [70,000 + 70,000 \times 2.2764] \times 7.336 \\
 &= -11,10,900 + 16,82,497 \\
 &= \text{Rs. } 5,71,596.93
 \end{aligned}$$

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

**(b) (i) Evaluation at  $i = 9\%$ : Alternative 1**

$$\begin{aligned}
 P &= \text{Rs. } 5,00,000 \\
 A1 &= \text{Rs. } 50,000 \\
 G &= \text{Rs. } 50,000 \\
 n &= 6 \text{ years}
 \end{aligned}$$

The formula for the future worth of alternative 1 is as follows:

$$\begin{aligned}
 FW_1(9\%) &= -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] \times (F/A, 9\%, 6) \\
 &= -5,00,000 (1.677) + [50,000 + 50,000 (2.2498)] \times 7.523 \\
 &= -8,38,500 + 12,22,412.27 \\
 &= \text{Rs. } 3,83,912.27
 \end{aligned}$$

**Alternative 2**

$$\begin{aligned}
 P &= \text{Rs. } 7,00,000 \\
 A1 &= \text{Rs. } 70,000 \\
 G &= \text{Rs. } 70,000 \\
 n &= 6 \text{ years}
 \end{aligned}$$

The formula for the future worth of the alternative 2 is

$$\begin{aligned}
 FW_2(9\%) &= -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] \times (F/A, 9\%, 6) \\
 &= -7,00,000 \times 1.677 + [70,000 + 70,000 \times 2.2498] \times 7.523 \\
 &= -11,73,900 + 17,11,377.18 \\
 &= \text{Rs. } 5,37,477.18
 \end{aligned}$$

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

**(ii) Evaluation at  $i = 20\%$ : Alternative 1**

$$\begin{aligned}
 P &= \text{Rs. } 5,00,000 \\
 A1 &= \text{Rs. } 50,000 \\
 G &= \text{Rs. } 50,000 \\
 n &= 6 \text{ years}
 \end{aligned}$$

The formula for the future worth of alternative 1 is

$$\begin{aligned}
 FW_1(20\%) &= -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \times (F/A, 20\%, 6) \\
 &= -5,00,000(2.986) + [50,000 + 50,000(1.9788)] \times 9.93 \\
 &= -14,93,000 + 14,78,974.20 \\
 &= \text{Rs. } -14,025.80
 \end{aligned}$$

The negative sign of the future worth amount indicates that alternative 1 incurs loss.

#### **Alternative 2**

$$P = \text{Rs. } 7,00,000$$

$$A1 = \text{Rs. } 70,000$$

$$G = \text{Rs. } 70,000$$

$$n = 6 \text{ years}$$

The formula for the future worth of alternative 2 is

$$\begin{aligned}
 FW_2(20\%) &= -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \times (F/A, 20\%, 6) \\
 &= -7,00,000 \times 2.986 + [70,000 + 70,000 \times 1.9788] \times 9.93 \\
 &= -20,90,200 + 20,70,563.88 \\
 &= \text{Rs. } -19,636.12
 \end{aligned}$$

The negative sign of the above future worth amount indicates that alternative 2 incurs loss. Thus, none of the two alternatives should be selected.

**EXAMPLE 5.4** M/S Krishna Castings Ltd. is planning to replace its annealing furnace. It has received tenders from three different original manufacturers of annealing furnace. The details are as follows.

|                                             | <i>Manufacturer</i> |           |           |
|---------------------------------------------|---------------------|-----------|-----------|
|                                             | 1                   | 2         | 3         |
| Initial cost (Rs.)                          | 80,00,000           | 70,00,000 | 90,00,000 |
| Life (years)                                | 12                  | 12        | 12        |
| Annual operation and maintenance cost (Rs.) | 8,00,000            | 9,00,000  | 8,50,000  |
| Salvage value after 12 years                | 5,00,000            | 4,00,000  | 7,00,000  |

Which is the best alternative based on future worth method at  $i = 20\%$ ?

#### **Solution Alternative 1—Manufacturer 1**

$$\text{First cost, } P = \text{Rs. } 80,00,000$$

$$\text{Life, } n = 12 \text{ years}$$

Annual operating and maintenance cost,  $A = \text{Rs. } 8,00,000$

Salvage value at the end of furnace life = Rs. 5,00,000

The cash flow diagram for this alternative is shown in Fig. 5.9.

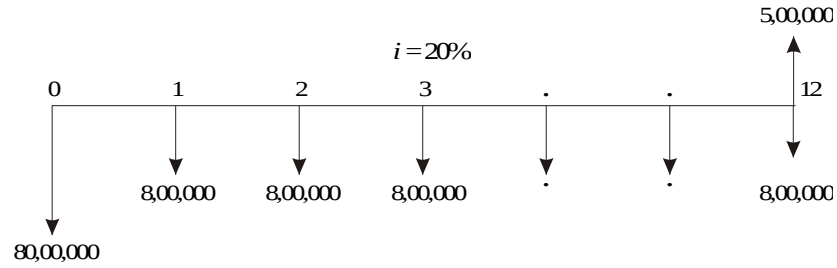


Fig. 5.9 Cash flow diagram for manufacturer 1.

The future worth amount of alternative 1 is computed as

$$\begin{aligned} FW_1(20\%) &= 80,00,000(F/P, 20\%, 12) + 8,00,000(F/A, 20\%, 12) - 5,00,000 \\ &= 80,00,000(8.916) + 8,00,000(39.581) - 5,00,000 \\ &= \text{Rs. } 10,24,92,800 \end{aligned}$$

#### Alternative 2—Manufacturer 2

First cost,  $P = \text{Rs. } 70,00,000$

Life,  $n = 12$  years

Annual operating and maintenance cost,  $A = \text{Rs. } 9,00,000$

Salvage value at the end of furnace life = Rs. 4,00,000

The cash flow diagram for this alternative is given in Fig. 5.10.

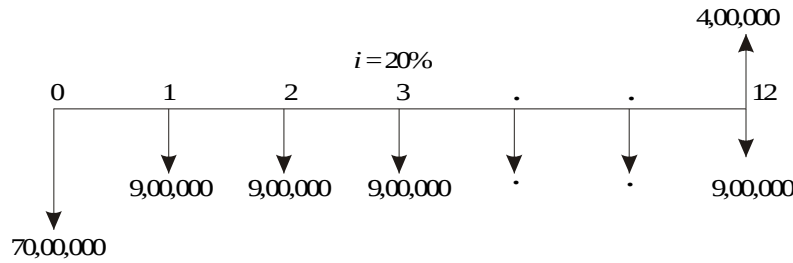


Fig. 5.10 Cash flow diagram for manufacturer 2.

The future worth amount of alternative 2 is computed as

$$\begin{aligned} FW_2(20\%) &= 70,00,000(F/P, 20\%, 12) + 9,00,000(F/A, 20\%, 12) - 4,00,000 \\ &= 70,00,000(8.916) + 9,00,000(39.581) - 4,00,000 \\ &= \text{Rs. } 9,76,34,900 \end{aligned}$$

#### Alternative 3—Manufacturer 3

First cost,  $P = \text{Rs. } 90,00,000$

Life,  $n = 12$  years

Annual operating and maintenance cost,  $A = \text{Rs. } 8,50,000$

Salvage value at the end of furnace life = Rs. 7,00,000

The cash flow diagram for this alternative is illustrated in Fig. 5.11.

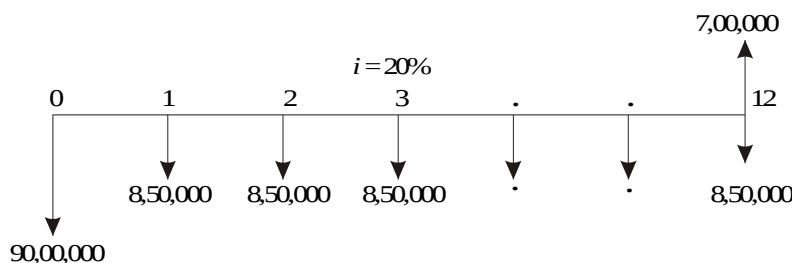


Fig. 5.11 Cash flow diagram for manufacturer 3.

The future worth amount of alternative 3 is calculated as

$$\begin{aligned}
 FW_3(20\%) &= 90,00,000(F/P, 20\%, 12) + 8,50,000(F/A, 20\%, 12) - 7,00,000 \\
 &= 90,00,000(8.916) + 8,50,000(39.581) - 7,00,000 \\
 &= \text{Rs. } 11,31,87,850
 \end{aligned}$$

The future worth cost of alternative 2 is less than that of the other two alternatives. Therefore, M/s. Krishna castings should buy the annealing furnace from manufacturer 2.

**EXAMPLE 5.5** A company must decide whether to buy machine A or machine B:

|                                          | Machine A    | Machine B    |
|------------------------------------------|--------------|--------------|
| Initial cost                             | Rs. 4,00,000 | Rs. 8,00,000 |
| Useful life, in years                    | 4            | 4            |
| Salvage value at the end of machine life | Rs. 2,00,000 | Rs. 5,50,000 |
| Annual maintenance cost                  | Rs. 40,000   | 0            |

At 12% interest rate, which machine should be selected? (Use future worth method of comparison).

**Solution Machine A**

Initial cost of the machine,  $P = \text{Rs. } 4,00,000$

Life,  $n = 4$  years

Salvage value at the end of machine life,  $S = \text{Rs. } 2,00,000$

Annual maintenance cost,  $A = \text{Rs. } 40,000$

Interest rate,  $i = 12\%$ , compounded annually.

The cash flow diagram of machine A is given in Fig. 5.12.

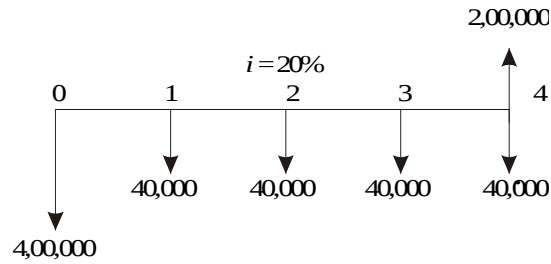


Fig. 5.12 Cash flow diagram for machine A.

The future worth function of Fig. 5.12 is

$$\begin{aligned}
 FW_A(12\%) &= 4,00,000 \times (F/P, 12\%, 4) + 40,000 \times (F/A, 12\%, 4) - 2,00,000 \\
 &= 4,00,000 \times (1.574) + 40,000 \times (4.779) - 2,00,000 \\
 &= \text{Rs. } 6,20,760
 \end{aligned}$$

### Machine B

Initial cost of the machine,  $P = \text{Rs. } 8,00,000$

Life,  $n = 4$  years

Salvage value at the end of machine life,  $S = \text{Rs. } 5,50,000$

Annual maintenance cost,  $A = \text{zero}$ .

Interest rate,  $i = 12\%$ , compounded annually.

The cash flow diagram of the machine B is illustrated in Fig. 5.13.

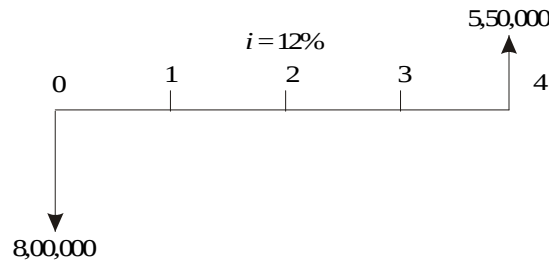


Fig. 5.13 Cash flow diagram for machine B.

The future worth function of Fig 5.13 is

$$\begin{aligned}
 FW_B(12\%) &= 8,00,000 \times (F/P, 12\%, 4) - 5,50,000 \\
 &= 8,00,000 \times (1.574) - 5,50,000 \\
 &= \text{Rs. } 7,09,200
 \end{aligned}$$

The future worth cost of machine A is less than that of machine B. Therefore, machine A should be selected.

## QUESTIONS

1. A suburban taxi company is considering buying taxis with diesel engines instead of petrol engines. The cars average 50,000 km a year, with a useful life of three years for the taxi with the petrol engine and four years for the diesel taxi. Other comparative information are as follows:

|                                             | <i>Diesel</i> | <i>Petrol</i> |
|---------------------------------------------|---------------|---------------|
| Vehicle cost                                | Rs. 5,00,000  | Rs. 4,00,000  |
| Fuel cost per litre                         | Rs. 9.00      | Rs. 24.00     |
| Mileage, in km/litre                        | 30            | 20            |
| Annual insurance premium                    | Rs. 500       | Rs. 500       |
| Salvage value at the end<br>of vehicle life | Rs. 70,000    | Rs. 1,00,000  |

Determine the more economical choice based on the future worth method of comparison if the interest rate is 15%, compounded annually.

2. A motorcycle is sold for Rs. 50,000. The motorcycle dealer is willing to sell it on the following terms:
- Make no down payment but pay Rs. 1,500 at the end of each of the first four months and Rs. 3,000 at the end of each month after that for 18 continuous months.
  - Make no down payment but pay a total amount of Rs. 90,000 at the end of the 22nd month; till that time the buyer should mortgage property worth of Rs. 50,000, at present.

Based on these terms and a 12% annual interest rate compounded monthly, find the best alternative for the buyer based on the future worth method of comparison.

3. Consider the following two mutually exclusive alternatives.

|                        | <i>A</i>  | <i>B</i>  |
|------------------------|-----------|-----------|
| Cost                   | Rs. 4,000 | Rs. 6,000 |
| Uniform annual benefit | Rs. 640   | Rs. 960   |
| Useful life (years)    | 20        | 20        |

Using a 15% interest rate, determine which alternative should be selected based on the future worth method of comparison.

4. A company must decide whether to buy machine A or machine B:

|                                             | <i>Machine A</i> | <i>Machine B</i> |
|---------------------------------------------|------------------|------------------|
| Initial cost                                | Rs. 4,00,000     | Rs. 8,00,000     |
| Useful life, (years)                        | 5                | 5                |
| Salvage value at the end<br>of machine life | Rs. 2,00,000     | Rs. 5,50,000     |
| Annual maintenance cost                     | Rs. 40,000       | 0                |

At 15% interest rate, which machine should be selected? (Use the future worth method of comparison.)

5. Due to increasing awareness of customers, two different television manufacturing companies started a marketing war. The details of advertisements of the companies are as follows:

|                                        | <i>Brand X</i> | <i>Brand Y</i> |
|----------------------------------------|----------------|----------------|
| Selling price of a TV set              | Rs. 15,000     | Rs. 10,000     |
| Amount returned to buyer after 5 years | Rs. 8,000      | –              |

Select the most economical brand from the customer's point of view using the future worth method of comparison, assuming an interest rate of 15%, compounded annually.

6. Alpha Finance Company is coming with an option of accepting Rs. 10,000 now and paying a sum of Rs. 1,60,000 after 20 years. Beta Finance Company is coming with a similar option of accepting Rs. 10,000 now and paying a sum of Rs. 3,00,000 after 25 years. Compare and select the best alternative based on the future worth method of comparison with 15% interest rate, compounded annually.
7. An insurance company gives an endowment policy for a person aged 30 years. The yearly premium for an insured sum of Rs. 1,00,000 is Rs. 4,000. The policy will mature after 25 years. Also, the person is entitled for a bonus of Rs. 75 per thousand per year at the end of the policy. If a person survives till the end of the 25th year:
- What will be the total sum that he will get from the insurance company at that time?
  - Instead of paying the premiums for the insurance policy, if the person invests an equal sum of Rs. 4,000 at the end of each year for the next 25 years in some other scheme which is having similar tax benefit, find the future worth of the investment at 15% interest rate, compounded annually.
  - Rate the above alternatives assuming that the person is sure of living for the next 25 years.

# 6

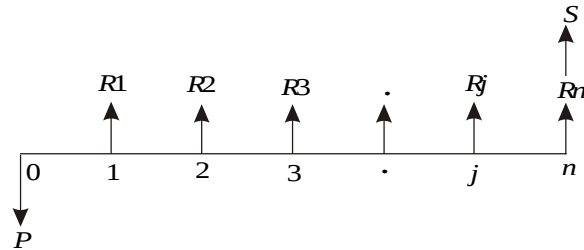
## ANNUAL EQUIVALENT METHOD

### 6.1 INTRODUCTION

In the annual equivalent method of comparison, first the annual equivalent cost or the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent revenue in the case of revenue-based comparison or with the minimum annual equivalent cost in the case of cost-based comparison will be selected as the best alternative.

### 6.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is presented in Fig. 6.1.



**Fig. 6.1** Revenue-dominated cash flow diagram.

In Fig. 6.1,  $P$  represents an initial investment,  $R_j$  the net revenue at the end of the  $j$ th year, and  $S$  the salvage value at the end of the  $n$ th year.

The first step is to find the net present worth of the cash flow diagram using the following expression for a given interest rate,  $i$ :

$$PW(i) = -P + R1/(1 + i)^1 + R2/(1 + i)^2 + \dots + Rj/(1 + i)^j + \dots + Rn/(1 + i)^n + S/(1 + i)^n$$

In the above formula, the expenditure is assigned with a negative sign and the revenues are assigned with a positive sign.

In the second step, the annual equivalent revenue is computed using the following formula:

$$A = PW(i) \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= PW(i) (A/P, i, n)$$

where  $(A/P, i, n)$  is called *equal payment series capital recovery factor*.

If we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent revenues are to be computed and compared. Finally, the alternative with the maximum annual equivalent revenue should be selected as the best alternative.

### 6.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is illustrated in Fig. 6.2.

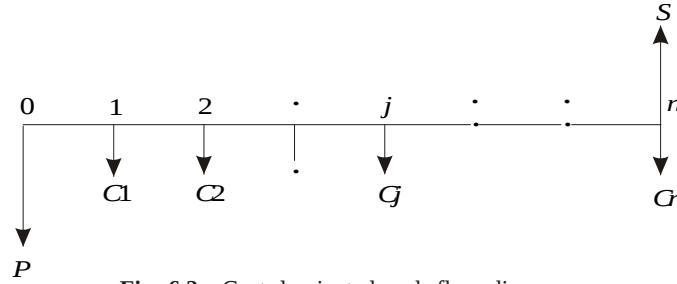


Fig. 6.2 Cost-dominated cash flow diagram.

In Fig. 6.2,  $P$  represents an initial investment,  $C_j$  the net cost of operation and maintenance at the end of the  $j$ th year, and  $S$  the salvage value at the end of the  $n$ th year.

The first step is to find the net present worth of the cash flow diagram using the following relation for a given interest rate,  $i$ .

$$PW(i) = P + C1/(1+i)^1 + C2/(1+i)^2 + \dots$$

$$+ Cj/(1+i)^j + \dots + Cn/(1+i)^n - S/(1+i)^n$$

In the above formula, each expenditure is assigned with positive sign and the salvage value with negative sign. Then, in the second step, the annual equivalent cost is computed using the following equation:

$$A = PW(i) \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= PW(i) (A/P, i, n)$$

where  $(A/P, i, n)$  is called as equal-payment series capital recovery factor.

As in the previous case, if we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent costs are to be computed and compared. Finally, the alternative with the minimum annual equivalent cost should be selected as the best alternative.

If we have some non-standard cash flow diagram, then we will have to follow the general procedure for converting each and every transaction to time zero and then convert the net present worth into an annual equivalent cost/revenue depending on the type of the cash flow diagram. Such procedure is to be applied to all the alternatives and finally, the best alternative is to be selected.

## 6.4 ALTERNATE APPROACH

Instead of first finding the present worth and then figuring out the annual equivalent cost/revenue, an alternate method which is as explained below can be used. In each of the cases presented in Sections 6.2 and 6.3, in the first step, one can find the future worth of the cash flow diagram of each of the alternatives. Then, in the second step, the annual equivalent cost/revenue can be obtained by using the equation:

$$A = F \frac{i}{(1+i)^n - 1}$$

$$= F(A/F, i, n)$$

where  $(A/F, i, n)$  is called *equal-payment series sinking fund factor*.

## 6.5 EXAMPLES

In this section, the application of the annual equivalent method is demonstrated with several numerical examples.

**EXAMPLE 6.1** A company provides a car to its chief executive. The owner of the company is concerned about the increasing cost of petrol. The cost per litre of petrol for the first year of operation is Rs. 21. He feels that the cost of petrol will be increasing by Re.1 every year. His experience with his company car indicates that it averages 9 km per litre of petrol. The executive expects to drive an average of 20,000 km each year for the next four years. What is the annual equivalent cost of fuel over this period of time?. If he is offered similar service with the same quality on rental basis at Rs. 60,000 per year, should the owner continue to provide company car for his executive or alternatively provide a rental car to his executive? Assume  $i = 18\%$ . If the rental car is preferred, then the company car will find some other use within the company.

### **Solution**

Average number of km run/year = 20,000 km

Number of km/litre of petrol = 9 km

Therefore,

$$\text{Petrol consumption/year} = 20,000/9 = 2222.2 \text{ litre}$$

$$\text{Cost/litre of petrol for the 1st year} = \text{Rs. } 21$$

$$\begin{aligned} \text{Cost/litre of petrol for the 2nd year} &= \text{Rs. } 21.00 + \text{Re. } 1.00 \\ &= \text{Rs. } 22.00 \end{aligned}$$

$$\begin{aligned} \text{Cost/litre of petrol for the 3rd year} &= \text{Rs. } 22.00 + \text{Re. } 1.00 \\ &= \text{Rs. } 23.00 \end{aligned}$$

$$\begin{aligned} \text{Cost/litre of petrol for the 4th year} &= \text{Rs. } 23.00 + \text{Re. } 1.00 \\ &= \text{Rs. } 24.00 \end{aligned}$$

$$\text{Fuel expenditure for 1st year} = 2222.2 \times 21 = \text{Rs. } 46,666.20$$

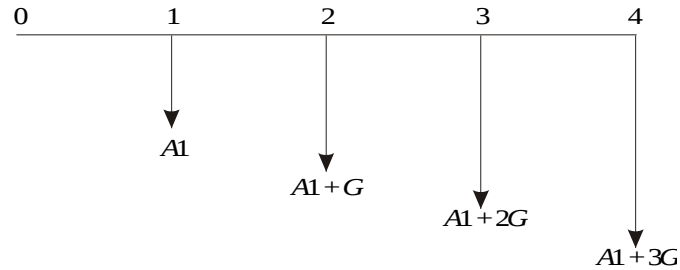
$$\text{Fuel expenditure for 2nd year} = 2222.2 \times 22 = \text{Rs. } 48,888.40$$

$$\text{Fuel expenditure for 3rd year} = 2222.2 \times 23 = \text{Rs. } 51,110.60$$

$$\text{Fuel expenditure for 4th year} = 2222.2 \times 24 = \text{Rs. } 53,332.80$$

The annual equal increment of the above expenditures is Rs. 2,222.20 (G).

The cash flow diagram for this situation is depicted in Fig. 6.3.



**Fig. 6.3** Uniform gradient series cash flow diagram.

In Fig. 6.3,  $A_1 = \text{Rs. } 46,666.20$  and  $G = \text{Rs. } 2,222.20$

$$\begin{aligned} A &= A_1 + G(A/G, 18\%, 4) \\ &= 46,666.20 + 2222.2(1.2947) \\ &= \text{Rs. } 49,543.28 \end{aligned}$$

The proposal of using the company car by spending for petrol by the company will cost an annual equivalent amount of Rs. 49,543.28 for four years. This amount is less than the annual rental value of Rs. 60,000. Therefore, the company should continue to provide its own car to its executive.

**EXAMPLE 6.2** A company is planning to purchase an advanced machine centre. Three original manufacturers have responded to its tender whose particulars are tabulated as follows:

| Manufacturer | Down payment<br>(Rs.) | Yearly equal<br>installment<br>(Rs.) | No. of<br>installments |
|--------------|-----------------------|--------------------------------------|------------------------|
| 1            | 5,00,000              | 2,00,000                             | 15                     |
| 2            | 4,00,000              | 3,00,000                             | 15                     |
| 3            | 6,00,000              | 1,50,000                             | 15                     |

Determine the best alternative based on the annual equivalent method by assuming  $i = 20\%$ , compounded annually.

**Solution Alternative 1**

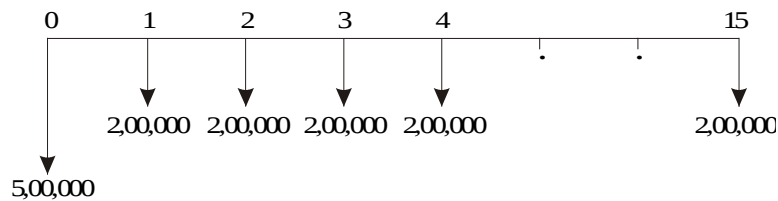
Down payment,  $P = \text{Rs. } 5,00,000$

Yearly equal installment,  $A = \text{Rs. } 2,00,000$

$n = 15$  years

$i = 20\%$ , compounded annually

The cash flow diagram for manufacturer 1 is shown in Fig. 6.4.



**Fig. 6.4** Cash flow diagram for manufacturer 1.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_1(20\%) &= 5,00,000(A/P, 20\%, 15) + 2,00,000 \\ &= 5,00,000(0.2139) + 2,00,000 \\ &= 3,06,950 \end{aligned}$$

**Alternative 2**

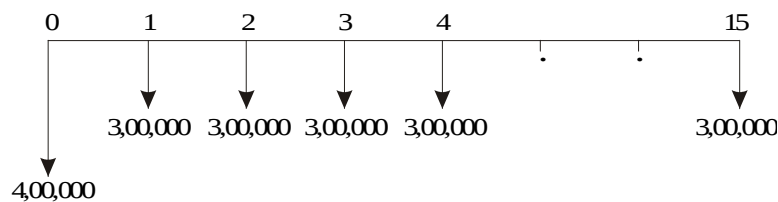
Down payment,  $P = \text{Rs. } 4,00,000$

Yearly equal installment,  $A = \text{Rs. } 3,00,000$

$n = 15$  years

$i = 20\%$ , compounded annually

The cash flow diagram for the manufacturer 2 is shown in Fig. 6.5.



**Fig. 6.5** Cash flow diagram for manufacturer 2.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_2(20\%) &= 4,00,000(A/P, 20\%, 15) + 3,00,000 \\ &= 4,00,000(0.2139) + 3,00,000 \\ &= \text{Rs. } 3,85,560. \end{aligned}$$

**Alternative 3**

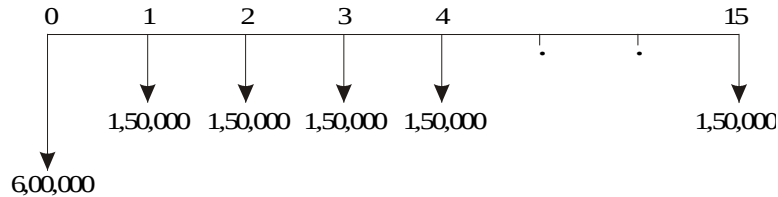
Down payment,  $P = \text{Rs. } 6,00,000$

Yearly equal installment,  $A = \text{Rs. } 1,50,000$

$n = 15$  years

$i = 20\%$ , compounded annually

The cash flow diagram for manufacturer 3 is shown in Fig. 6.6.



**Fig. 6.6** Cash flow diagram for manufacturer 3.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_3(20\%) &= 6,00,000(A/P, 20\%, 15) + 1,50,000 \\ &= 6,00,000(0.2139) + 1,50,000 \\ &= \text{Rs. } 2,78,340. \end{aligned}$$

The annual equivalent cost of manufacturer 3 is less than that of manufacturer 1 and manufacturer 2. Therefore, the company should buy the advanced machine centre from manufacturer 3.

**EXAMPLE 6.3** A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values.

|                           | Alternative |            |
|---------------------------|-------------|------------|
|                           | A           | B          |
| Investment (Rs.)          | – 1,50,000  | – 1,75,000 |
| Annual equal return (Rs.) | + 60,000    | + 70,000   |
| Salvage value (Rs.)       | + 15,000    | + 35,000   |

Determine the best alternative based on the annual equivalent method by assuming  $i = 25\%$ .

**Solution Alternative A**

Initial investment,  $P = \text{Rs. } 1,50,000$

Annual equal return,  $A = \text{Rs. } 60,000$

Salvage value at the end of machine life,  $S = \text{Rs. } 15,000$

Life = 5 years

Interest rate,  $i = 25\%$ , compounded annually

The cash flow diagram for alternative A is shown in Fig. 6.7.

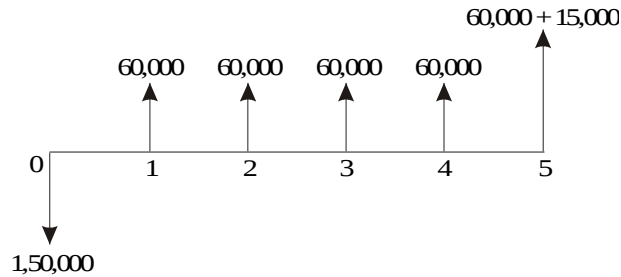


Fig. 6.7 Cash flow diagram for alternative A.

The annual equivalent revenue expression of the above cash flow diagram is as follows:

$$\begin{aligned}
 AE_A(25\%) &= -1,50,000(A/P, 25\%, 5) + 60,000 + 15,000(A/F, 25\%, 5) \\
 &= -1,50,000(0.3718) + 60,000 + 15,000(0.1218) \\
 &= \text{Rs. } 6,057
 \end{aligned}$$

#### Alternative B

Initial investment,  $P = \text{Rs. } 1,75,000$

Annual equal return,  $A = \text{Rs. } 70,000$

Salvage value at the end of machine life,  $S = \text{Rs. } 35,000$

Life = 5 years

Interest rate,  $i = 25\%$ , compounded annually

The cash flow diagram for alternative B is shown in Fig. 6.8.

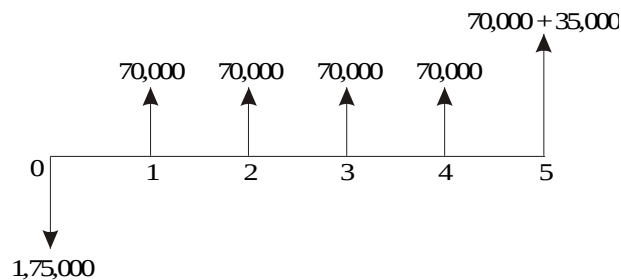


Fig. 6.8 Cash flow diagram for alternative B.

The annual equivalent revenue expression of the above cash flow diagram is

$$\begin{aligned}
 AE_B(25\%) &= -1,75,000(A/P, 25\%, 5) + 70,000 + 35,000(A/F, 25\%, 5) \\
 &= -1,75,000(0.3718) + 70,000 + 35,000(0.1218) \\
 &= \text{Rs. } 9,198
 \end{aligned}$$

The annual equivalent net return of alternative B is more than that of alternative A. Thus, the company should select alternative B.

**EXAMPLE 6.4** A certain individual firm desires an economic analysis to determine which of the two machines is attractive in a given interval of time. The minimum attractive rate of return for the firm is 15%. The following data are to be used in the analysis:

|                         | <i>Machine X</i> | <i>Machine Y</i> |
|-------------------------|------------------|------------------|
| First cost              | Rs. 1,50,000     | Rs. 2,40,000     |
| Estimated life          | 12 years         | 12 years         |
| Salvage value           | Rs. 0            | Rs. 6,000        |
| Annual maintenance cost | Rs. 0            | Rs. 4,500        |

Which machine would you choose? Base your answer on annual equivalent cost.

**Solution Machine X**

First cost,  $P = \text{Rs. } 1,50,000$

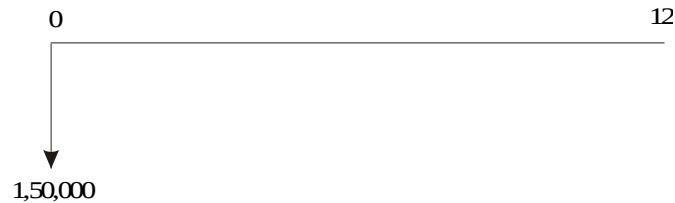
Life,  $n = 12$  years

Estimated salvage value at the end of machine life,  $S = \text{Rs. } 0$ .

Annual maintenance cost,  $A = \text{Rs. } 0$ .

Interest rate,  $i = 15\%$ , compounded annually.

The cash flow diagram of machine X is illustrated in Fig. 6.9.



**Fig. 6.9** Cash flow diagram for machine X.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE_X(15\%) &= 1,50,000(A/P, 15\%, 12) \\
 &= 1,50,000(0.1845) \\
 &= \text{Rs. } 27,675
 \end{aligned}$$

**Machine Y**

First cost,  $P = \text{Rs. } 2,40,000$

Life,  $n = 12$  years

Estimated salvage value at the end of machine life,  $S = \text{Rs. } 60,000$

Annual maintenance cost,  $A = \text{Rs. } 4,500$

Interest rate,  $i = 15\%$ , compounded annually.

The cash flow diagram of machine Y is depicted in Fig. 6.10.

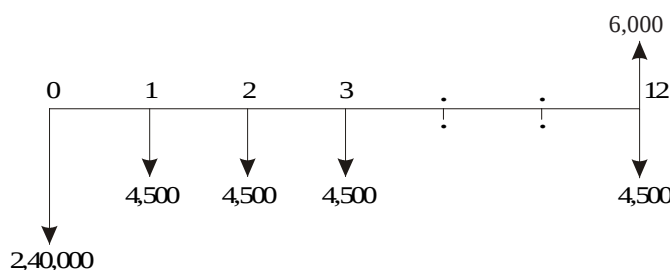


Fig. 6.10 Cash flow diagram for machine Y.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE_Y(15\%) &= 2,40,000(A/P, 15\%, 12) + 4,500 - 6,000(A/F, 15\%, 12) \\
 &= 2,40,000(0.1845) + 4,500 - 6,000(0.0345) \\
 &= \text{Rs. } 48,573
 \end{aligned}$$

The annual equivalent cost of machine X is less than that of machine Y. So, machine X is the more cost effective machine.

**EXAMPLE 6.5** Two possible routes for laying a power line are under study. Data on the routes are as follows:

|                   |         | <i>Around the lake</i> | <i>Under the lake</i> |
|-------------------|---------|------------------------|-----------------------|
| Length            |         | 15 km                  | 5 km                  |
| First cost        | (Rs.)   | 1,50,000/km            | 7,50,000/km           |
| Useful life       | (years) | 15                     | 15                    |
| Maintenance cost  | (Rs.)   | 6,000/km/yr            | 12,000/km/yr          |
| Salvage value     | (Rs.)   | 90,000/km              | 1,50,000/km           |
| Yearly power loss | (Rs.)   | 15,000/km              | 15,000/km             |

If 15% interest is used, should the power line be routed around the lake or under the lake?

**Solution Alternative 1—Around the lake**

$$\text{First cost} = 1,50,000 \times 15 = \text{Rs. } 22,50,000$$

$$\text{Maintenance cost/yr} = 6,000 \times 15 = \text{Rs. } 90,000$$

$$\text{Power loss/yr} = 15,000 \times 15 = \text{Rs. } 2,25,000$$

$$\begin{aligned}
 \text{Maintenance cost and power loss/yr} &= \text{Rs. } 90,000 + \text{Rs. } 2,25,000 \\
 &= \text{Rs. } 3,15,000
 \end{aligned}$$

$$\text{Salvage value} = 90,000 \times 15 = \text{Rs. } 13,50,000$$

The cash flow diagram for this alternative is shown in Fig. 6.11.

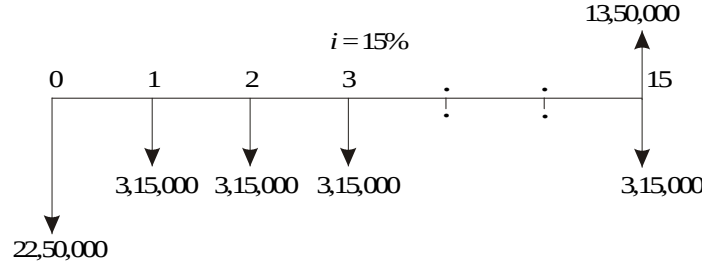


Fig. 6.11 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_1(15\%) &= 22,50,000(A/P, 15\%, 15) + 3,15,000 - 13,50,000(A/F, 15\%, 15) \\ &= 22,50,000(0.1710) + 3,15,000 - 13,50,000(0.0210) \\ &= \text{Rs. } 6,71,400 \end{aligned}$$

#### Alternative 2—Under the lake

First cost =  $7,50,000 \times 5 = \text{Rs. } 37,50,000$

Maintenance cost/yr =  $12,000 \times 5 = \text{Rs. } 60,000$

Power loss/yr =  $15,000 \times 5 = \text{Rs. } 75,000$

Maintenance cost and power loss/yr =  $\text{Rs. } 60,000 + \text{Rs. } 75,000$   
 $= \text{Rs. } 1,35,000$

Salvage value =  $1,50,000 \times 5 = \text{Rs. } 7,50,000$

The cash flow diagram for this alternative is shown in Fig. 6.12.

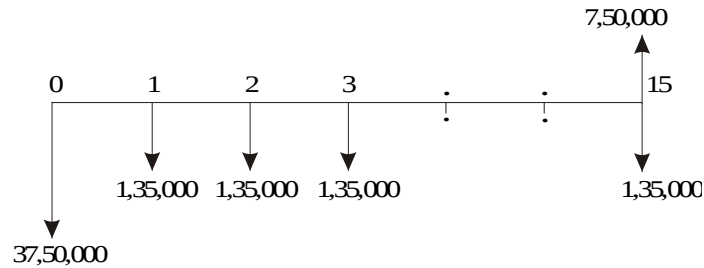


Fig. 6.12 Cash flow diagram for alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_2(15\%) &= 37,50,000(A/P, 15\%, 15) + 1,35,000 - 7,50,000(A/F, 15\%, 15) \\ &= 37,50,000(0.1710) + 1,35,000 - 7,50,000(0.0210) \\ &= \text{Rs. } 7,60,500 \end{aligned}$$

The annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, select the route around the lake for laying the power line.

**EXAMPLE 6.6** A suburban taxi company is analyzing the proposal of buying

cars with diesel engines instead of petrol engines. The cars average 60,000 km a year with a useful life of three years for the petrol taxi and four years for the diesel taxi. Other comparative details are as follows:

|                                               | <i>Diesel</i> | <i>Petrol</i> |
|-----------------------------------------------|---------------|---------------|
| Vehicle cost (Rs.)                            | 3,90,000      | 3,60,000      |
| Fuel cost per litre (Rs.)                     | 8             | 20            |
| Mileage in km/litre                           | 30            | 20            |
| Annual repairs (Rs.)                          | 9,000         | 6,000         |
| Annual insurance premium (Rs.)                | 15,000        | 15,000        |
| Resale value at the end of vehicle life (Rs.) | 60,000        | 90,000        |

Determine the more economical choice if interest rate is 20%, compounded annually.

**Solution Alternative 1— Purchase of diesel taxi**

Vehicle cost = Rs. 3,90,000

Life = 4 years

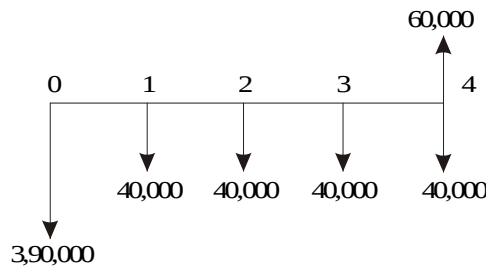
Number of litres/year  $60,000/30 = 2,000$  litres

Fuel cost/yr =  $2,000 \times 8 = \text{Rs. } 16,000$

Fuel cost, annual repairs and insurance premium/yr  
= Rs. 16,000 + Rs. 9,000 + Rs. 15,000 = Rs. 40,000

Salvage value at the end of vehicle life = Rs. 60,000

The cash flow diagram for alternative 1 is shown in Fig. 6.13.



**Fig. 6.13** Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE(20\%) &= 3,90,000(A/P, 20\%, 4) + 40,000 - 60,000(A/F, 20\%, 4) \\
 &= 3,90,000(0.3863) + 40,000 - 60,000(0.1863) \\
 &= \text{Rs. } 1,79,479
 \end{aligned}$$

**Alternative 2— Purchase of petrol taxi**

Vehicle cost = Rs. 3,60,000

Life = 3 years

Number of litres/year  $60,000/20 = 3,000$  litres

Fuel cost/yr =  $3,000 \times 20 = \text{Rs. } 60,000$

Fuel cost, annual repairs and insurance premium/yr  
 = Rs. 60,000 + Rs. 6,000 + Rs. 15,000 = Rs. 81,000  
 Salvage value at the end of vehicle life = Rs. 90,000

The cash flow diagram for alternative 2 is shown in Fig. 6.14.

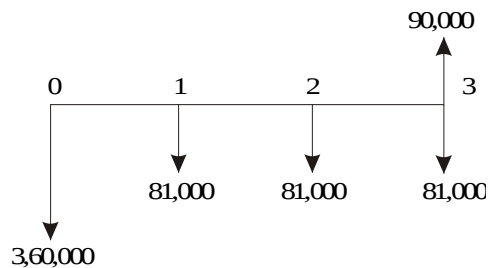


Fig. 6.14 Cash flow diagram for alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE(20\%) &= 3,60,000(A/P, 20\%, 3) + 81,000 - 90,000(A/F, 20\%, 3) \\
 &= 3,60,000(0.4747) + 81,000 - 90,000(0.2747) \\
 &= \text{Rs. } 2,27,169
 \end{aligned}$$

The annual equivalent cost of purchase and operation of the cars with diesel engine is less than that of the cars with petrol engine. Therefore, the taxi company should buy cars with diesel engine. (Note: Comparison is done on common multiple lives of 12 years.)

**EXAMPLE 6.7** Ramu, a salesman, needs a new car for use in his business. He expects that he will be promoted to a supervisory job at the end of third year and so his concern now is to have a car for the three years he expects to be “on the road”. The company will reimburse their salesman each month the fuel cost and maintenance cost. Ramu has decided to drive a low-priced automobile. He finds, however, that there are two different ways of obtaining the automobile. In either case, the fuel cost and maintenance cost are borne by the company.

- (a) Purchase for cash at Rs. 3,90,000.
- (b) Lease a car. The monthly charge is Rs. 10,500 on a 36-month lease payable at the end of each month. At the end of the three-year period, the car is returned to the leasing company.

Ramu believes that he should use a 12% interest rate compounded monthly in determining which alternative to select. If the car could be sold for Rs. 1,20,000 at the end of the third year, which option should he use to obtain it?

**Alternative 1—Purchase car for cash**

Purchase price of the car = Rs. 3,90,000

Life = 3 years = 36 months

Salvage value after 3 years = Rs. 1,20,000

Interest rate = 12% (nominal rate, compounded annually)  
 = 1% compounded monthly

The cash flow diagram for alternative 1 is shown in Fig. 6.15.

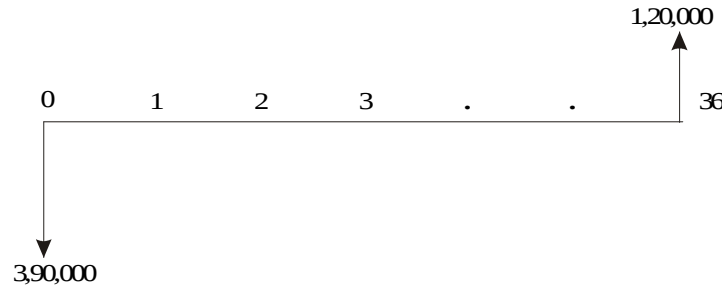


Fig. 6.15 Cash flow diagram for alternative 1.

The monthly equivalent cost expression [ $ME(1\%)$ ] of the above cash flow diagram is

$$\begin{aligned} ME(1\%) &= 3,90,000(A/P, 1\%, 36) - 1,20,000(A/F, 1\%, 36) \\ &= 3,90,000(0.0332) - 1,20,000(0.0232) \\ &= \text{Rs. } 10,164 \end{aligned}$$

**Alternative 2—Use of car under lease**

Monthly lease amount for 36 months = Rs. 10,500

The cash flow diagram for alternative 2 is illustrated in Fig. 6.16.

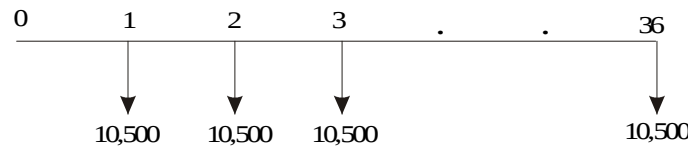


Fig. 6.16 Cash flow diagram for alternative 2.

Monthly equivalent cost = Rs.10,500.

The monthly equivalent cost of alternative 1 is less than that of alternative 2. Hence, the salesman should purchase the car for cash.

**EXAMPLE 6.8** A company must decide whether to buy machine A or machine B.

|                                                | Machine A | Machine B |
|------------------------------------------------|-----------|-----------|
| Initial cost (Rs.)                             | 3,00,000  | 6,00,000  |
| Useful life (years)                            | 4         | 4         |
| Salvage value at the end of machine life (Rs.) | 2,00,000  | 3,00,000  |
| Annual maintenance (Rs.)                       | 30,000    | 0         |

At 15% interest rate, which machine should be purchased?

**Solution Machine A**

Initial cost = Rs. 3,00,000

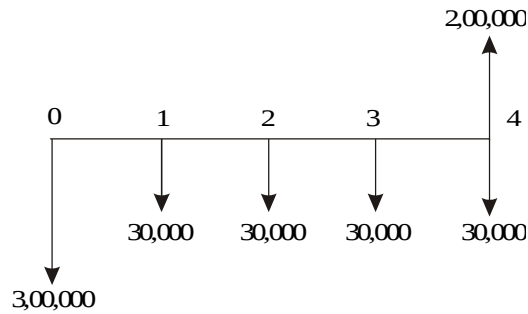
Useful life (years) = 4

Salvage value at the end of machine life = Rs. 2,00,000

Annual maintenance = Rs. 30,000

Interest rate = 15%, compounded annually

The cash flow diagram of machine A is depicted in Fig. 6.17.



**Fig. 6.17** Cash flow diagram for machine A.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE(15\%) &= 3,00,000(A/P, 15\%, 4) + 30,000 - 2,00,000(A/F, 15\%, 4) \\
 &= 3,00,000(0.3503) + 30,000 - 2,00,000(0.2003) \\
 &= \text{Rs. } 95,030
 \end{aligned}$$

**Machine B**

Initial cost = Rs. 6,00,000

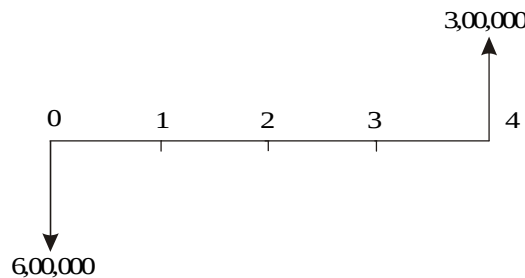
Useful life (years) = 4

Salvage value at the end of machine life = Rs. 3,00,000

Annual maintenance = Rs. 0.

Interest rate = 15%, compounded annually

The cash flow diagram of machine B is illustrated in Fig. 6.18.



**Fig. 6.18** Cash flow diagram for machine B.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(15\%) &= 6,00,000(A/P, 15\%, 4) - 3,00,000(A/F, 15\%, 4) \\ &= 6,00,000(0.3503) - 3,00,000(0.2003) \\ &= \text{Rs. } 1,50,090 \end{aligned}$$

Since the annual equivalent cost of machine A is less than that of machine B, it is advisable to buy machine A.

**EXAMPLE 6.9** Jothi Lakshmi has arranged to buy some home recording equipment. She estimates that it will have a five year useful life and no salvage value at the end of equipment life. The dealer, who is a friend has offered Jothi Lakshmi two alternative ways to pay for the equipment.

- (a) Pay Rs. 60,000 immediately and Rs. 15,000 at the end of one year.
- (b) Pay nothing until the end of fourth year when a single payment of Rs. 90,000 must be made.

If Jothi Lakshmi believes 12% is a suitable interest rate, which alternative is the best for her?

**Solution Alternative 1**

Down payment = Rs. 60,000

Payment after one year = Rs. 15,000

The cash flow diagram for alternative 1 is shown in Fig. 6.19.

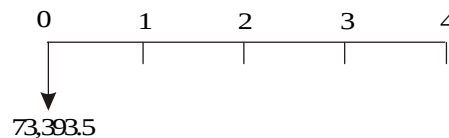


**Fig. 6.19** Cash flow diagram for alternative 1.

The present worth equation of the above cash flow diagram is

$$\begin{aligned} PW(12\%) &= 60,000 + 15,000(P/F, 12\%, 1) \\ &= 60,000 + 15,000(0.8929) \\ &= 73,393.50 \end{aligned}$$

The above present worth is represented in Fig. 6.20.



**Fig. 6.20** Resultant cash flow diagram.

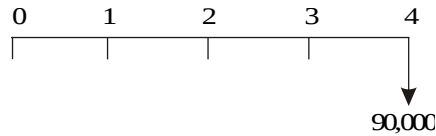
The annual equivalent expression of the above cash flow diagram is

$$\begin{aligned} AE(12\%) &= 73,393.5(A/P, 12\%, 4) \\ &= 73,393.5(0.3292) \\ &= \text{Rs. } 24,161.14 \end{aligned}$$

**Alternative 2**

Payment after four years = Rs. 90,000

The cash flow diagram for alternative 2 is shown in Fig. 6.21.



**Fig. 6.21** Cash flow diagram of alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(12\%) &= 90,000(A/F, 12\%, 4) \\ &= 90,000(0.2092) \\ &= \text{Rs. } 18,828 \end{aligned}$$

The annual equivalent cost of alternative 2 is less than that of alternative 1. Hence, Jothi Lakshmi should select alternative 2 for purchasing the home equipment.

**EXAMPLE 6.10** A transport company has been looking for a new tyre for its truck and has located the following alternatives:

| Brand | Tyre warranty<br>(months) | Price per tyre<br>(Rs.) |
|-------|---------------------------|-------------------------|
| A     | 12                        | 1,200                   |
| B     | 24                        | 1,800                   |
| C     | 36                        | 2,100                   |
| D     | 48                        | 2,700                   |

If the company feels that the warranty period is a good estimate of the tyre life and that a nominal interest rate (compounded annually) of 12% is appropriate, which tyre should it buy?

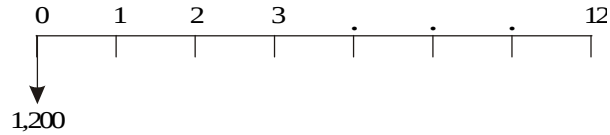
**Solution** In all the cases, the interest rate is 12%. This is equivalent to 1% per month.

**Brand A**

Tyre warranty = 12 months

Price/tyre = Rs. 1,200

The cash flow diagram for brand A is shown in Fig. 6.22.



**Fig. 6.22** Cash flow diagram of brand A.

The annual equivalent cost expression of the above cash flow diagram is

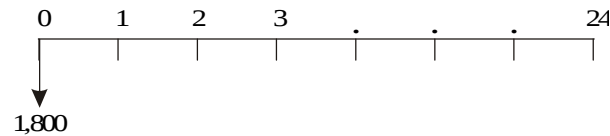
$$\begin{aligned} AE(1\%) &= 1,200(A/P, 1\%, 12) \\ &= 1,200(0.0888) \\ &= \text{Rs. } 106.56 \end{aligned}$$

### **Brand B**

Tyre warranty = 24 months

Price/tyre = Rs. 1,800

The cash flow diagram for brand B is shown in Fig. 6.23.



**Fig. 6.23** Cash flow diagram of brand B.

The annual equivalent cost expression of the above cash flow diagram is

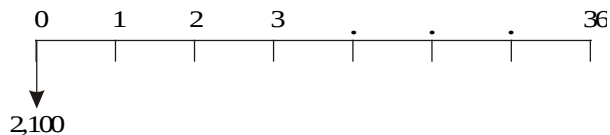
$$\begin{aligned} AE(1\%) &= 1,800(A/P, 1\%, 24) \\ &= 1,800(0.0471) \\ &= \text{Rs. } 84.78 \end{aligned}$$

### **Brand C**

Tyre warranty = 36 months

Price/tyre = Rs. 2,100

The cash flow diagram for brand C is shown in Fig. 6.24.



**Fig. 6.24** Cash flow diagram of brand C.

The annual equivalent expression of the above cash flow diagram is

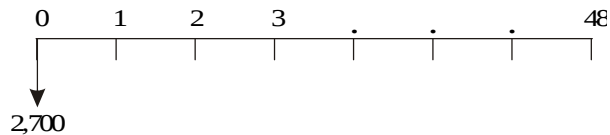
$$\begin{aligned} AE(1\%) &= 2,100(A/P, 1\%, 36) \\ &= 2,100(0.0332) \\ &= \text{Rs. } 69.72 \end{aligned}$$

**Brand D**

Tyre warranty = 48 months

Price/tyre = Rs. 2,700

The cash flow diagram for brand D is shown in Fig. 6.25.



**Fig. 6.25** Cash flow diagram of brand D.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(1\%) &= 2,700(A/P, 1\%, 48) \\ &= 2,700(0.0263) \\ &= \text{Rs. } 71.01 \end{aligned}$$

Here, minimum common multiple lives of tyres is considered. This is 144 months. Therefore, the comparison is made on 144 month's basis.

The annual equivalent cost of brand C is less than that of other brands. Hence, it should be used in the vehicles of the trucking company. It should be replaced four times during the 144-month period.

## QUESTIONS

1. A company has three proposals for expanding its business operations. The details are as follows:

| Alternative | Initial cost<br>(Rs.) | Annual revenue<br>(Rs.) | Life<br>(years) |
|-------------|-----------------------|-------------------------|-----------------|
| A1          | 25,00,000             | 8,00,000                | 10              |
| A2          | 20,00,000             | 6,00,000                | 10              |
| A3          | 30,00,000             | 10,00,000               | 10              |

Each alternative has insignificant salvage value at the end of its life. Assuming an interest rate of 15%, compounded annually, find the best alternative for expanding the business operations of the company using the annual equivalent method.

2. An automobile dealer has recently advertised for its new car. There are three alternatives of purchasing the car which are explained below.

**Alternative 1** The customer can take delivery of a car after making a down payment of Rs. 25,000. The remaining money should be paid in 36 equal monthly installments of Rs. 10,000 each.

**Alternative 2** The customer can take delivery of the car after making a down payment of Rs. 1,00,000. The remaining money should be paid in 36 equal monthly installments of Rs. 7,000 each.

**Alternative 3** The customer can take delivery of the car by making full payment of Rs. 3,00,000.

Suggest the best alternative of buying the cars for the customers by assuming an interest rate of 20% compounded annually. Use the annual equivalent method.

3. A small-scale industry is in the process of buying a milling machine. The purchase value of the milling machine is Rs. 60,000. It has identified two banks for loan to purchase the milling machine. The banks can give only 80% of the purchase value of the milling machine as loan. In Urban Bank, the loan is to be repaid in 60 equal monthly installments of Rs. 2,500 each. In State Bank, the loan is to be repaid in 40 equal monthly installments of Rs. 4,500 each. Suggest the most economical loan scheme for the company, based on the annual equivalent method of comparison. Assume a nominal rate of 24%, compounded monthly.
4. There are two alternatives of replacing a machine. The details of the alternatives are as follows:

**Alternative 1**

|                                                         |                |
|---------------------------------------------------------|----------------|
| Purchase value of the new machine                       | : Rs. 2,00,000 |
| Life of the machine                                     | : 10 years     |
| Salvage value of the new machine at the end of its life | : Rs. 20,000   |
| Annual operation and maintenance cost                   | : Rs. 40,000   |
| Buyback price of the existing machine                   | : Rs. 25,000   |

**Alternative 2**

|                                                         |                |
|---------------------------------------------------------|----------------|
| Purchase value of the new machine                       | : Rs. 3,00,000 |
| Life of the machine                                     | : 10 years     |
| Salvage value of the new machine at the end of its life | : Rs. 15,000   |
| Annual operation and maintenance cost                   | : Rs. 35,000   |
| Buyback price of the existing machine                   | : Rs. 5,000    |

Suggest the best replacement option for the company using the annual equivalent cost method of comparison by assuming 20% interest rate, compounded annually.

5. A company receives two options for purchasing a copier machine for its office.

**Option 1** Make a down payment of Rs. 30,000 and take delivery of the copier machine. The remaining money is to be paid in 24 equal monthly installments of Rs. 4,500 each.

**Option 2** Make a full payment of Rs. 1,00,000 and take delivery of the copier machine.

Suggest the best option for the company to buy the copier machine based on the annual equivalent method of comparison by assuming 15% interest rate, compounded annually.

6. Find the best alternative using the annual equivalent method of comparison. Assume an interest rate of 15% compounded annually.

| <i>Alternative</i>   | <i>A</i> | <i>B</i> | <i>C</i> |
|----------------------|----------|----------|----------|
| Initial cost (Rs.)   | 5,00,000 | 8,00,000 | 6,00,000 |
| Annual receipt (Rs.) | 2,00,000 | 1,50,000 | 1,20,000 |
| Life (years)         | 10       | 10       | 10       |
| Salvage value (Rs.)  | 1,00,000 | 50,000   | 30,000   |

# 7

## RATE OF RETURN METHOD

### 7.1 INTRODUCTION

The rate of return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero. In this method of comparison, the rate of return for each alternative is computed. Then the alternative which has the highest rate of return is selected as the best alternative.

In this type of analysis, the expenditures are always assigned with a negative sign and the revenues/inflows are assigned with a positive sign.

A generalized cash flow diagram to demonstrate the rate of return method of comparison is presented in Fig. 7.1.

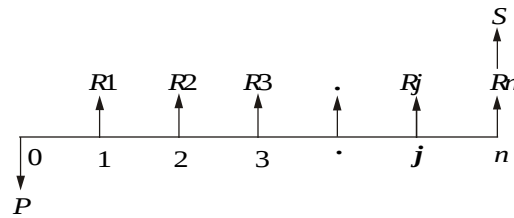


Fig. 7.1 Generalized cash flow diagram.

In the above cash flow diagram,  $P$  represents an initial investment,  $R_j$  the net revenue at the end of the  $j$ th year, and  $S$  the salvage value at the end of the  $n$ th year.

The first step is to find the net present worth of the cash flow diagram using the following expression at a given interest rate,  $i$ .

$$PW(i) = -P + R1/(1 + i)^1 + R2/(1 + i)^2 + ... + Rj/(1 + i)^j + ... + Rn/(1 + i)^n + S/(1 + i)^n$$

Now, the above function is to be evaluated for different values of  $i$  until the present worth function reduces to zero, as shown in Fig. 7.2.

In the figure, the present worth goes on decreasing when the interest rate is increased. The value of  $i$  at which the present worth curve cuts the  $X$ -axis is the rate of return of the given proposal/project. It will be very difficult to find the exact value of  $i$  at which the present worth function reduces to zero.

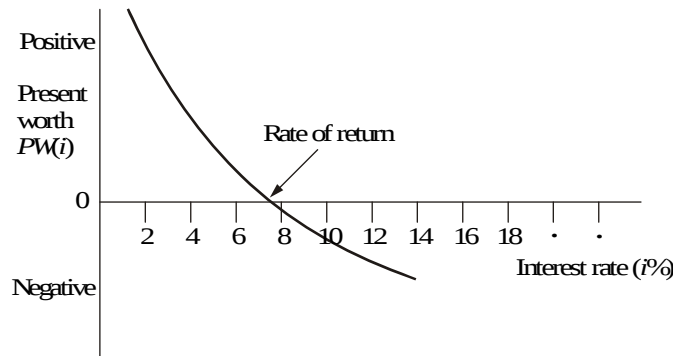


Fig. 7.2 Present worth function graph.

So, one has to start with an intuitive value of  $i$  and check whether the present worth function is positive. If so, increase the value of  $i$  until  $PW(i)$  becomes negative. Then, the rate of return is determined by interpolation method in the range of values of  $i$  for which the sign of the present worth function changes from positive to negative.

## 7.2 EXAMPLES

In this section, the concept of rate of return calculation is demonstrated with suitable examples.

**EXAMPLE 7.1** A person is planning a new business. The initial outlay and cash flow pattern for the new business are as listed below. The expected life of the business is five years. Find the rate of return for the new business.

| Period          | 0         | 1      | 2      | 3      | 4      | 5      |
|-----------------|-----------|--------|--------|--------|--------|--------|
| Cash flow (Rs.) | -1,00,000 | 30,000 | 30,000 | 30,000 | 30,000 | 30,000 |

### Solution

Initial investment = Rs. 1,00,000

Annual equal revenue = Rs. 30,000

Life = 5 years

The cash flow diagram for this situation is illustrated in Fig. 7.3.

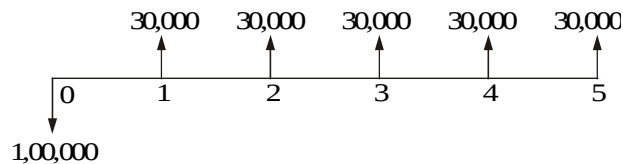


Fig. 7.3 Cash flow diagram.

The present worth function for the business is

$$PW(i) = -1,00,000 + 30,000(P/A, i, 5)$$

When  $i = 10\%$ ,

$$\begin{aligned} PW(10\%) &= -1,00,000 + 30,000(P/A, 10\%, 5) \\ &= -1,00,000 + 30,000(3.7908) \\ &= \text{Rs. } 13,724. \end{aligned}$$

When  $i = 15\%$ ,

$$\begin{aligned} PW(15\%) &= -1,00,000 + 30,000(P/A, 15\%, 5) \\ &= -1,00,000 + 30,000(3.3522) \\ &= \text{Rs. } 566. \end{aligned}$$

When  $i = 18\%$ ,

$$\begin{aligned} PW(18\%) &= -1,00,000 + 30,000(P/A, 18\%, 5) \\ &= -1,00,000 + 30,000(3.1272) \\ &= \text{Rs. } -6,184 \end{aligned}$$

$$\begin{aligned} i &= 15\% + \frac{566 - 0}{566 - (-6184)} \times (3\%) \\ &= 15\% + 0.252\% \\ &= 15.252\% \end{aligned}$$

Therefore, the rate of return for the new business is 15.252%.

**EXAMPLE 7.2** A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is Rs. 20,00,000. The annual net profit is Rs. 3,50,000. Find the rate of return for the new business.

**Solution**

Life of the product line ( $n$ ) = 10 years

Initial outlay = Rs. 20,00,000

Annual net profit = Rs. 3,50,000

Scrap value after 10 years = 0

The cash flow diagram for this situation is shown in Fig. 7.4.

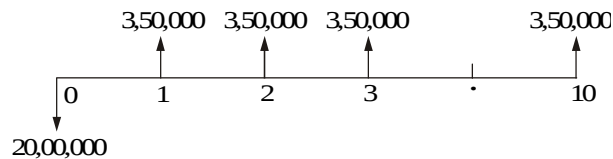


Fig. 7.4 Cash flow diagram.

The formula for the net present worth function of the situation is

$$PW(i) = -20,00,000 + 3,50,000(P/A, i, 10)$$

When  $i = 10\%$ ,

$$\begin{aligned} PW(10\%) &= -20,00,000 + 3,50,000(P/A, 10\%, 10) \\ &= -20,00,000 + 3,50,000(6.1446) \\ &= \text{Rs. } 1,50,610. \end{aligned}$$

When  $i = 12\%$ ,

$$\begin{aligned} PW(12\%) &= -20,00,000 + 3,50,000(P/A, 12\%, 10) \\ &= -20,00,000 + 3,50,000(5.6502) \\ &= \text{Rs. } -22,430. \end{aligned}$$

$$\begin{aligned} i &= 10\% + \frac{1,50,610 - 0}{1,50,610 - (-22,430)} \times (2\%) \\ &= 11.74\% \end{aligned}$$

Therefore, the rate of return of the new product line is 11.74%

**EXAMPLE 7.3** A firm has identified three mutually exclusive investment proposals whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%.

|                   | Alternative  |              |              |
|-------------------|--------------|--------------|--------------|
|                   | A1           | A2           | A3           |
| Investment        | Rs. 1,50,000 | Rs. 2,10,000 | Rs. 2,55,000 |
| Annual net income | Rs. 45,570   | Rs. 58,260   | Rs. 69,000   |

Find the best alternative based on the rate of return method of comparison.

**Solution** Calculation of rate of return for alternative A1

Initial outlay = Rs. 1,50,000

Annual profit = Rs. 45,570

Life = 5 years

The cash flow diagram for alternative A1 is shown in Fig. 7.5.

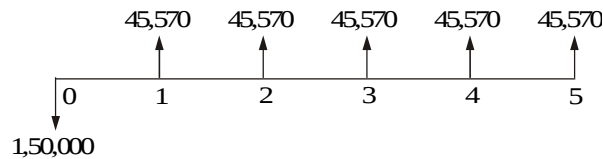


Fig. 7.5 Cash flow diagram for alternative A1.

The formula for the net present worth of alternative A1 is given as

$$PW(i) = -1,50,000 + 45,570(P/A, i, 5)$$

When  $i = 10\%$ ,

$$\begin{aligned} PW(10\%) &= -1,50,000 + 45,570(P/A, 10\%, 5) \\ &= -1,50,000 + 45,570(3.7908) \\ &= \text{Rs. } 22,746.76 \end{aligned}$$

When  $i = 12\%$ ,

$$\begin{aligned} PW(12\%) &= -1,50,000 + 45,570(P/A, 12\%, 5) \\ &= -1,50,000 + 45,570(3.6048) \\ &= \text{Rs. } 14,270.74 \end{aligned}$$

When  $i = 15\%$ ,

$$\begin{aligned} PW(15\%) &= -1,50,000 + 45,570(P/A, 15\%, 5) \\ &= -1,50,000 + 45,570(3.3522) \\ &= \text{Rs. } 2,759.75 \end{aligned}$$

When  $i = 18\%$ ,

$$\begin{aligned} PW(18\%) &= -1,50,000 + 45,570(P/A, 18\%, 5) \\ &= -1,50,000 + 45,570(3.1272) \\ &= \text{Rs. } -7,493.50 \end{aligned}$$

Therefore, the rate of return of the alternative A1 is

$$\begin{aligned} i &= 15\% + \frac{2,759.75 - 0}{2,759.75 - (-7,493.50)} \times (3\%) \\ &= 15\% + 0.81\% \\ &= 15.81\% \end{aligned}$$

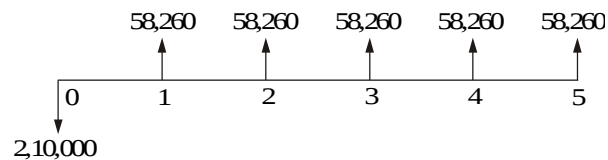
#### **Calculation of rate of return for alternative A2**

Initial outlay = Rs. 2,10,000

Annual profit = Rs. 58,260

Life of alternative A2 = 5 years

The cash flow diagram for alternative A2 is shown in Fig. 7.6.



**Fig. 7.6** Cash flow diagram for alternative A2.

The formula for the net present worth of this alternative is

$$PW(i) = -2,10,000 + 58,260(P/A, i, 5)$$

When  $i = 12\%$ ,

$$\begin{aligned} PW(12\%) &= -2,10,000 + 58,260(P/A, 12\%, 5) \\ &= -2,10,000 + 58,260(3.6048) \\ &= \text{Rs. } 15.65 \end{aligned}$$

When  $i = 13\%$ ,

$$\begin{aligned} PW(13\%) &= -2,10,000 + 58,260(P/A, 13\%, 5) \\ &= -2,10,000 + 58,260(3.5172) \\ &= \text{Rs. } -5,087.93 \end{aligned}$$

Therefore, the rate of return of alternative A2 is

$$\begin{aligned} i &= 12\% + \frac{15.65 - 0}{15.65 - (-5,087.93)} \times (1\%) \\ &= 12\% + 0\% \\ &= 12\% \end{aligned}$$

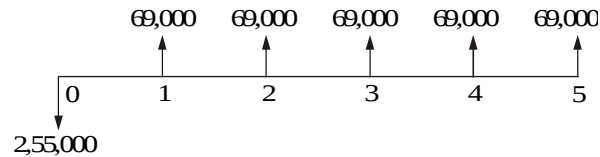
#### **Calculation of rate of return for alternative A3**

Initial outlay = Rs. 2,55,000

Annual profit = Rs. 69,000

Life of alternative A3 = 5 years

The cash flow diagram for alternative A3 is depicted in Fig. 7.7.



**Fig. 7.7** Cash flow diagram for alternative A3.

The formula for the net present worth of this alternative A3 is

$$PW(i) = -2,55,000 + 69,000(P/A, i, 5)$$

When  $i = 11\%$ ,

$$\begin{aligned} PW(11\%) &= -2,55,000 + 69,000(P/A, 11\%, 5) \\ &= -2,55,000 + 69,000(3.6959) \\ &= \text{Rs. } 17.1 \end{aligned}$$

When  $i = 12\%$ ,

$$\begin{aligned} PW(12\%) &= -2,55,000 + 69,000(P/A, 12\%, 5) \\ &= -2,55,000 + 69,000(3.6048) \\ &= \text{Rs. } -6,268.80 \end{aligned}$$

Therefore, the rate of return for alternative A3 is

$$i = 11\% + \frac{17.1 - 0}{17.1 - (-6,268.80)} \times 1\%$$

$$= 11\%$$

The rates of return for the three alternatives are now tabulated.

| Alternative    | A1     | A2  | A3  |
|----------------|--------|-----|-----|
| Rate of return | 15.81% | 12% | 11% |

From the above data, it is clear that the rate of return for alternative A3 is less than the minimum attractive rate of return of 12%. So, it should not be considered for comparison. The remaining two alternatives are qualified for consideration. Among the alternatives A1 and A2, the rate of return of alternative A1 is greater than that of alternative A2. Hence, alternative A1 should be selected.

**EXAMPLE 7.4** For the cash flow diagram shown in Fig. 7.8, compute the rate of return. The amounts are in rupees.

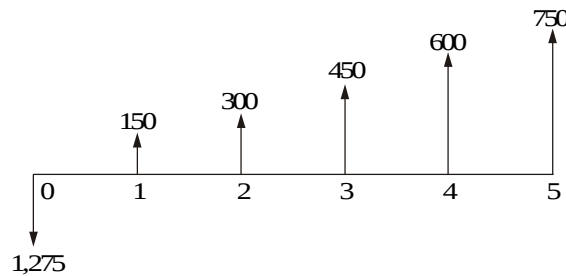


Fig. 7.8 Cash flow diagram.

**Solution** For the positive cash flows of the problem,

$$A1 = \text{Rs. } 150, \quad G = \text{Rs. } 150$$

The annual equivalent of the positive cash flows of the uniform gradient series is given by

$$A = A1 + G(A/G, i, n)$$

$$= 150 + 150(A/G, i, 5)$$

The formula for the present worth of the whole diagram

$$= -1,275 + [150 + 150(A/G, i, 5)] \times (P/A, i, 5)$$

$$PW(10\%) = -1,275 + [150 + 150(A/G, 10\%, 5)] \times (P/A, 10\%, 5)$$

$$= -1,275 + [150 + 150(1.8101)] \times (3.7908)$$

$$= \text{Rs. } 322.88$$

$$\begin{aligned}
 PW(12\%) &= -1,275 + [150 + 150(A/G, 12\%, 5)] \times (P/A, 12\%, 5) \\
 &= -1,275 + [150 + 150(1.7746)] \times (3.6048) \\
 &= \text{Rs. } 225.28
 \end{aligned}$$

$$\begin{aligned}
 PW(15\%) &= -1,275 + [150 + 150(A/G, 15\%, 5)] \times (P/A, 15\%, 5) \\
 &= -1,275 + [150 + 150(1.7228)] \times (3.3522) \\
 &= \text{Rs. } 94.11
 \end{aligned}$$

$$\begin{aligned}
 PW(18\%) &= -1,275 + [150 + 150(A/G, 18\%, 5)] \times (P/A, 18\%, 5) \\
 &= -1,275 + [150 + 150(1.6728)] \times (3.1272) \\
 &= \text{Rs. } -21.24
 \end{aligned}$$

Therefore, the rate of return for the cash flow diagram is

$$\begin{aligned}
 i &= 15\% + \frac{94.11 - 0}{94.11 - (-21.24)} \times 3\% \\
 &= 15\% + 2.45\% = 17.45\%
 \end{aligned}$$

**EXAMPLE 7.5** A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are tabulated below. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 12%. Suggest the best alternative to the company.

|               | Initial investment<br>(Rs.) | Yearly revenue<br>(Rs.) |
|---------------|-----------------------------|-------------------------|
| Alternative 1 | 5,00,000                    | 1,70,000                |
| Alternative 2 | 8,00,000                    | 2,70,000                |

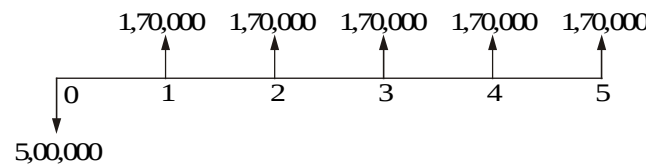
**Solution Alternative 1**

Initial outlay = Rs. 5,00,000

Annual revenue = Rs. 1,70,000

Life of alternative 1 = 5 years

The cash flow diagram for alternative 1 is illustrated in Fig. 7.9.



**Fig. 7.9** Cash flow diagram for alternative 1.

The formulae for the net present worth of alternative 1 are as follows:

$$PW_1(i) = -5,00,000 + 1,70,000(P/A, i, 5)$$

$$\begin{aligned} PW_1(15\%) &= -5,00,000 + 1,70,000(P/A, 15\%, 5) \\ &= -5,00,000 + 1,70,000(3.3522) \\ &= \text{Rs. } 69,874 \end{aligned}$$

$$\begin{aligned} PW_1(17\%) &= -5,00,000 + 1,70,000(P/A, 17\%, 5) \\ &= -5,00,000 + 1,70,000(3.1993) \\ &= \text{Rs. } 43,881 \end{aligned}$$

$$\begin{aligned} PW_1(20\%) &= -5,00,000 + 1,70,000(P/A, 20\%, 5) \\ &= -5,00,000 + 1,70,000(2.9906) \\ &= \text{Rs. } 8,402 \end{aligned}$$

$$\begin{aligned} PW_1(22\%) &= -5,00,000 + 1,70,000(P/A, 22\%, 5) \\ &= -5,00,000 + 1,70,000(2.8636) \\ &= \text{Rs. } -13,188 \end{aligned}$$

Therefore, the rate of return of alternative 1 is

$$\begin{aligned} i &= 20\% + \frac{8,402 - 0}{8,402 - (-13,188)} \times 2\% \\ &= 20.78\% \end{aligned}$$

### Alternative 2

Initial outlay = Rs. 8,00,000

Annual revenue = Rs. 2,70,000

Life = 5 years

The cash flow diagram for alternative 2 is depicted in Fig. 7.10.

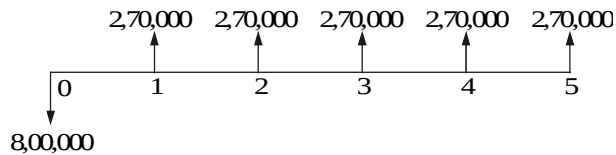


Fig. 7.10 Cash flow diagram for alternative 2.

The formula for the net present worth of alternative 2 is:

$$PW_2(i) = -8,00,000 + 2,70,000(P/A, i, 5)$$

$$\begin{aligned} PW_2(20\%) &= -8,00,000 + 2,70,000(P/A, 20\%, 5) \\ &= -8,00,000 + 2,70,000(2.9906) \\ &= \text{Rs. } 7,462 \end{aligned}$$

$$\begin{aligned}
 PW_2(22\%) &= -8,00,000 + 2,70,000(P/A, 22\%, 5) \\
 &= -8,00,000 + 2,70,000(2.8636) \\
 &= \text{Rs. } -26,828
 \end{aligned}$$

Thus, the rate of return of alternative 2 is

$$\begin{aligned}
 i &= 20\% + \frac{7,462 - 0}{7,462 - (-26,828)} \times 2\% \\
 &= 20.435\%
 \end{aligned}$$

Since the rate of return of alternative 1 is greater than that of the alternative 2, select alternative 1.

### QUESTIONS

1. Consider the following cash flow of a project:

| Year      | 0       | 1     | 2     | 3     | 4     | 5     |
|-----------|---------|-------|-------|-------|-------|-------|
| Cash flow | -10,000 | 4,000 | 4,500 | 5,000 | 5,500 | 6,000 |

Find the rate of return of the project.

2. A person invests a sum of Rs. 2,00,000 in a business and receives equal net revenue of Rs. 50,000 for the next 10 years. At the end of the 10th year, the salvage value of the business is Rs. 25,000. Find the rate of return of the business.
3. A company is in the process of selecting the best alternative among the following three mutually exclusive alternatives:

| Alternative | Initial investment | Annual revenue (Rs.) | Life (years) |
|-------------|--------------------|----------------------|--------------|
| A1          | Rs. 5,00,000       | 1,00,000             | 10           |
| A2          | Rs. 8,00,000       | 1,40,000             | 10           |
| A3          | Rs. 3,00,000       | 70,000               | 10           |

Find the best alternative based on the rate of return method of comparison.

4. A shipping firm is considering the purchase of a materials handling system for unloading ships at a dock. The firm has reduced their choice to three different systems, all of which are expected to provide the same unloading speed. The initial costs and the operating costs estimated for each system are now tabulated.

| <i>System</i> | <i>Initial cost</i> | <i>Annual operating expenses</i> |
|---------------|---------------------|----------------------------------|
| S1            | Rs. 6,50,000        | Rs. 91,810                       |
| S2            | Rs. 7,80,000        | Rs. 52,600                       |
| S3            | Rs. 7,50,000        | Rs. 68,417                       |

The life of each system is estimated to be five years and the firm's minimum attractive rate of return is 15%. If the firm must select one of the materials handling systems, which one is the most desirable?

5. A firm has identified three mutually exclusive alternatives. The life of all three alternatives is estimated to be five years. The minimum attractive rate of return is 12%. Find the best alternative based on the rate of return method.

| <i>Alternative</i>       | <i>A1</i> | <i>A2</i> | <i>A3</i> |
|--------------------------|-----------|-----------|-----------|
| Initial investment (Rs.) | 2,00,000  | 2,80,000  | 3,60,000  |
| Annual income (Rs.)      | 52,000    | 72,000    | 1,00,000  |

6. An automobile company is planning to buy a robot for its forging unit. It has identified two different companies for the supply of the robot. The details of cost and incremental revenue of using robots are summarized in the following table:

|                                  | <i>Brand</i>   |              |
|----------------------------------|----------------|--------------|
|                                  | <i>Speedex</i> | <i>Giant</i> |
| Initial cost (Rs.)               | 5,00,000       | 9,00,000     |
| Annual incremental revenue (Rs.) | 80,000         | 2,50,000     |
| Life (years)                     | 8              | 8            |
| Life-end salvage value (Rs.)     | 40,000         | 60,000       |

The minimum attractive return for the company is 12%. Suggest the best brand of robot to the company based on the rate of return method.

7. A bank introduces two different investment schemes whose details are as follows: Find the best investment alternative from the investor's point of view.

|                           | <i>Alpha Bank</i> | <i>Beta Bank</i> |
|---------------------------|-------------------|------------------|
| Deposit amount (Rs.)      | 1,00,000          | 2,00,000         |
| Period of deposit (years) | 5 years           | 3 years          |
| Maturity amount (Rs.)     | 3,00,000          | 4,50,000         |

8. A company is planning for its expansion programme which will take place after five years. The expansion requires an equal sum of Rs. 5,00,000 for consecutive three years. Gamma Bank has recently introduced a scheme in this line. If the company invests Rs. 7,00,000 now with this bank, it will make equal repayments of Rs. 5,00,000 for three consecutive years starting from the end of the fifth year from now. The minimum attractive rate of return for the company is 12%. Suggest whether the company should invest with the Gamma Bank for its expansion programme.
9. Consider the following table which summarizes data of two alternatives.

|               | <i>First cost</i> | <i>Annual return</i> | <i>Life</i> |
|---------------|-------------------|----------------------|-------------|
| Alternative 1 | Rs. 5,00,000      | Rs. 1,50,000         | 10 yrs      |
| Alternative 2 | Rs. 8,00,000      | Rs. 2,50,000         | 10 yrs      |

Find the best alternative based on the rate of return method of comparison.

10. A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are given in the following table. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 15%. Suggest the best alternative to the company.

|               | <i>Initial investment<br/>(Rs.)</i> | <i>Yearly revenue<br/>(Rs.)</i> |
|---------------|-------------------------------------|---------------------------------|
| Alternative 1 | 4,50,000                            | 1,50,000                        |
| Alternative 2 | 7,50,000                            | 2,50,000                        |

# 8

## REPLACEMENT AND MAINTENANCE ANALYSIS

### 8.1 INTRODUCTION

Organizations providing goods/services use several facilities like equipment and machinery which are directly required in their operations. In addition to these facilities, there are several other items which are necessary to facilitate the functioning of organizations.

All such facilities should be continuously monitored for their efficient functioning; otherwise, the quality of service will be poor. Besides the quality of service of the facilities, the cost of their operation and maintenance would increase with the passage of time. Hence, it is an absolute necessity to maintain the equipment in good operating conditions with economical cost. Thus, we need an integrated approach to minimize the cost of maintenance. In certain cases, the equipment will be obsolete over a period of time.

If a firm wants to be in the same business competitively, it has to take decision on whether to replace the old equipment or to retain it by taking the cost of maintenance and operation into account.

There are two basic reasons for considering the replacement of an equipment—physical impairment of the various parts or obsolescence of the equipment.

Physical impairment refers only to changes in the physical condition of the machine itself. This would lead to a decline in the value of the service rendered, increased operating cost, increased maintenance cost or a combination of these.

Obsolescence is due to improvement of the tools of production, mainly improvement in technology.

So, it would be uneconomical to continue production with the same machine under any of the above situations. Hence, the machines are to be periodically replaced.

Sometimes, the capacity of existing facilities may be inadequate to meet the current demand. Under such situation, the following alternatives will be considered.

- Replacement of the existing equipment with a new one.
- Augmenting the existing one with an additional equipment.

## 8.2 TYPES OF MAINTENANCE

Maintenance activity can be classified into two types: preventive maintenance and breakdown maintenance. *Preventive maintenance* (PM) is the periodical inspection and service activities which are aimed to detect potential failures and perform minor adjustments or repairs which will prevent major operating problems in future. *Breakdown maintenance* is the repair which is generally done after the equipment has attained down state. It is often of an emergency nature which will have associated penalty in terms of expediting cost of maintenance and down time cost of equipment. Preventive maintenance will reduce such cost up to a point. Beyond that point, the cost of preventive maintenance will be more when compared to the breakdown maintenance cost. The total cost, which is the sum of the preventive maintenance cost and the breakdown maintenance cost, will go on decreasing with an increase in the level of maintenance up to a point. Beyond that point, the total cost will start increasing. The level of maintenance corresponding to the minimum total cost is the optimal level of maintenance. The concepts are demonstrated in Fig. 8.1.

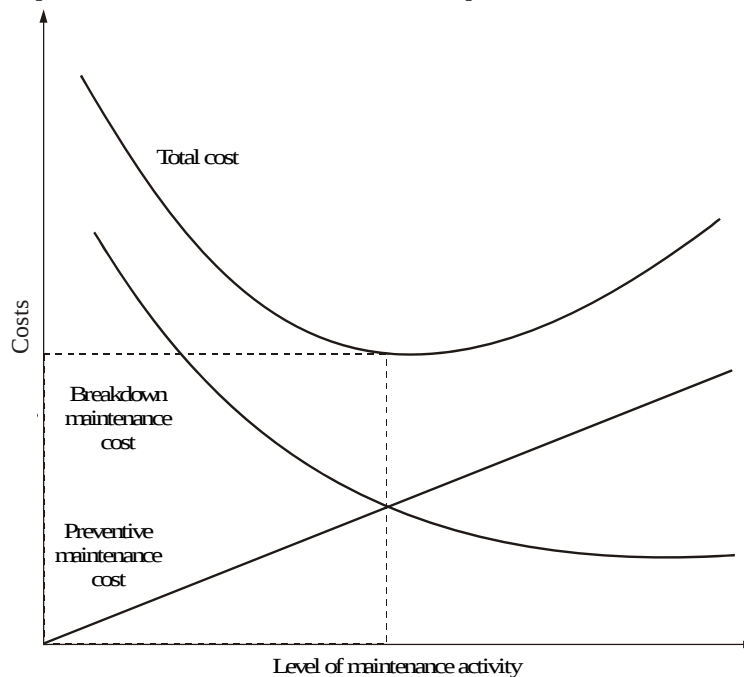


Fig. 8.1 Maintenance costs.

## 8.3 TYPES OF REPLACEMENT PROBLEM

Replacement study can be classified into two categories:

- (a) Replacement of assets that deteriorate with time (Replacement due to gradual failure, or wear and tear of the components of the machines).

This can be further classified into the following types:

- (i) Determination of economic life of an asset.
  - (ii) Replacement of an existing asset with a new asset.
- (b) Simple probabilistic model for assets which fail completely (replacement due to sudden failure).

## 8.4 DETERMINATION OF ECONOMIC LIFE OF AN ASSET

Any asset will have the following cost components:

- Capital recovery cost (average first cost), computed from the first cost (purchase price) of the machine.
- Average operating and maintenance cost (O & M cost)
- Total cost which is the sum of capital recovery cost (average first cost) and average maintenance cost.

A typical shape of each of the above costs with respect to life of the machine is shown in Fig. 8.2.

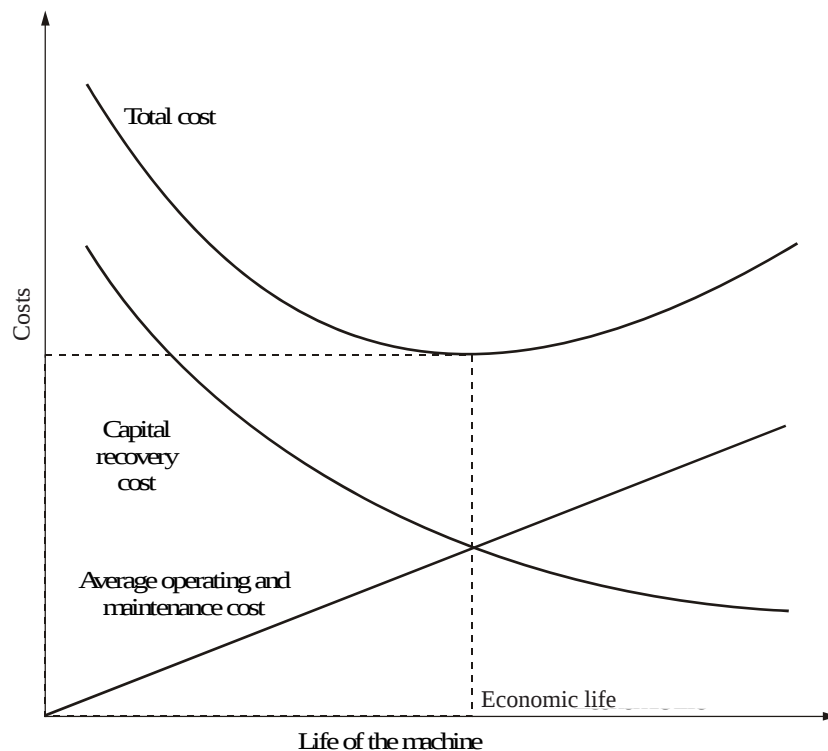


Fig. 8.2 Chart showing economic life.

From Fig. 8.2, it is clear that the capital recovery cost (average first cost) goes on decreasing with the life of the machine and the average operating and maintenance cost goes on increasing with the life of the machine. From the beginning, the total cost continues to decrease up to a particular life and then it starts increasing. The point where the total cost is minimum is called the *economic life* of the machine.

If the interest rate is more than zero per cent, then we use interest formulas to determine the economic life.

The replacement alternatives can be evaluated based on the present worth criterion and annual equivalent criterion. The basics of these criteria are already presented in Chapter 3.

**EXAMPLE 8.1** A firm is considering replacement of an equipment, whose first cost is Rs. 4,000 and the scrap value is negligible at the end of any year. Based on experience, it was found that the maintenance cost is zero during the first year and it increases by Rs. 200 every year thereafter.

(a) When should the equipment be replaced if  $i = 0\%$ ?

(b) When should the equipment be replaced if  $i = 12\%$ ?

(a) When  $i = 0\%$ . In this problem,

(i) First cost = Rs. 4,000

(ii) Maintenance cost is Rs. 0 during the first year and it increases by Rs. 200 every year thereafter.

This is summarized in column B of Table 8.1.

**Table 8.1** Calculations to Determine Economic Life (First cost = Rs. 4,000, Interest = 0%)

| End of year<br>(n) | Maintenance cost at end of year | Summation of maintenance costs | Average cost of maintenance through year given | Average first cost if replaced at year end given | Average total cost through year given |
|--------------------|---------------------------------|--------------------------------|------------------------------------------------|--------------------------------------------------|---------------------------------------|
|                    |                                 | $\Sigma B$                     | C/A                                            | 4,000/A                                          | D + E                                 |
| A                  | B (Rs.)                         | C (Rs.)                        | D (Rs.)                                        | E (Rs.)                                          | F (Rs.)                               |
| 1                  | 0                               | 0                              | 0                                              | 4,000.00                                         | 4,000.00                              |
| 2                  | 200                             | 200                            | 100                                            | 2,000.00                                         | 2,100.00                              |
| 3                  | 400                             | 600                            | 200                                            | 1,333.33                                         | 1,533.33                              |
| 4                  | 600                             | 1,200                          | 300                                            | 1,000.00                                         | 1,300.00                              |
| 5                  | 800                             | 2,000                          | 400                                            | 800.00                                           | 1,200.00                              |
| 6                  | 1,000                           | 3,000                          | 500                                            | 666.67                                           | 1,166.67*                             |
| 7                  | 1,200                           | 4,200                          | 600                                            | 571.43                                           | 1,171.43                              |

\*Economic life of the machine = 6 years

Column C summarizes the summation of maintenance costs for each replacement period. The value corresponding to any end of year in this column represents the total maintenance cost of using the equipment till the end of that particular year.

$$\begin{aligned}
 \text{Average total cost} &= \frac{\text{First cost (FC)} + \text{Summation of maintenance cost}}{\text{Replacement period}} \\
 &= \frac{\text{FC}}{n} + \frac{\text{Column C}}{n} \\
 &= \text{Average first cost for the given period} + \text{Average maintenance cost for the given period}
 \end{aligned}$$

$$\text{Column F} = \text{Column E} + \text{Column D}$$

The value corresponding to any end of year ( $n$ ) in Column F represents the average total cost of using the equipment till the end of that particular year.

For this problem, the average total cost decreases till the end of year 6 and then it increases. Therefore, the optimal replacement period is six years, i.e. economic life of the equipment is six years.

(b) When interest rate,  $i = 12\%$ . When the interest rate is more than 0%, the steps to be taken for getting the economic life are summarized with reference to Table 8.2.

**Table 8.2** Calculations to Determine Economic Life (First cost = Rs. 4,000, Interest = 12%)

| End of year<br>(n) | Maintenance cost at end of year | P/F, 12%,<br>n | Present worth as of beginning of year 1 of maintenance costs | Summation of present worth of maintenance costs through year given | Present worth of cumulative maintenance cost & first cost | A/P, 12%,<br>n | Annual equivalent total cost through year given |
|--------------------|---------------------------------|----------------|--------------------------------------------------------------|--------------------------------------------------------------------|-----------------------------------------------------------|----------------|-------------------------------------------------|
|                    |                                 |                | (B × C)                                                      | Σ D                                                                | E + Rs. 4,000                                             |                | F × G                                           |
| A                  | B (Rs.)                         | C              | D (Rs.)                                                      | E (Rs.)                                                            | F (Rs.)                                                   | G              | H (Rs.)                                         |
| 1                  | 0                               | 0.8929         | 0.00                                                         | 0.00                                                               | 4,000.00                                                  | 1.1200         | 4,480.00                                        |
| 2                  | 200                             | 0.7972         | 159.44                                                       | 159.44                                                             | 4,159.44                                                  | 0.5917         | 2,461.14                                        |
| 3                  | 400                             | 0.7118         | 284.72                                                       | 444.16                                                             | 4,444.16                                                  | 0.4163         | 1,850.10                                        |
| 4                  | 600                             | 0.6355         | 381.30                                                       | 825.46                                                             | 4,825.46                                                  | 0.3292         | 1,588.54                                        |
| 5                  | 800                             | 0.5674         | 453.92                                                       | 1,279.38                                                           | 5,279.38                                                  | 0.2774         | 1,464.50                                        |
| 6                  | 1,000                           | 0.5066         | 506.60                                                       | 1,785.98                                                           | 5,785.98                                                  | 0.2432         | 1,407.15                                        |
| 7                  | 1,200                           | 0.4524         | 542.88                                                       | 2,328.86                                                           | 6,328.86                                                  | 0.2191         | 1,386.65*                                       |
| 8                  | 1,400                           | 0.4039         | 565.46                                                       | 2,894.32                                                           | 6,894.32                                                  | 0.2013         | 1,387.83                                        |
| 9                  | 1,600                           | 0.3606         | 576.96                                                       | 3,471.28                                                           | 7,471.28                                                  | 0.1877         | 1,402.36                                        |
| 10                 | 1,800                           | 0.3220         | 579.60                                                       | 4,050.88                                                           | 8,050.88                                                  | 0.1770         | 1,425.00                                        |

\*Economic life of the machine = 7 years

The steps are summarized now:

1. Discount the maintenance costs to the beginning of year 1.

$$\begin{aligned}
 \text{Column D} &= \text{Column B} \times \frac{1}{(1+i)^n} \\
 &= \text{Column B} \times (P/F, i, n) = \text{Column B} \times \text{Column C}.
 \end{aligned}$$

2. Find the summation of present worth of maintenance costs through the year given (Column E = Σ Column D).

3. Find Column F by adding the first cost of Rs. 4,000 to Column E.
4. Find the annual equivalent total cost through the years given.

$$\begin{aligned}\text{Column H} &= \text{Column F} \times \frac{i(1+i)^n}{(1+i)^n - 1} \\ &= \text{Column F} \times (A/P, 12\%, n) = \text{Column F} \times \text{Column G}\end{aligned}$$

5. Identify the end of year for which the annual equivalent total cost is minimum.

For this problem, the annual equivalent total cost is minimum at the end of year 7. Therefore, the economic life of the equipment is seven years.

**EXAMPLE 8.2** The following table gives the operation cost, maintenance cost and salvage value at the end of every year of a machine whose purchase value is Rs. 20,000.

Find the economic life of the machine assuming interest rate,  $i = 15\%$ .

| End of year<br>(n) | Operation cost<br>at the end of<br>year (Rs.) | Maintenance<br>cost at the<br>end of year (Rs.) | Salvage value<br>at the end of<br>year (Rs.) |
|--------------------|-----------------------------------------------|-------------------------------------------------|----------------------------------------------|
| 1                  | 3,000                                         | 300                                             | 9,000                                        |
| 2                  | 4,000                                         | 400                                             | 8,000                                        |
| 3                  | 5,000                                         | 500                                             | 7,000                                        |
| 4                  | 6,000                                         | 600                                             | 6,000                                        |
| 5                  | 7,000                                         | 700                                             | 5,000                                        |
| 6                  | 8,000                                         | 800                                             | 4,000                                        |
| 7                  | 9,000                                         | 900                                             | 3,000                                        |
| 8                  | 10,000                                        | 1,000                                           | 2,000                                        |
| 9                  | 11,000                                        | 1,100                                           | 1,000                                        |
| 10                 | 12,000                                        | 1,200                                           | 0                                            |

### Solution

First cost = Rs. 20,000

Interest rate = 15%

The other details are summarized in Table 8.3 along with regular calculations for determining the economic life.

Table 8.3 Calculations to Determine Economic Life

| (First Cost = Rs. 20,000, Interest Rate = 15%) |                                   |                                     |                                                           |             |                                                                                                        |                     |                                  |                                                          |                     |             |                                                 |
|------------------------------------------------|-----------------------------------|-------------------------------------|-----------------------------------------------------------|-------------|--------------------------------------------------------------------------------------------------------|---------------------|----------------------------------|----------------------------------------------------------|---------------------|-------------|-------------------------------------------------|
| End of year (n)                                | Operation cost at the end of year | Maintenance cost at the end of year | Sum of operation and maintenance costs at the end of year | P/F, 15%, n | Present worth as of beginning of year 1 of sum of operation & through year maintenance cost designated | Cumulative sum of F | Salvage value at the end of year | Present worth as of beginning of year 1 of salvage value | Total present worth | A/P, 15%, n | Annual equivalent total cost through year given |
| A                                              | B (Rs.)                           | C (Rs.)                             | D (Rs.)                                                   | E           | F (Rs.)                                                                                                | G (Rs.)             | H (Rs.)                          | I (Rs.)                                                  | J (Rs.)             | K           | L (Rs.)                                         |
| 1                                              | 3,000                             | 300                                 | 3,300                                                     | 0.8696      | 2,869.68                                                                                               | 2,869.68            | 9,000                            | 7,826.40                                                 | 15,043.28           | 1.1500      | 17,299.77                                       |
| 2                                              | 4,000                             | 400                                 | 4,400                                                     | 0.7562      | 3,326.84                                                                                               | 6,196.52            | 8,000                            | 6,048.80                                                 | 20,147.72           | 0.6151      | 12,392.86                                       |
| 3                                              | 5,000                             | 500                                 | 5,500                                                     | 0.6575      | 3,616.25                                                                                               | 9,812.77            | 7,000                            | 4,602.50                                                 | 25,210.27           | 0.4380      | 11,042.01                                       |
| 4                                              | 6,000                             | 600                                 | 6,600                                                     | 0.5718      | 3,773.88                                                                                               | 13,586.65           | 6,000                            | 3,430.80                                                 | 30,155.85           | 0.3503      | 10,563.59                                       |
| 5                                              | 7,000                             | 700                                 | 7,700                                                     | 0.4972      | 3,828.44                                                                                               | 17,415.09           | 5,000                            | 2,486.00                                                 | 34,929.09           | 0.2983      | 10,419.35 *                                     |
| 6                                              | 8,000                             | 800                                 | 8,800                                                     | 0.4323      | 3,804.24                                                                                               | 21,219.33           | 4,000                            | 1,729.20                                                 | 39,490.13           | 0.2642      | 10,433.29                                       |
| 7                                              | 9,000                             | 900                                 | 9,900                                                     | 0.3759      | 3,721.41                                                                                               | 24,940.74           | 3,000                            | 1,127.70                                                 | 43,813.04           | 0.2404      | 10,532.66                                       |

\*Economic Life = 5 years

$$\left[ \begin{array}{l} \text{Cumulative sum of} \\ \text{present worth as of} \\ \text{beginning of year 1} \\ \text{of operation and} \\ \text{maintenance costs} \end{array} \right] + \begin{array}{l} \text{First} \\ \text{cost} \end{array} - \left[ \begin{array}{l} \text{Present worth} \\ \text{as of beginning} \\ \text{of year 1 of} \\ \text{salvage value} \end{array} \right] \times (A/P, 15\%, n)$$

In Column L, the annual equivalent total cost is minimum for  $n = 5$ . Therefore, the economic life of the machine is five years.

Now, the manufacturer of machine B has approached the company. Machine B, which has the same capacity as that of machine A, is priced at Rs. 6,000. The maintenance cost of machine B is estimated at Rs. 1,500 for the first year and an equal yearly increment of Rs. 300 thereafter.

If the money is worth 15% per year, which machine should be purchased? (Assume that the scrap value of each of the machines is negligible at any year.)

*Determination of economic life and corresponding annual equivalent total cost of machine B.* The details of machine B are summarized in Table 8.4 along with the usual calculations to determine the economic life.

| End of year (n) | Maintenance cost for end of year | P/F, 15%, n | Present worth as of beginning of year 1 of maintenance costs | Summation of present worth of maintenance costs through year given | Column E + Rs. 6000 | A/P, 15%, n | Annual equivalent total cost through year given |
|-----------------|----------------------------------|-------------|--------------------------------------------------------------|--------------------------------------------------------------------|---------------------|-------------|-------------------------------------------------|
|                 |                                  |             | B × C                                                        | Σ D                                                                |                     |             |                                                 |
| A               | B (Rs.)                          | C           | D (Rs.)                                                      | E (Rs.)                                                            | F (Rs.)             | G           | H (Rs.)                                         |
| 1               | 1,500                            | 0.8696      | 1,304.40                                                     | 1,304.40                                                           | 7,304.40            | 1.1500      | 8,400.06                                        |
| 2               | 1,800                            | 0.7561      | 1,360.98                                                     | 2,665.38                                                           | 8,665.38            | 0.6151      | 5,330.08                                        |
| 3               | 2,100                            | 0.6575      | 1,380.75                                                     | 4,046.13                                                           | 10,046.13           | 0.4380      | 4,400.21                                        |
| 4               | 2,400                            | 0.5718      | 1,372.32                                                     | 5,418.45                                                           | 11,418.45           | 0.3503      | 3,999.88                                        |
| 5               | 2,700                            | 0.4972      | 1,342.44                                                     | 6,760.89                                                           | 12,760.89           | 0.2983      | 3,806.57                                        |
| 6               | 3,000                            | 0.4323      | 1,296.90                                                     | 8,057.79                                                           | 14,057.79           | 0.2642      | 3,714.07                                        |
| 7               | 3,300                            | 0.3759      | 1,240.47                                                     | 9,298.26                                                           | 15,298.26           | 0.2404      | 3,677.70                                        |
| 8               | 3,600                            | 0.3269      | 1,176.84                                                     | 10,475.10                                                          | 16,475.10           | 0.2229      | 3,672.30 *                                      |
| 9               | 3,900                            | 0.2843      | 1,108.77                                                     | 11,583.87                                                          | 17,583.87           | 0.2096      | 3,685.58                                        |
| 10              | 4,200                            | 0.2472      | 1,038.24                                                     | 12,622.11                                                          | 18,622.11           | 0.1993      | 3,711.39                                        |

\*Economic life of the machine = 8 years

Column B of Table 8.4 summarizes the yearly maintenance costs of machine B. The first cost of machine B is equal to Rs. 6,000.

Annual equivalent total cost

$$\begin{aligned}
 &= \left[ \begin{array}{c} \text{Summation of present worth} \\ \text{of maintenance cost through} \\ \text{year} \end{array} + \begin{array}{c} \text{First} \\ \text{cost} \end{array} \right] \times (A/P, 15\%, n) \\
 &= (\text{Column E} + \text{Rs. 6,000}) \times \text{Column G} \\
 &= \text{Column F} \times \text{Column G}
 \end{aligned}$$

In Column H, the minimum annual equivalent total cost occurs when  $n$  is equal to 8. Hence the economic life of machine B is 8 years and the corresponding annual equivalent total cost is Rs. 3,672.30.

#### RESULT

Minimum annual equivalent total cost for machine A = Rs. 2,780

Minimum annual equivalent total cost for machine B = Rs. 3,672.30

Since the minimum annual equivalent total cost of machine A is less than that of machine B, machine A is selected as the best machine which has the economic life of four years. (*Note:* Selection of the best machine is based on the minimum annual equivalent total cost. The comparison is made over the minimum common multiple of the lives of machine A and machine B, i.e.  $4 \times 2 = 8$  years).

## 8.5 REPLACEMENT OF EXISTING ASSET WITH A NEW ASSET

In this section, the concept of comparison of replacement of an existing asset with a new asset is presented. In this analysis, the annual equivalent cost of each alternative should be computed first. Then the alternative which has the least cost should be selected as the best alternative. Before discussing details, some preliminary concepts which are essential for this type of replacement analysis are presented.

### 8.5.1 Capital Recovery with Return

Consider the following data of a machine. Let

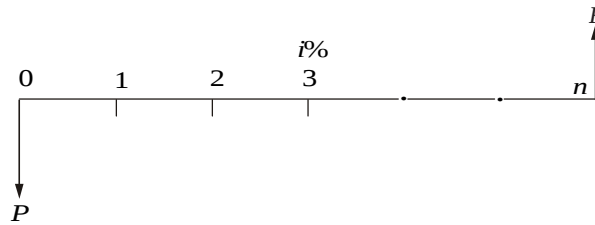
$P$  = purchase price of the machine,

$F$  = salvage value of the machine at the end of machine life,

$n$  = life of the machine in years, and

$i$  = interest rate, compounded annually

The corresponding cash flow diagram is shown in Fig. 8.3.



**Fig. 8.3** Cash flow diagram of machine.

The equation for the annual equivalent amount for the above cash flow diagram is

$$AE(i) = (P - F) \times (A/P, i, n) + F \times i$$

This equation represents the *capital recovery with return*.

### 8.5.2 Concept of Challenger and Defender

If an existing equipment is considered for replacement with a new equipment, then the existing equipment is known as the *defender* and the new equipment is known as *challenger*.

Assume that an equipment has been purchased about three years back for Rs. 5,00,000 and it is considered for replacement with a new equipment. The supplier of the new equipment will take the old one for some money, say, Rs. 3,00,000. This should be treated as the present value of the existing equipment and it should be considered for all further economic analysis. The purchase value of the existing equipment before three years is now known as *sunk cost*, and it should not be considered for further analysis.

**EXAMPLE 8.4** Two years ago, a machine was purchased at a cost of Rs. 2,00,000 to be useful for eight years. Its salvage value at the end of its life is Rs. 25,000. The annual maintenance cost is Rs. 25,000. The market value of the present machine is Rs. 1,20,000. Now, a new machine to cater to the need of the present machine is available at Rs. 1,50,000 to be useful for six years. Its annual maintenance cost is Rs. 14,000. The salvage value of the new machine is Rs. 20,000. Using an interest rate of 12%, find whether it is worth replacing the present machine with the new machine.

#### **Solution** *Alternative 1—Present machine*

Purchase price = Rs. 2,00,000

Present value ( $P$ ) = Rs. 1,20,000

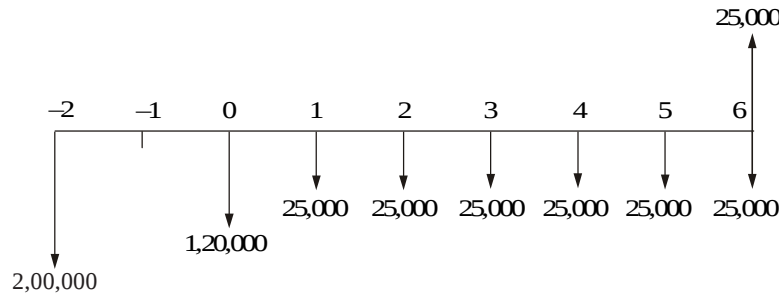
Salvage value ( $F$ ) = Rs. 25,000

Annual maintenance cost ( $A$ ) = Rs. 25,000

Remaining life = 6 years

Interest rate = 12%

The cash flow diagram of the present machine is illustrated in Fig. 8.4. The



**Fig. 8.4** Cash flow diagram for alternative 1.

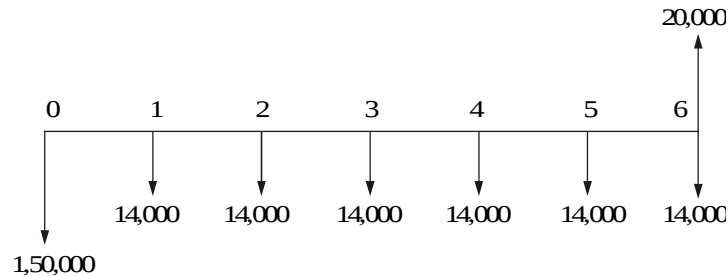
annual maintenance cost for the preceding periods are not shown in this figure. The annual equivalent cost is computed as

$$\begin{aligned}
 AE(12\%) &= (P - F)(A/P, 12\%, 6) + F \times i + A \\
 &= (1,20,000 - 25,000)(0.2432) + 25,000 \times 0.12 + 25,000 \\
 &= \text{Rs. } 51,104
 \end{aligned}$$

#### **Alternative 2—New machine**

Purchase price ( $P$ ) = Rs. 1,50,000  
 Salvage value ( $F$ ) = Rs. 20,000  
 Annual maintenance cost ( $A$ ) = Rs. 14,000  
 Life = 6 years  
 Interest rate = 12%

The cash flow diagram of the new machine is depicted in Fig. 8.5.



**Fig. 8.5** Cash flow diagram for alternative 2.

The formula for the annual equivalent cost is

$$\begin{aligned}
 AE(12\%) &= (P - F)(A/P, 12\%, 6) + F \times i + A \\
 &= (1,50,000 - 20,000)(0.2432) + 20,000 \times 0.12 + 14,000 \\
 &= \text{Rs. } 48,016
 \end{aligned}$$

Since the annual equivalent cost of the new machine is less than that of the present machine, it is suggested that the present machine be replaced with the new machine.

**EXAMPLE 8.5** A diesel engine was installed 10 years ago at a cost of Rs. 50,000. It has a present realizable market value of Rs. 15,000. If kept, it can be expected to last five years more, with operating and maintenance cost of Rs. 14,000 per year and to have a salvage value of Rs. 8,000 at the end of the fifth year. This engine can be replaced with an improved version costing Rs. 65,000 which has an expected life of 20 years. This improved version will have an estimated annual operating and maintenance cost of Rs. 9,000 and ultimate salvage value of Rs. 13,000. Using an interest rate of 15%, make an annual equivalent cost analysis to determine whether to keep or replace the old engine.

**Solution Alternative 1—Old diesel engine**

Purchase price = Rs. 50,000

Present value ( $P$ ) = Rs. 15,000

Salvage value ( $F$ ) = Rs. 8,000

Annual operating and maintenance cost ( $A$ ) = Rs. 14,000

Remaining life ( $n$ ) = 5 years

Interest rate = 15%

The cash flow diagram of the old diesel engine is shown in Fig. 8.6.

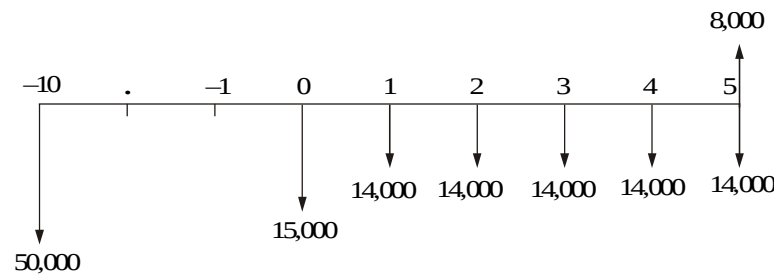


Fig. 8.6 Cash flow diagram for alternative 1.

The formula for the annual equivalent cost is

$$\begin{aligned}
 AE(15\%) &= (P - F)(A/P, 15\%, 5) + F \times i + A \\
 &= (15,000 - 8,000)(0.2983) + 8,000 \times 0.15 + 14,000 \\
 &= \text{Rs. } 17,288.10
 \end{aligned}$$

**Alternative 2—New diesel engine**

Present value ( $P$ ) = Rs. 65,000

Salvage value ( $F$ ) = Rs. 13,000

Annual operating and maintenance cost ( $A$ ) = Rs. 9,000

Life ( $n$ ) = 20 years

Interest rate = 15%

The cash flow diagram of the new diesel engine is shown in Fig. 8.7.

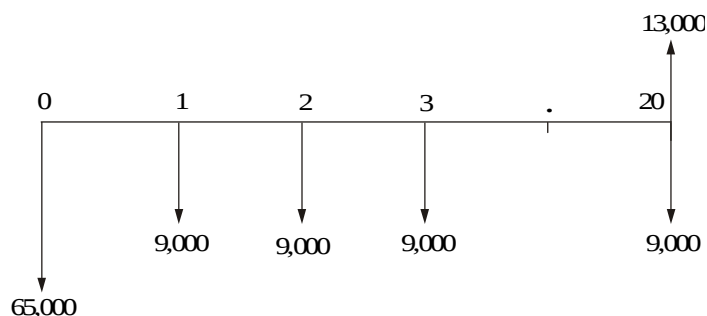


Fig. 8.7 Cash flow diagram for alternative 2.

The formula for the annual equivalent cost is

$$\begin{aligned}
 AE(15\%) &= (P - F)(A/P, 15\%, 20) + F \times i + A \\
 &= (65,000 - 13,000)(0.1598) + 13,000 \times 0.15 + 9,000 \\
 &= \text{Rs. } 19,259.60
 \end{aligned}$$

For comparing the engines based on equal lives (20 years), the annual equivalent figures are given in Fig. 8.8. Equal lives are nothing but the least common multiple of the lives of the alternatives.

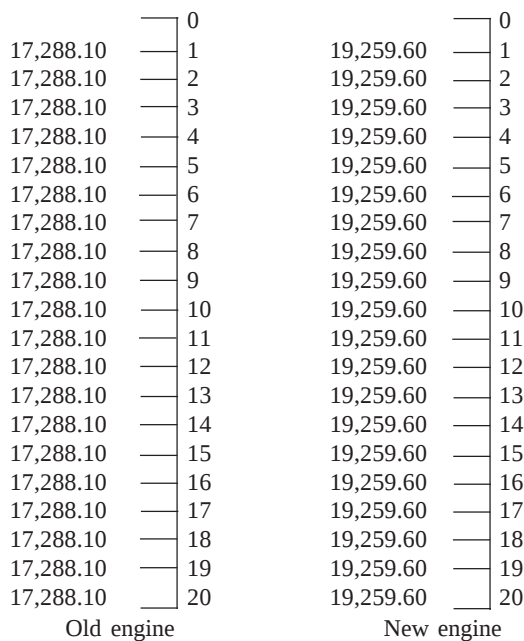


Fig. 8.8 Cash flow diagram of alternatives based on common lives.

Since the annual equivalent cost of the old diesel engine is less than that of the new diesel engine, it is suggested to keep the old diesel engine. Here, an important assumption is that the old engine will be replaced four times during the 20 years period of comparison.

**EXAMPLE 8.6** A steel highway bridge must either be reinforced or replaced. Reinforcement would cost Rs. 6,60,000 and would make the bridge fit for an additional five years of service. If it is reinforced, it is estimated that its net salvage value would be Rs. 4,00,000 at the time it is retired from service. The new prestressed concrete bridge would cost Rs. 15,00,000 and would meet the foreseeable requirements of the next 40 years. Such a bridge would have no salvage value. It is estimated that the annual maintenance cost of the reinforced bridge would exceed that of the concrete bridge by Rs. 96,000. If the bridge is replaced by a new prestressed concrete bridge, the scrap value of the steel would exceed the demolition cost by Rs. 4,20,000. Assume that the money costs the state 10%. What would you recommend?

**Solution** There are two alternatives:

1. Reinforce the existing bridge.
2. Replace the existing bridge by a new prestressed concrete bridge.

**Alternative 1—Reinforce the existing bridge**

Cost of reinforcement ( $P$ ) = Rs. 6,60,000

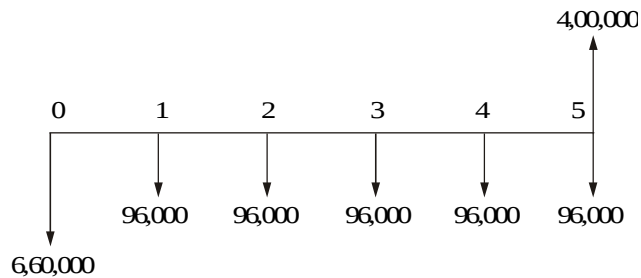
Salvage value after 5 years ( $F$ ) = Rs. 4,00,000

The excess annual maintenance cost over prestressed concrete bridge ( $A$ )  
= Rs. 96,000

Life ( $n$ ) = 5 years

Interest rate ( $i$ ) = 10%

The cash flow diagram of alternative 1 is illustrated in Fig. 8.9.



**Fig. 8.9** Cash flow diagram for alternative 1.

The annual equivalent cost of the alternative 1 is computed as

$$\begin{aligned}
 AE(10\%) &= (P - F)(A/P, 10\%, 5) + F \times i + A \\
 &= (6,60,000 - 4,00,000)(0.2638) + 4,00,000 \times 0.10 + 96,000 \\
 &= \text{Rs. } 2,04,588
 \end{aligned}$$

**Alternative 2—Replace the existing bridge by a new prestressed concrete bridge**

Cost of prestressed concrete bridge ( $P$ ) = Rs. 15,00,000

Excess scrap value of steel over the demolition cost of the current bridge ( $X$ ) = Rs. 4,20,000

Life ( $n$ ) = 40 years  
Interest rate ( $i$ ) = 10%

Note that the excess maintenance cost of the reinforced bridge over the prestressed concrete bridge is included in alternative 1.

The cash flow diagram for alternative 2 is shown in Fig. 8.10.

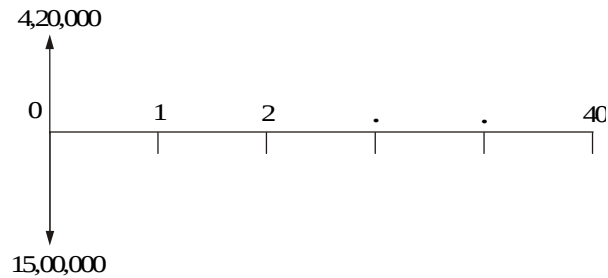


Fig. 8.10 Cash flow diagram for alternative 2.

The annual equivalent cost of alternative 2 is calculated as

$$\begin{aligned} AE(10\%) &= (P - X) (A/P, 10\%, 40) \\ &= (15,00,000 - 4,20,000) \times 0.1023 \\ &= \text{Rs. } 1,10,484 \end{aligned}$$

The annual equivalent cost of alternative 2 is less than that of alternative 1. Based on equal lives comparison over 40 years, alternative 2 is selected as the best alternative.

Thus, it is suggested to go in for prestressed concrete bridge.

**EXAMPLE 8.7** Three years back, a municipality purchased a 10 hp motor for pumping drinking water. Its useful life was estimated to be 10 years. Due to the fast development of that locality, the municipality is unable to meet the current demand for water with the existing motor. The municipality can cope with the situation either by augmenting an additional 5 hp motor or replacing the existing 10 hp motor with a new 15 hp motor. The details of these motors are now tabulated.

|                                                   | Old 10 hp<br>motor | New 5 hp<br>motor | New 15 hp<br>motor |
|---------------------------------------------------|--------------------|-------------------|--------------------|
| Purchase cost ( $P$ ) Rs.                         | 25,000             | 10,000            | 35,000             |
| Life in years ( $n$ )                             | 10                 | 7                 | 7                  |
| Salvage value at the end of<br>machine life (Rs.) | 1,500              | 800               | 4,000              |
| Annual operating & maintenance<br>cost (Rs.)      | 1,600              | 1,000             | 500                |

The current market value of the 10 hp motor is Rs. 10,000. Using an interest rate of 15%, find the best alternative.

**Solution** There are two alternatives to cope with the situation:

1. Augmenting the present 10 hp motor with an additional 5 hp motor.
2. Replacing the present 10 hp motor with a new 15 hp motor.

**Alternative 1—Augmenting the present 10 hp motor with an additional 5 hp motor**

Total annual equivalent cost = Annual equivalent cost of 10 hp motor + Annual equivalent cost of 5 hp motor

**Calculation of annual equivalent cost of 10 hp Motor**

Present market value of the 10 hp motor ( $P$ ) = Rs. 10,000

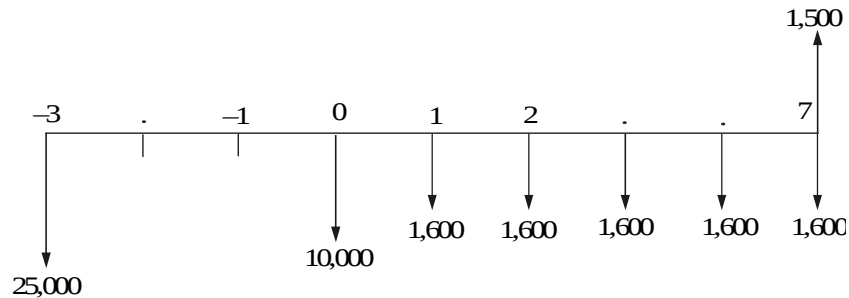
Remaining life ( $n$ ) = 7 years

Salvage value at the end of motor life ( $F$ ) = Rs. 1,500

Annual operation and maintenance cost ( $A$ ) = Rs. 1,600

Interest rate,  $i$  = 15%

The cash flow diagram of this alternative is shown in Fig. 8.11.



**Fig. 8.11** Cash flow diagram for 10 hp motor.

The annual equivalent cost of the 10 hp motor is calculated as

$$\begin{aligned}
 AE(15\%) &= (P - F)(A/P, 15\%, 7) + F \times i + A \\
 &= (10,000 - 1,500)(0.2404) + 1,500 \times 0.15 + 1,600 \\
 &= \text{Rs. } 3,868.40
 \end{aligned}$$

**Calculation of annual equivalent cost of 5 hp motor**

Purchase value of the 5 hp motor ( $P$ ) = Rs. 10,000

Life ( $n$ ) = 7 years

Salvage value at the end of motor life ( $F$ ) = Rs. 800

Annual operation and maintenance cost ( $A$ ) = Rs. 1,000

Interest rate,  $i$  = 15%

The cash flow diagram of the 5 hp motor is illustrated in Fig. 8.12.

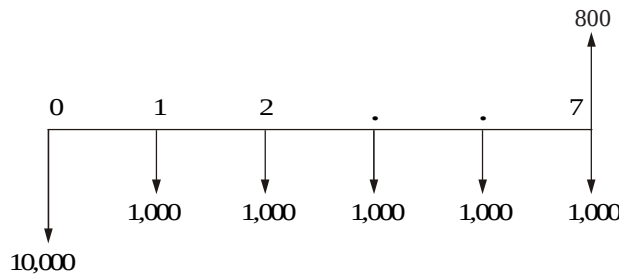


Fig. 8.12 Cash flow diagram for 5 hp motor.

The annual equivalent cost of the 5 hp motor is computed as

$$\begin{aligned}
 AE(15\%) &= (P - F)(A/P, 15\%, 7) + F \times i + A \\
 &= (10,000 - 800)(0.2404) + 800 \times 0.15 + 1,000 \\
 &= \text{Rs. } 3,331.68
 \end{aligned}$$

$$\begin{aligned}
 \text{Total annual equivalent cost of the alternative 1} &= \text{Rs. } 3,868.40 \\
 &+ \text{Rs. } 3,331.68 \\
 &= \text{Rs. } 7,200.08
 \end{aligned}$$

**Alternative 2—Replacing the present 10 hp motor with a new 15 hp motor**

Purchase value of the 15 hp motor ( $P$ ) = Rs. 35,000

Life ( $n$ ) = 7 years

Salvage value at the end of motor life ( $F$ ) = Rs. 4,000

Annual operation and maintenance cost ( $A$ ) = Rs. 500

Interest rate,  $i$  = 15%

The cash flow diagram of this alternative is shown in Fig. 8.13.

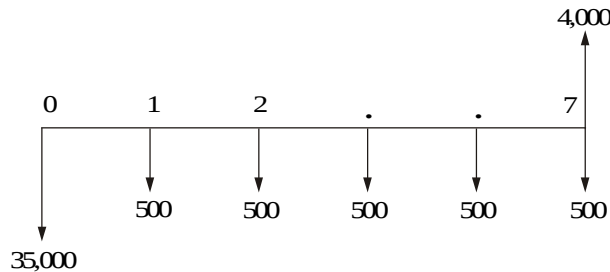


Fig. 8.13 Cash flow diagram for alternative 2.

The annual equivalent cost of alternative 2 is

$$\begin{aligned}
 AE(15\%) &= (P - F)(A/P, 15\%, 7) + F \times i + A \\
 &= (35,000 - 4,000)(0.2404) + 4,000 \times 0.15 + 500 \\
 &= \text{Rs. } 8,552.40
 \end{aligned}$$

The total annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, it is suggested that the present 10 hp motor be augmented with a new 5 hp motor.

**EXAMPLE 8.8** A machine was purchased two years ago for Rs. 10,000. Its annual maintenance cost is Rs. 750. Its life is six years and its salvage value at the end of its life is Rs. 1,000. Now, a company is offering a new machine at a cost of Rs. 10,000. Its life is four years and its salvage value at the end of its life is Rs. 4,000. The annual maintenance cost of the new machine is Rs. 500. The company which is supplying the new machine is willing to take the old machine for Rs. 8,000 if it is replaced by the new machine. Assume an interest rate of 12%, compounded annually.

- Find the comparative use value of the old machine.
- Is it advisable to replace the old machine?

**Solution Old machine** Let the comparative use value of the old machine be  $X$ .

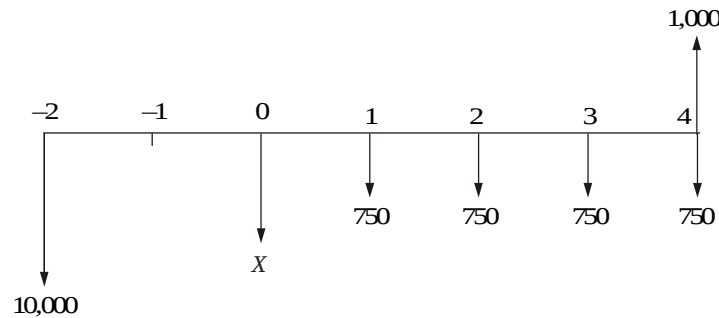
Remaining life ( $n$ ) = 4 years.

Salvage value of the old machine ( $F$ ) = Rs. 1,000

Annual maintenance cost ( $A$ ) = Rs. 750

Interest rate,  $i$  = 12%

The cash flow diagram of the old machine is depicted in Fig. 8.14.



**Fig. 8.14** Cash flow diagram for old machine.

The annual equivalent cost of the old machine is computed as

$$\begin{aligned} AE(12\%) &= (X - F)(A/P, 12\%, 4) + F \times i + A \\ &= (X - 1,000)(0.3292) + 1,000 \times 0.12 + 750 \end{aligned}$$

#### **New machine**

Cost of the new Machine ( $P$ ) = Rs. 10,000

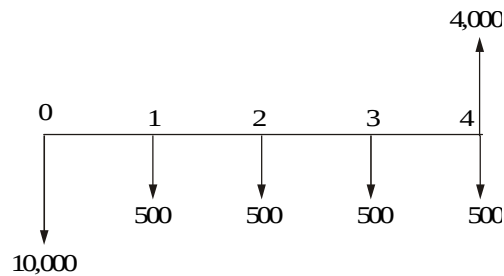
Life ( $n$ ) = 4 years.

Salvage value of the new machine ( $F$ ) = Rs. 4,000

Annual Maintenance cost ( $A$ ) = Rs. 500

Interest rate,  $i$  = 12%

The cash flow diagram of the new machine is illustrated in Fig. 8.15.



**Fig. 8.15** Cash flow diagram for new machine.

The annual equivalent cost of the new machine is illustrated as

$$\begin{aligned}
 AE(12\%) &= (P - F) (A/P, 12\%, 4) + F \times i + A \\
 &= (10,000 - 4,000)(0.3292) + 4,000 \times 0.12 + 500 \\
 &= \text{Rs. } 2,955.20
 \end{aligned}$$

Now, equate the annual equivalent costs of the two alternatives and solve for  $X$ .

$$\begin{aligned}
 (X - 1,000)(0.3292) + 1,000 \times 0.12 + 750 &= 2,955.20 \\
 X &= \text{Rs. } 7,334.14
 \end{aligned}$$

The comparative use value of the old machine is Rs. 7,334.14, which is less than the price (Rs. 8,000) offered by the company which is supplying the new machine in the event of replacing the old machine by the new machine.

Therefore, it is advisable to replace the old machine with the new one.

## 8.6 SIMPLE PROBABILISTIC MODEL FOR ITEMS WHICH FAIL COMPLETELY

Electronic items like transistors, resistors, tubelights, bulbs, etc. could fail all of a sudden, instead of gradual deterioration. The failure of the item may result in complete breakdown of the system. The system may contain a collection of such items or just one item, say a tubelight. Therefore, we use some replacement policy for such items which would avoid the possibility of a complete breakdown.

The following are the replacement policies which are applicable for this situation.

**(i) Individual replacement policy.** Under this policy, an item is replaced immediately after its failure.

**(ii) Group replacement policy.** Under this policy, the following decision is made:

At what equal intervals are all the items to be replaced simultaneously with a provision to replace the items individually which fail during a fixed group replacement period?

There is a trade-off between the individual replacement policy and the group replacement policy. Hence, for a given problem, each of the replacement policies is evaluated and the most economical policy is selected for implementation. This is explained with two numerical problems.

**EXAMPLE 8.9** The failure rates of transistors in a computer are summarized in Table 8.5.

**Table 8.5** Failure Rates of Transistors in Computers

| End of week                    | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|--------------------------------|------|------|------|------|------|------|------|
| Probability of failure to date | 0.07 | 0.18 | 0.30 | 0.48 | 0.69 | 0.89 | 1.00 |

The cost of replacing an individual failed transistor is Rs. 9. If all the transistors are replaced simultaneously, it would cost Rs. 3.00 per transistor. Any one of the following two options can be followed to replace the transistors:

- Replace the transistors individually when they fail (individual replacement policy).
- Replace all the transistors simultaneously at fixed intervals and replace the individual transistors as they fail in service during the fixed interval (group replacement policy).

Find out the optimal replacement policy, i.e. individual replacement policy or group replacement policy. If group replacement policy is optimal, then find at what equal intervals should all the transistors be replaced.

**Solution** Assume that there are 100 transistors in use.

Let,  $p_i$  be the probability that a transistor which was new when placed in position for use, fails during the  $i$ th week of its life. Hence,

$$p_1 = 0.07, \quad p_2 = 0.11, \quad p_3 = 0.12, \quad p_4 = 0.18, \\ p_5 = 0.21, \quad p_6 = 0.20, \quad p_7 = 0.11$$

Since the sum of  $p_i$ s is equal to 1 at the end of the 7th week, the transistors are sure to fail during the seventh week.

Assume that

- transistors that fail during a week are replaced just before the end of the week, and
- the actual percentage of failures during a week for a sub-group of transistors with the same age is same as the expected percentage of failures during the week for that sub-group of transistors.

Let

$$N_i = \text{the number of transistors replaced at the end of the } i\text{th week} \\ N_0 = \text{number of transistors replaced at the end of the week 0 (or at the} \\ \text{beginning of the first week).} \\ = 100$$

$$\begin{aligned}
N1 &= \text{number of transistors replaced at the end of the 1st week} \\
&= N0 \times p1 = 100 \times 0.07 = 7 \\
N2 &= \text{number of transistors replaced at the end of the 2nd week} \\
&= N0 \times p2 + N1 \times p1 \\
&= 100 \times 0.11 + 7 \times 0.07 = 12 \\
N3 &= N0 \times p3 + N1 \times p2 + N2 \times p1 \\
&= 100 \times 0.12 + 7 \times 0.11 + 12 \times 0.07 \\
&= 14 \\
N4 &= N0 \times p4 + N1 \times p3 + N2 \times p2 + N3 \times p1 \\
&= 100 \times 0.18 + 7 \times 0.12 + 12 \times 0.11 + 14 \times 0.07 \\
&= 21 \\
N5 &= N0 \times p5 + N1 \times p4 + N2 \times p3 + N3 \times p2 + N4 \times p1 \\
&= 100 \times 0.21 + 7 \times 0.18 + 12 \times 0.12 + 14 \times 0.11 + 21 \times 0.07 \\
&= 27 \\
N6 &= N0 \times p6 + N1 \times p5 + N2 \times p4 + N3 \times p3 + N4 \times p2 + N5 \times p1 \\
&= 100 \times 0.2 + 7 \times 0.21 + 12 \times 0.18 + 14 \times 0.12 + 21 \times 0.11 + 27 \times 0.07 \\
&= 30 \\
N7 &= N0 \times p7 + N1 \times p6 + N2 \times p5 + N3 \times p4 + N4 \times p3 + N5 \times p2 \\
&\quad + N6 \times p1 \\
&= 100 \times 0.11 + 7 \times 0.2 + 12 \times 0.21 + 14 \times 0.18 + 21 \times 0.12 \\
&\quad + 27 \times 0.11 + 30 \times 0.07 \\
&= 25
\end{aligned}$$

#### Calculation of individual replacement cost

$$\begin{aligned}
\text{Expected life of each transistor} &= \sum_{i=1}^7 i \times p_i \\
&= 1 \times 0.07 + 2 \times 0.11 + 3 \times 0.12 + 4 \times 0.18 \\
&\quad + 5 \times 0.21 + 6 \times 0.2 + 7 \times 0.11 \\
&= 4.39 \text{ weeks}
\end{aligned}$$

$$\text{Average No. of failures/week} = 100/4.39 = 23 \text{ (approx.)}$$

Therefore,

$$\begin{aligned}
&\text{Cost of individual replacement} \\
&= (\text{No. of failures/week} \times \text{Individual replacement cost/transistor}) \\
&= 23 \times 9 = \text{Rs. 207.}
\end{aligned}$$

#### Determination of group replacement cost

$$\begin{aligned}
&\text{Cost of transistor when replaced simultaneously} = \text{Rs. 3} \\
&\text{Cost of transistor when replaced individually} = \text{Rs. 9}
\end{aligned}$$

The costs of group replacement policy for several replacement periods are summarized in Table 8.6.

**Table 8.6** Calculations of Cost for Preventive Maintenance

| End of week | Cost of replacing 100 transistors at a time<br>B (Rs.) | Cost of replacing transistors individually during given replacement period<br>C (Rs.) | Total cost<br>(B + C)<br>D (Rs.) | Average cost/week<br>(D/A)<br>E (Rs.) |
|-------------|--------------------------------------------------------|---------------------------------------------------------------------------------------|----------------------------------|---------------------------------------|
| A           | B (Rs.)                                                | C (Rs.)                                                                               | D (Rs.)                          | E (Rs.)                               |
| 1           | 300                                                    | $7 \times 9 = 63$                                                                     | 363                              | 363.00                                |
| 2           | 300                                                    | $(7 + 12) \times 9 = 171$                                                             | 471                              | 235.50                                |
| 3           | 300                                                    | $(7 + 12 + 14) \times 9 = 297$                                                        | 597                              | 199.00                                |
| 4           | 300                                                    | $(7 + 12 + 14 + 21) \times 9 = 486$                                                   | 786                              | 196.50*                               |
| 5           | 300                                                    | $(7 + 12 + 14 + 21 + 27) \times 9 = 729$                                              | 1,029                            | 205.80                                |
| 6           | 300                                                    | $(7 + 12 + 14 + 21 + 27 + 30) \times 9 = 999$                                         | 1,299                            | 216.50                                |
| 7           | 300                                                    | $(7 + 12 + 14 + 21 + 27 + 30 + 25) \times 9 = 1,224$                                  | 1,524                            | 217.71                                |

\*Indicates the minimum average cost/week.

From Table 8.6, it is clear that the average cost/week is minimum for the fourth week. Hence, the group replacement period is four weeks.

Individual replacement cost/week = Rs. 207

Minimum group replacement cost/week = Rs. 196.50

Since the minimum group replacement cost/week is less than the individual replacement cost/week, the group replacement policy is the best, and hence all the transistors should be replaced once in four weeks and the transistors which fail during this four-week period are to be replaced individually.

**EXAMPLE 8.10** An electronic equipment contains 1,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs. 10. If all the resistors are replaced at the same time, the cost per resistor is Rs. 4. The per cent surviving,  $S(i)$  at the end of month  $i$  is tabulated as follows:

| $i$    | 0   | 1  | 2  | 3  | 4  | 5  | 6 |
|--------|-----|----|----|----|----|----|---|
| $S(i)$ | 100 | 96 | 89 | 68 | 37 | 13 | 0 |

What is the optimum replacement plan?

**Solution** Let  $p_i$  be the probability of failure during the month  $i$ . Then,

$$p_1 = (100 - 96)/100 = 0.04 \quad p_4 = (68 - 37)/100 = 0.31$$

$$p_2 = (96 - 89)/100 = 0.07 \quad p_5 = (37 - 13)/100 = 0.24$$

$$p_3 = (89 - 68)/100 = 0.21 \quad p_6 = (13 - 0)/100 = 0.13$$

It is clear that no resistor can survive beyond six months. Hence, a resistor which has survived for five months would certainly fail during the sixth month. We assume that the resistors failing during a month are accounted at the end of the month.

Let

$N_i$  = No. of resistors replaced at the end of the  $i$ th month.

$$\begin{aligned}
N_0 &= 1,000 \\
N_1 &= N_0 p_1 = 1,000 \times 0.04 = 40 \\
N_2 &= N_0 p_2 + N_1 p_1 = 1,000 \times 0.07 + 40 \times 0.04 = 72 \\
N_3 &= N_0 p_3 + N_1 p_2 + N_2 p_1 = 1,000 \times 0.21 + 40 \times 0.07 \\
&\quad + 72 \times 0.04 = 216 \\
N_4 &= N_0 p_4 + N_1 p_3 + N_2 p_2 + N_3 p_1 \\
&= 1,000 \times 0.31 + 40 \times 0.21 + 72 \times 0.07 + 216 \times 0.04 = 332 \\
N_5 &= N_0 p_5 + N_1 p_4 + N_2 p_3 + N_3 p_2 + N_4 p_1 \\
&= 1,000 \times 0.24 + 40 \times 0.31 + 72 \times 0.21 + 216 \times 0.07 \\
&\quad + 332 \times 0.04 = 296 \\
N_6 &= N_0 p_6 + N_1 p_5 + N_2 p_4 + N_3 p_3 + N_4 p_2 + N_5 p_1 \\
&= 1,000 \times 0.13 + 40 \times 0.24 + 72 \times 0.31 + 216 \times 0.21 + 332 \times 0.07 \\
&\quad + 296 \times 0.04 = 242
\end{aligned}$$

#### Determination of individual replacement cost

$$\begin{aligned}
\text{Expected life of each resistor} &= \sum_{i=1}^6 i \times p_i \\
&= 1 \times 0.04 + 2 \times 0.07 + 3 \times 0.21 + 4 \times 0.31 \\
&\quad + 5 \times 0.24 + 6 \times 0.13 \\
&= 4.03 \text{ months.}
\end{aligned}$$

$$\text{Average number of failures/month} = 1,000/4.03 = 248 \text{ (approx.)}$$

Therefore,

$$\begin{aligned}
&\text{Cost of individual replacement} \\
&= (\text{No. of failures/month} \times \text{individual replacement cost/resistor}) \\
&= 248 \times 10 = \text{Rs. } 2,480.
\end{aligned}$$

#### Determination of group replacement cost

$$\text{Cost/resistor when replaced simultaneously} = \text{Rs. } 4.00$$

$$\text{Cost/resistor when replaced individually} = \text{Rs. } 10.00$$

The costs of group replacement policy for several replacement periods are summarized in Table 8.7.

**Table 8.7** Calculations of Costs for Preventive Maintenance

| End of month | Cost of replacing 1,000 resistors at a time | Cost of replacing resistors individually during given replacement period | Total cost (B + C) | Average cost/month (D/A) |
|--------------|---------------------------------------------|--------------------------------------------------------------------------|--------------------|--------------------------|
| A            | B (Rs.)                                     | C (Rs.)                                                                  | D (Rs.)            | E (Rs.)                  |
| 1            | 4,000                                       | $40 \times 10 = 400$                                                     | 4,400              | 4,400.00                 |
| 2            | 4,000                                       | $(40 + 72)10 = 1,120$                                                    | 5,120              | 2,560.00                 |
| 3            | 4,000                                       | $(40 + 72 + 216)10 = 3,280$                                              | 7,280              | 2,426.67*                |
| 4            | 4,000                                       | $(40 + 72 + 216 + 332)10 = 6,600$                                        | 10,600             | 2,650.00                 |

\*Indicates the minimum average cost/month.

From Table 8.7, it is clear that the average cost/month is minimum for the third month. Thus, the group replacement period is three months.

### Summary

Individual replacement cost/month = Rs. 2,480.00

Minimum group replacement cost/month = Rs. 2,426.67

Since the minimum group replacement cost/month is less than the individual replacement cost/month, the group replacement policy is the best and hence all the resistors are to be replaced once in three months and the resistors which fail during this three months period are to be replaced individually.

## QUESTIONS

1. List and explain the different types of maintenance.
2. Discuss the reasons for replacement.
3. Define 'economic life' of an equipment.
4. Distinguish between breakdown maintenance and preventive maintenance.
5. A firm is considering replacement of an equipment, whose first cost is Rs. 1,750 and the scrap value is negligible at any year. Based on experience, it was found that the maintenance cost is zero during the first year and it increases by Rs. 100 every year thereafter.
  - (a) When should the equipment be replaced if  $i = 0\%$ ?
  - (b) When should the equipment be replaced if  $i = 12\%$ ?
6. The following table gives the operation cost, maintenance cost and salvage value at the end of every year of a machine whose purchase value is Rs. 20,000.
  - (a) Find the economic life of the machine assuming interest rate ( $i$ ) of 0%
  - (b) Find the economic life of the machine assuming interest rate of 15%.

| End of year ( $n$ ) | Operation cost | Maintenance cost | Salvage value |
|---------------------|----------------|------------------|---------------|
| 1                   | 2,000          | 200              | 10,000        |
| 2                   | 3,000          | 300              | 9,000         |
| 3                   | 4,000          | 400              | 8,000         |
| 4                   | 5,000          | 500              | 7,000         |
| 5                   | 6,000          | 600              | 6,000         |
| 6                   | 7,000          | 700              | 5,000         |
| 7                   | 8,000          | 800              | 4,000         |
| 8                   | 9,000          | 900              | 3,000         |
| 9                   | 10,000         | 1,000            | 2,000         |
| 10                  | 11,000         | 1,100            | 1,000         |

7. A manufacturer is offered two machines A and B. A is priced at Rs. 8,000 and maintenance costs are estimated at Rs. 500 for the first year and an equal increment of Rs. 100 from year 2 to year 5, and Rs. 1,500 for the sixth year and an equal increment of Rs. 500 from year 7 onwards.

Machine B which has the same capacity is priced at Rs. 6,000. The maintenance costs of the machine B are estimated at Rs. 1,000 for the first year and an equal yearly increment of Rs. 200 thereafter.

If the money is worth 15% per year, which machine should be purchased? (Assume that the scrap value of each of the machines is negligible at any year.)

8. Three years back, a machine was purchased at a cost of Rs. 3,00,000 to be useful for 10 years. Its salvage value at the end of its estimated life is Rs. 50,000. Its annual maintenance cost is Rs. 40,000. The market value of the present machine is Rs. 2,00,000. A new machine to cater to the need of the present machine is available at Rs. 2,50,000 to be useful for 7 years. Its annual maintenance cost is Rs. 14,000. The salvage value of the new machine is Rs. 20,000. Using an interest rate of 15%, find whether it is worth replacing the present machine with the new one.
9. A steel highway bridge must either be reinforced or replaced. Reinforcement would cost Rs. 8,60,000 and would make the bridge adequate for an additional seven years of service. If it is reinforced, it is estimated that its net salvage value would be Rs. 5,00,000 at the time it is retired from service. The new prestressed concrete bridge would cost Rs. 18,00,000 and would meet the foreseeable requirements of the next 35 years. Such a bridge would have no salvage value. It is estimated that the annual maintenance cost of the reinforced bridge would exceed that of the concrete bridge by Rs. 1,00,000. If the bridge is replaced by a new prestressed concrete bridge, the scrap value of the steel would exceed the demolition cost by Rs. 5,20,000. Assume that the money costs the state 12%. What would you recommend?
10. Three years back, a municipality purchased a 10 hp motor for pumping drinking water. Its useful life was estimated to be 10 years. Its annual operation and maintenance cost is Rs. 1,500. Due to rapid development of that locality, the municipality is unable to meet the current demand for water with the existing motor. The municipality can cope with the situation either by augmenting an additional 5 hp motor or replacing the existing 10 hp motor with a new 15 hp motor. The details of these motors are given in the following table.

|                                                   | <i>Old 10 hp<br/>motor</i> | <i>New 5 hp<br/>motor</i> | <i>New 15 hp<br/>motor</i> |
|---------------------------------------------------|----------------------------|---------------------------|----------------------------|
| Purchase cost (P) Rs.                             | 20,000                     | 8,000                     | 30,000                     |
| Life in years (n)                                 | 10                         | 7                         | 7                          |
| Salvage value at the<br>end of machine life (Rs.) | 1,200                      | 800                       | 3,500                      |
| Annual operating &<br>maintenance cost (Rs.)      | 1,500                      | 900                       | 450                        |

The current market value of the 10 hp motor is Rs. 10,000. Using an interest rate of 18% find the best alternative.

11. The failure rates of transistors in a computer are summarized in the following table.

| End of week                    | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|--------------------------------|------|------|------|------|------|------|------|
| Probability of failure to date | 0.09 | 0.17 | 0.27 | 0.50 | 0.65 | 0.90 | 1.00 |

The cost of replacing an individual failed transistor is Rs. 8. If all the transistors are replaced simultaneously, it would cost Rs. 4 per transistor. Any one of the following two options can be followed to replace the transistors:

- Replace the transistors individually when they fail (individual replacement policy).
- Replace all the transistors simultaneously at fixed intervals and replace the individual transistors as they fail in service during the fixed interval (Group replacement policy).

Find out which is the optimal replacement policy, i.e. the individual replacement policy or the group replacement policy. If the group replacement policy is optimal, then find at what equal intervals should all the transistors be replaced.

12. An electronic equipment contains 1,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs. 7. If all the resistors are replaced at the same time, the cost per resistor is Rs. 4. The per cent surviving,  $S(i)$  at the end of month  $i$  is tabulated now.

| $i$    | 0   | 1  | 2  | 3  | 4  | 5  | 6 |
|--------|-----|----|----|----|----|----|---|
| $S(i)$ | 100 | 90 | 85 | 65 | 35 | 15 | 0 |

What is the optimum replacement plan?

# 9

## DEPRECIATION

### 9.1 INTRODUCTION

Any equipment which is purchased today will not work for ever. This may be due to wear and tear of the equipment or obsolescence of technology. Hence, it is to be replaced at the proper time for continuance of any business. The replacement of the equipment at the end of its life involves money. This must be internally generated from the earnings of the equipment. The recovery of money from the earnings of an equipment for its replacement purpose is called *depreciation fund* since we make an assumption that the value of the equipment decreases with the passage of time. Thus, the word “depreciation” means *decrease* in value of any physical asset with the passage of time.

### 9.2 METHODS OF DEPRECIATION

There are several methods of accounting depreciation fund. These are as follows:

1. Straight line method of depreciation
2. Declining balance method of depreciation
3. Sum of the years—digits method of depreciation
4. Sinking-fund method of depreciation
5. Service output method of depreciation

These are now discussed in detail.

#### 9.2.1 Straight Line Method of Depreciation

In this method of depreciation, a fixed sum is charged as the depreciation amount throughout the lifetime of an asset such that the accumulated sum at the end of the life of the asset is exactly equal to the purchase value of the asset. Here, we make an important assumption that inflation is absent.

Let

$P$  = first cost of the asset,

$F$  = salvage value of the asset,

$n$  = life of the asset,

$B_t$  = book value of the asset at the end of the period  $t$ ,

$D_t$  = depreciation amount for the period  $t$ .

The formulae for depreciation and book value are as follows:

$$D_t = (P - F)/n$$

$$B_t = B_{t-1} - D_t = P - t \times [(P - F)/n]$$

**EXAMPLE 9.1** A company has purchased an equipment whose first cost is Rs. 1,00,000 with an estimated life of eight years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Determine the depreciation charge and book value at the end of various years using the straight line method of depreciation.

**Solution**

$$P = \text{Rs. } 1,00,000$$

$$F = \text{Rs. } 20,000$$

$$n = 8 \text{ years}$$

$$D_t = (P - F)/n$$

$$= (1,00,000 - 20,000)/8$$

$$= \text{Rs. } 10,000$$

In this method of depreciation, the value of  $D_t$  is the same for all the years. The calculations pertaining to  $B_t$  for different values of  $t$  are summarized in Table 9.1.

**Table 9.1**  $D_t$  and  $B_t$  Values under Straight line Method of Depreciation

| End of year<br>( $t$ ) | Depreciation<br>( $D_t$ ) | Book value<br>( $B_t = B_{t-1} - D_t$ ) |
|------------------------|---------------------------|-----------------------------------------|
| 0                      |                           | 1,00,000                                |
| 1                      | 10,000                    | 90,000                                  |
| 2                      | 10,000                    | 80,000                                  |
| 3                      | 10,000                    | 70,000                                  |
| 4                      | 10,000                    | 60,000                                  |
| 5                      | 10,000                    | 50,000                                  |
| 6                      | 10,000                    | 40,000                                  |
| 7                      | 10,000                    | 30,000                                  |
| 8                      | 10,000                    | 20,000                                  |

If we are interested in computing  $D_t$  and  $B_t$  for a specific period ( $t$ ), the formulae can be used. In this approach, it should be noted that the depreciation is the same for all the periods.

**EXAMPLE 9.2** Consider Example 9.1 and compute the depreciation and the book value for period 5.

$$P = \text{Rs. } 1,00,000$$

$$F = \text{Rs. } 20,000$$

$$n = 8 \text{ years}$$

$$D_5 = (P - F)/n$$

$$= (1,00,000 - 20,000)/8$$

$$= \text{Rs. } 10,000 \text{ (This is independent of the time period.)}$$

$$B_t = P - t \times (P - F)/n$$

$$B_5 = 1,00,000 - 5 \times (1,00,000 - 20,000)/8$$

$$= \text{Rs. } 50,000$$

### 9.2.2 Declining Balance Method of Depreciation

In this method of depreciation, a constant percentage of the book value of the previous period of the asset will be charged as the depreciation amount for the current period. This approach is a more realistic approach, since the depreciation charge decreases with the life of the asset which matches with the earning potential of the asset. The book value at the end of the life of the asset may not be exactly equal to the salvage value of the asset. This is a major limitation of this approach.

Let

$P$  = first cost of the asset,

$F$  = salvage value of the asset,

$n$  = life of the asset,

$B_t$  = book value of the asset at the end of the period  $t$ ,

$K$  = a fixed percentage, and

$D_t$  = depreciation amount at the end of the period  $t$ .

The formulae for depreciation and book value are as follows:

$$D_t = K \times B_{t-1}$$

$$B_t = B_{t-1} - D_t = B_{t-1} - K \times B_{t-1}$$

$$= (1 - K) \times B_{t-1}$$

The formulae for depreciation and book value in terms of  $P$  are as follows:

$$D_t = K(1 - K)^{t-1} \times P$$

$$B_t = (1 - K)^t \times P$$

While availing income-tax exception for the depreciation amount paid in each year, the rate  $K$  is limited to at the most  $2/n$ . If this rate is used, then the corresponding approach is called the *double declining balance method of depreciation*.

**EXAMPLE 9.3** Consider Example 9.1 and demonstrate the calculations of the declining balance method of depreciation by assuming 0.2 for  $K$ .

**Solution**

$$P = \text{Rs. } 1,00,000$$

$$F = \text{Rs. } 20,000$$

$$n = 8 \text{ years}$$

$$K = 0.2$$

The calculations pertaining to  $D_t$  and  $B_t$  for different values of  $t$  are summarized in Table 9.2 using the following formulae:

$$D_t = K \times B_{t-1}$$

$$B_t = B_{t-1} - D_t$$

**Table 9.2**  $D_t$  and  $B_t$  according to Declining Balance Method of Depreciation

| End of year<br>( $n$ ) | Depreciation<br>( $D_t$ ) | Book value<br>( $B_t$ ) |
|------------------------|---------------------------|-------------------------|
| 0                      |                           | 1,00,000.00             |
| 1                      | 20,000.00                 | 80,000.00               |
| 2                      | 16,000.00                 | 64,000.00               |
| 3                      | 12,800.00                 | 51,200.00               |
| 4                      | 10,240.00                 | 40,960.00               |
| 5                      | 8,192.00                  | 32,768.00               |
| 6                      | 6,553.60                  | 26,214.40               |
| 7                      | 5,242.88                  | 20,971.52               |
| 8                      | 4,194.30                  | 16,777.22               |

If we are interested in computing  $D_t$  and  $B_t$  for a specific period  $t$ , the respective formulae can be used.

**EXAMPLE 9.4** Consider Example 9.1 and calculate the depreciation and the book value for period 5 using the declining balance method of depreciation by assuming 0.2 for  $K$ .

**Solution**

$$P = \text{Rs. } 1,00,000$$

$$F = \text{Rs. } 20,000$$

$$n = 8 \text{ years}$$

$$K = 0.2$$

$$D_t = K(1 - K)^{t-1} \times P$$

$$\begin{aligned} D_5 &= 0.2(1 - 0.2)^4 \times 1,00,000 \\ &= \text{Rs. } 8,192 \end{aligned}$$

$$\begin{aligned}
 B_t &= (1 - K)^t \times P \\
 B_5 &= (1 - 0.2)^5 \times 1,00,000 \\
 &= \text{Rs. } 32,768
 \end{aligned}$$

### 9.2.3 Sum-of-the-Years-Digits Method of Depreciation

In this method of depreciation also, it is assumed that the book value of the asset decreases at a decreasing rate. If the asset has a life of eight years, first the sum of the years is computed as

$$\begin{aligned}
 \text{Sum of the years} &= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \\
 &= 36 = n(n + 1)/2
 \end{aligned}$$

The rate of depreciation charge for the first year is assumed as the highest and then it decreases. The rates of depreciation for the years 1–8, respectively are as follows: 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36, and 1/36.

For any year, the depreciation is calculated by multiplying the corresponding rate of depreciation with  $(P - F)$ .

$$\begin{aligned}
 D_t &= \text{Rate} \times (P - F) \\
 B_t &= B_{t-1} - D_t
 \end{aligned}$$

The formulae for  $D_t$  and  $B_t$  for a specific year  $t$  are as follows:

$$\begin{aligned}
 D_t &= \frac{n - t + 1}{n(n + 1)/2} (P - F) \\
 B_t &= (P - F) \frac{(n - t)}{n} \frac{(n - t + 1)}{(n + 1)} + F
 \end{aligned}$$

**EXAMPLE 9.5** Consider Example 9.1 and demonstrate the calculations of the sum-of-the-years-digits method of depreciation.

#### Solution

$$P = \text{Rs. } 1,00,000$$

$$F = \text{Rs. } 20,000$$

$$n = 8 \text{ years}$$

$$\text{Sum} = n(n + 1)/2 = 8 \times 9/2 = 36$$

The rates for years 1–8, are respectively 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36 and 1/36.

The calculations of  $D_t$  and  $B_t$  for different values of  $t$  are summarized in Table 9.3 using the following formulae:

$$\begin{aligned}
 D_t &= \text{Rate} \times (P - F) \\
 B_t &= B_{t-1} - D_t
 \end{aligned}$$

**Table 9.3**  $D_t$  and  $B_t$  under Sum-of-the-years-digits Method of Depreciation

| End of year<br>( $n$ ) | Depreciation<br>( $D_t$ ) | Book value<br>( $B_t$ ) |
|------------------------|---------------------------|-------------------------|
| 0                      |                           | 1,00,000.00             |
| 1                      | 17,777.77                 | 82,222.23               |
| 2                      | 15,555.55                 | 66,666.68               |
| 3                      | 13,333.33                 | 53,333.35               |
| 4                      | 11,111.11                 | 42,222.24               |
| 5                      | 8,888.88                  | 33,333.36               |
| 6                      | 6,666.66                  | 26,666.70               |
| 7                      | 4,444.44                  | 22,222.26               |
| 8                      | 2,222.22                  | 20,000.04               |

If we are interested in calculating  $D_t$  and  $B_t$  for a specific  $t$ , then the usage of the formulae would be better.

**EXAMPLE 9.6** Consider Example 9.1 and find the depreciation and book value for the 5th year using the sum-of-the-years-digits method of depreciation.

**Solution**

$$P = \text{Rs. } 1,00,000$$

$$F = \text{Rs. } 20,000$$

$$n = 8 \text{ years}$$

$$D_t = \frac{n - t + 1}{n(n + 1)/2} (P - F)$$

$$\begin{aligned} D_5 &= \frac{8 - 5 + 1}{8(8 + 1)/2} (1,00,000 - 20,000) \\ &= \text{Rs. } 8,888.88 \end{aligned}$$

$$B_t = (P - F) \frac{n - t}{n} \frac{n - t + 1}{n + 1} + F$$

$$\begin{aligned} B_5 &= (1,00,000 - 20,000) \frac{8 - 5}{8} \frac{8 - 5 + 1}{8 + 1} + 20,000 \\ &= 80,000 \times (3/8) \times (4/9) + 20,000 \\ &= \text{Rs. } 33,333.33 \end{aligned}$$

## 9.2.4 Sinking Fund Method of Depreciation

In this method of depreciation, the book value decreases at increasing rates with respect to the life of the asset. Let

$P$  = first cost of the asset,

$F$  = salvage value of the asset,

$n$  = life of the asset,

$i$  = rate of return compounded annually,

$A$  = the annual equivalent amount,

$B_t$  = the book value of the asset at the end of the period  $t$ , and

$D_t$  = the depreciation amount at the end of the period  $t$ .

The loss in value of the asset ( $P - F$ ) is made available in the form of cumulative depreciation amount at the end of the life of the asset by setting up an equal depreciation amount ( $A$ ) at the end of each period during the lifetime of the asset.

$$A = (P - F) \times [A/F, i, n]$$

The fixed sum depreciated at the end of every time period earns an interest at the rate of  $i\%$  compounded annually, and hence the actual depreciation amount will be in the increasing manner with respect to the time period. A generalized formula for  $D_t$  is

$$D_t = (P - F) \times (A/F, i, n) \times (F/P, i, t - 1)$$

The formula to calculate the book value at the end of period  $t$  is

$$B_t = P - (P - F) (A/F, i, n) (F/A, i, t)$$

The above two formulae are very useful if we have to calculate  $D_t$  and  $B_t$  for any specific period. If we calculate  $D_t$  and  $B_t$  for all the periods, then the tabular approach would be better.

**EXAMPLE 9.7** Consider Example 9.1 and give the calculations regarding the sinking fund method of depreciation with an interest rate of 12%, compounded annually.

**Solution**

$$P = \text{Rs. } 1,00,000$$

$$F = \text{Rs. } 20,000$$

$$n = 8 \text{ years}$$

$$i = 12\%$$

$$\begin{aligned} A &= (P - F) \times [A/F, 12\%, 8] \\ &= (1,00,000 - 20,000) \times 0.0813 \\ &= \text{Rs. } 6,504 \end{aligned}$$

In this method of depreciation, a fixed amount of Rs. 6,504 will be depreciated at the end of every year from the earning of the asset. The depreciated amount will earn interest for the remaining period of life of the asset at an interest rate of 12%, compounded annually. For example, the calculations of net depreciation for some periods are as follows:

$$\text{Depreciation at the end of year 1 } (D_1) = \text{Rs. } 6,504.$$

$$\begin{aligned} \text{Depreciation at the end of year 2 } (D_2) &= 6,504 + 6,504 \times 0.12 \\ &= \text{Rs. } 7,284.48 \end{aligned}$$

Depreciation at the end of the year 3 ( $D_3$ )

$$= 6,504 + (6,504 + 7,284.48) \times 0.12$$

$$= \text{Rs. } 8,158.62$$

Depreciation at the end of year 4 ( $D_4$ )

$$= 6,504 + (6,504 + 7,284.48 + 8,158.62) \times 0.12$$

$$= \text{Rs. } 9,137.65$$

These calculations along with book values are summarized in Table 9.4.

**Table 9.4**  $D_t$  and  $B_t$  according to Sinking Fund Method of Depreciation

| End of year<br>$t$    | Fixed<br>depreciation<br>(Rs.) | Net depreciation<br>$D_t$<br>(Rs.) | Book value<br>$B_t$<br>(Rs.) |
|-----------------------|--------------------------------|------------------------------------|------------------------------|
| 0                     | 6,504                          | —                                  | 1,00,000.00                  |
| 1                     | 6,504                          | 6,504.00                           | 93,496.00                    |
| 2                     | 6,504                          | 7,284.48                           | 86,211.52                    |
| 3                     | 6,504                          | 8,158.62                           | 78,052.90                    |
| 4                     | 6,504                          | 9,137.65                           | 68,915.25                    |
| 5                     | 6,504                          | 10,234.17                          | 58,681.08                    |
| 6                     | 6,504                          | 11,462.27                          | 47,218.81                    |
| 7                     | 6,504                          | 12,837.74                          | 34,381.07                    |
| 8                     | 6,504                          | 14,378.27                          | 20,002.80                    |
| $B_t = B_{t-1} - D_t$ |                                |                                    |                              |

**EXAMPLE 9.8** Consider Example 9.1 and compute  $D_5$  and  $B_7$  using the sinking fund method of depreciation with an interest rate of 12%, compounded annually.

**Solution**

$$P = \text{Rs. } 1,00,000$$

$$F = \text{Rs. } 20,000$$

$$n = 8 \text{ years}$$

$$i = 12\%$$

$$D_t = (P - F) (A/F, i, n) (F/P, i, t - 1)$$

$$D_5 = (P - F) (A/F, 12\%, 8) (F/P, 12\%, 4)$$

$$= (1,00,000 - 20,000) \times 0.0813 \times 1.574$$

$$= \text{Rs. } 10,237.30$$

This is almost the same as the corresponding value given in the table. The minor difference is due to truncation error.

$$B_t = P - (P - F) (A/F, i, n) (F/A, i, t)$$

$$B_7 = P - (P - F) (A/F, 12\%, 8) (F/A, 12\%, 7)$$

$$= 1,00,000 - (1,00,000 - 20,000) \times 0.0813 \times 10.089$$

$$= 34,381.10$$

### 9.2.5 Service Output Method of Depreciation

In some situations, it may not be realistic to compute depreciation based on time period. In such cases, the depreciation is computed based on service rendered by an asset. Let

$P$  = first cost of the asset

$F$  = salvage value of the asset

$X$  = maximum capacity of service of the asset during its lifetime

$x$  = quantity of service rendered in a period.

Then, the depreciation is defined per unit of service rendered:

$$\text{Depreciation/unit of service} = (P - F)/X$$

$$\text{Depreciation for } x \text{ units of service in a period} = \frac{P - F}{X}(x)$$

**EXAMPLE 9.9** The first cost of a road laying machine is Rs. 80,00,000. Its salvage value after five years is Rs. 50,000. The length of road that can be laid by the machine during its lifetime is 75,000 km. In its third year of operation, the length of road laid is 2,000 km. Find the depreciation of the equipment for that year.

**Solution**

$$P = \text{Rs. } 80,00,000$$

$$F = \text{Rs. } 50,000$$

$$X = 75,000 \text{ km}$$

$$x = 2,000 \text{ km}$$

$$\text{Depreciation for } x \text{ units of service in a period} = \frac{P - F}{X}x$$

$$\begin{aligned} \text{Depreciation for year 3} &= \frac{(80,00,000 - 50,000)}{75,000} \times 2,000 \\ &= \text{Rs. } 2,12,000 \end{aligned}$$

### QUESTIONS

1. Define the following:
  - (a) Depreciation
  - (b) Book value
2. Distinguish between declining balance method of depreciation and double declining balance method of depreciation.
3. The Alpha Drug Company has just purchased a capsulating machine for Rs. 20,00,000. The plant engineer estimates that the machine has a useful

life of five years and a salvage value of Rs. 25,000 at the end of its useful life. Compute the depreciation schedule for the machine by each of the following depreciation methods:

- (a) Straight line method of depreciation
  - (b) Sum-of-the-years digits method of depreciation
  - (c) Double declining balance method of depreciation
4. A company has recently purchased an overhead travelling crane for Rs. 25,00,000. Its expected life is seven years and the salvage value at the end of the life of the overhead travelling crane is Rs. 1,00,000. Using the straight line method of depreciation, find the depreciation and the book value at the end of third and fourth year after the crane is purchased.
  5. An automobile company has purchased a wheel alignment device for Rs. 10,00,000. The device can be used for 15 years. The salvage value at the end of the life of the device is 10% of the purchase value. Find the following using the double declining balance method of depreciation:
    - (a) Depreciation at the end of the seventh year
    - (b) Depreciation at the end of the twelfth year
    - (c) Book value at the end of the eighth year
  6. A company has purchased a bus for its officers for Rs. 10,00,000. The expected life of the bus is eight years. The salvage value of the bus at the end of its life is Rs. 1,50,000. Find the following using the sinking fund method of depreciation:
    - (a) Depreciation at the end of the third and fifth year
    - (b) Book value at the end of the second year and sixth year
  7. Consider Problem 4 and find the following using the sum-of-the-years-digits method of depreciation:
    - (a) Depreciation at the end of the fourth year
    - (b) Depreciation at the end of the seventh year
    - (c) Book value at the end of the fifth year
    - (d) Book value at the end of the eighth year
  8. A company has purchased a Xerox machine for Rs. 2,00,000. The salvage value of the machine at the end of its useful life would be insignificant. The maximum number of copies that can be taken during its lifetime is 1,00,00,000. During the fourth year of its operation, the number of copies taken is 9,00,000. Find the depreciation for the fourth year of operation of the Xerox machine using the service output method of depreciation.
  9. A heavy construction firm has been awarded a contract to build a large concrete dam. It is expected that a total of eight years will be required to

complete the work. The firm will buy Rs.1,80,00,000 worth of special equipment for the job. During the preparation of the job cost estimate, the following utilization schedule was computed for the special equipment:

| Year     | 1     | 2     | 3     | 4     | 5   | 6   | 7     | 8     |
|----------|-------|-------|-------|-------|-----|-----|-------|-------|
| Hours/yr | 6,000 | 4,000 | 4,000 | 1,600 | 800 | 800 | 2,200 | 2,200 |

At the end of the job, it is estimated that the equipment can be sold at auction for Rs. 18,00,000.

Prepare the depreciation schedule for all the years of operation of the equipment using the service output method of depreciation.

# 10

## EVALUATION OF PUBLIC ALTERNATIVES

### 10.1 INTRODUCTION

In evaluating alternatives of private organizations, the criterion is to select the alternative with the maximum profit. The profit maximization is the main goal of private organizations while providing goods/services as per specifications to their customers. But the same criterion cannot be used while evaluating public alternatives. Examples of some public alternatives are constructing bridges, roads, dams, establishing public utilities, etc.

The main objective of any public alternative is to provide goods/services to the public at the minimum cost. In this process, one should see whether the benefits of the public activity are at least equal to its costs. If yes, then the public activity can be undertaken for implementation. Otherwise, it can be cancelled. This is nothing but taking a decision based on Benefit-Cost ratio (BC) given by

$$\text{BC ratio} = \frac{\text{Equivalent benefits}}{\text{Equivalent costs}}$$

The benefits may occur at different time periods of the public activity. For the purpose of comparison, these are to be converted into a common time base (present worth or future worth or annual equivalent). Similarly, the costs consist of initial investment and yearly operation and maintenance cost. These are to be converted to a common time base as done in the equivalent benefits. Now the ratio between the equivalent benefits and equivalent costs is known as the “Benefit-Cost ratio”. If this ratio is at least one, the public activity is justified; otherwise, it is not justified. Let

$B_P$  = present worth of the total benefits

$B_F$  = future worth of the total benefits

$B_A$  = annual equivalent of the total benefits

$P$  = initial investment

$P_F$  = future worth of the initial investment

$P_A$  = annual equivalent of the initial investment

$C$  = yearly cost of operation and maintenance

$C_P$  = present worth of yearly cost of operation and maintenance

$C_F$  = future worth of yearly cost of operation and maintenance

$$\text{BC ratio} = \frac{B_P}{P + C_P} = \frac{B_F}{P_F + C_F} = \frac{B_A}{P_A + C}$$

## 10.2 EXAMPLES

**EXAMPLE 10.1** In a particular locality of a state, the vehicle users take a roundabout route to reach certain places because of the presence of a river. This results in excessive travel time and increased fuel cost. So, the state government is planning to construct a bridge across the river. The estimated initial investment for constructing the bridge is Rs. 40,00,000. The estimated life of the bridge is 15 years. The annual operation and maintenance cost is Rs. 1,50,000. The value of fuel savings due to the construction of the bridge is Rs. 6,00,000 in the first year and it increases by Rs. 50,000 every year thereafter till the end of the life of the bridge. Check whether the project is justified based on BC ratio by assuming an interest rate of 12%, compounded annually.

### Solution

Initial investment = Rs. 40,00,000

Annual operation and maintenance = Rs. 1,50,000

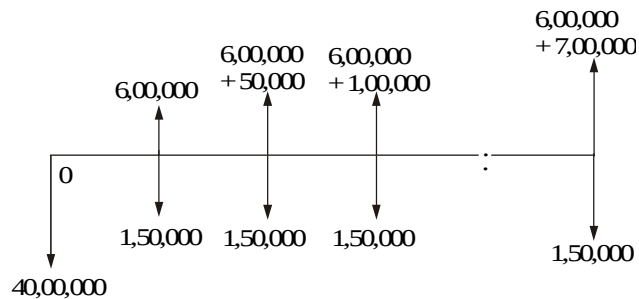
Annual fuel savings during the first year = Rs. 6,00,000

Equal increment in fuel savings in the following years = Rs. 50,000

Life of the project = 15 years

Interest rate = 12%

The cash flow diagram of the project is shown in Fig. 10.1.



**Fig. 10.1** Cash flow diagram for constructing bridge.

$$\begin{aligned} \text{Total present worth of costs} &= \text{Initial investment (P)} \\ &\quad + \text{Present worth of annual operating} \\ &\quad \text{and maintenance cost (C}_P\text{)} = P + C_P \\ &= \text{Rs. } 40,00,000 + 1,50,000 \times (P/A, 12\%, 15) \\ &= \text{Rs. } 40,00,000 + 1,50,000 \times 6.8109 \\ &= \text{Rs. } 50,21,635 \end{aligned}$$

Total present worth of fuel savings ( $B_P$ ):

$$A_1 = \text{Rs. } 6,00,000$$

$$G = \text{Rs. } 50,000$$

$$n = 15 \text{ years}$$

$$i = 12\%$$

$$\begin{aligned} \text{Annual equivalent fuel savings (A)} &= A_1 + G (A/G, 12\%, 15) \\ &= 6,00,000 + 50,000 (4.9803) \\ &= \text{Rs. } 8,49,015 \end{aligned}$$

$$\begin{aligned} \text{Present worth of the fuel savings (B}_P\text{)} &= A(P/A, 12\%, 15) \\ &= 8,49,015 (6.8109) \\ &= \text{Rs. } 57,82,556 \end{aligned}$$

$$\text{BC ratio} = \frac{B_P}{P + C_P} = \frac{57,82,556}{50,21,635} = 1.1515$$

Since the BC ratio is more than 1, the construction of the bridge across the river is justified.

**EXAMPLE 10.2** A state government is planning a hydroelectric project for a river basin. In addition to the production of electric power, this project will provide flood control, irrigation and recreation benefits. The estimated benefits and costs that are expected to be derived from this project are as follows:

Initial cost = Rs. 8,00,00,000

Annual power sales = Rs. 60,00,000

Annual flood control savings = Rs. 30,00,000

Annual irrigation benefits = Rs. 50,00,000

Annual recreation benefits = Rs. 20,00,000

Annual operating and maintenance costs = Rs. 30,00,000

Life of the project = 50 years

Check whether the state government should implement the project (Assume  $i = 12\%$ )

**Solution**

Initial cost = Rs. 8,00,00,000

Annual power sales = Rs. 60,00,000

Annual flood control savings = Rs. 30,00,000

Annual irrigation benefits = Rs. 50,00,000

Annual recreation benefits = Rs. 20,00,000

Annual operating and maintenance costs = Rs. 30,00,000

Life of the project = 50 years,  $i = 12\%$

$$\begin{aligned} \text{Total annual benefits} &= \text{Flood control savings} + \text{Irrigation benefits} \\ &\quad + \text{Recreation benefits} \\ &= \text{Rs. } 30,00,000 + \text{Rs. } 50,00,000 + \text{Rs. } 20,00,000 \\ &= \text{Rs. } 1,00,00,000 \end{aligned}$$

$$\begin{aligned}
 \text{Present worth of the benefits} &= \text{Total annual benefits} \times (P/A, 12\%, 50) \\
 &= 1,00,00,000 \times (8.3045) \\
 &= \text{Rs. } 8,30,45,000
 \end{aligned}$$

$$\begin{aligned}
 \text{Present worth of costs} &= \text{Initial cost} + \text{Present worth of annual operating} \\
 &\quad \text{and maintenance cost} \\
 &\quad - \text{Present worth of power sales} \\
 &= \text{Rs. } 8,00,00,000 + 30,00,000 \times (P/A, 12\%, 50) \\
 &\quad - 60,00,000 (P/A, 12\%, 50) \\
 &= \text{Rs. } 8,00,00,000 + 30,00,000 \times 8.3045 \\
 &\quad - 60,00,000 \times 8.3045 \\
 &= \text{Rs. } 5,50,86,500
 \end{aligned}$$

$$\begin{aligned}
 \text{BC ratio} &= \frac{\text{Present worth of benefits}}{\text{Present worth of costs}} \\
 &= \frac{8,30,45,000}{5,50,86,500} = 1.508
 \end{aligned}$$

Since, the BC ratio is more than 1, the state government can implement the hydroelectric project.

**EXAMPLE 10.3** Two mutually exclusive projects are being considered for investment. Project A1 requires an initial outlay of Rs. 30,00,000 with net receipts estimated as Rs. 9,00,000 per year for the next 5 years. The initial outlay for the project A2 is Rs. 60,00,000, and net receipts have been estimated at Rs. 15,00,000 per year for the next seven years. There is no salvage value associated with either of the projects. Using the benefit cost ratio, which project would you select? Assume an interest rate of 10%.

**Solution Alternative A1**

$$\text{Initial cost } (P) = \text{Rs. } 30,00,000$$

$$\text{Net benefits/year } (B) = \text{Rs. } 9,00,000$$

$$\text{Life } (n) = 5 \text{ years}$$

$$\begin{aligned}
 \text{Annual equivalent of initial cost} &= P \times (A/P, 10\%, 5) \\
 &= 30,00,000 \times 0.2638 \\
 &= \text{Rs. } 7,91,400
 \end{aligned}$$

$$\begin{aligned}
 \text{Benefit-cost ratio} &= \frac{\text{Annual equivalent benefit}}{\text{Annual equivalent cost}} \\
 &= 9,00,000/7,91,400 \\
 &= 1.137
 \end{aligned}$$

**Alternative A2**

Initial cost ( $P$ ) = Rs. 60,00,000

Net benefits/year ( $B$ ) = Rs. 15,00,000

Life ( $n$ ) = 7 years

Annual equivalent of initial cost =  $P \times (A/P, 10\%, 7)$   
 $= 60,00,000 \times 0.2054$   
 $= \text{Rs. } 12,32,400$

$$\text{BC ratio} = \frac{\text{Annual equivalent benefit}}{\text{Annual equivalent cost}}$$

$$= 15,00,000 / 12,32,400 = 1.217$$

The benefit-cost ratio of alternative A2 is more than that of alternative A1. Hence, alternative A2 is to be selected. The comparison is made on a 35-year period which is the minimum common multiple of the lives of alternatives 1 and 2.

**EXAMPLE 10.4** An inland state is presently connected to a seaport by means of a railroad system. The annual goods transported is 1,00,00,000 ton km. The average transport charge is Rs. 30/ton/km. Within the next 20 years, the transport is likely to increase by 10,00,000 ton km per year.

It is proposed to broaden a river flowing from the state to the seaport at a cost of Rs. 2,50,00,00,000. This will make the river navigable to barges and will reduce the transport cost to Rs. 10.00/ton/km. The project will be financed by 10% bond at par. There would be some side effects of the change-over as follows.

1. The railroad would be bankrupt and be sold for no salvage value. The right of way, worth about Rs. 3,00,00,000, will revert to the state.
2. 300 employees will be out of employment. The state will have to pay to each of them a welfare cheque of Rs. 48,000/year.
3. The reduction in the income from the taxes on the railroad will be compensated by the taxes on the barges.

What is the benefit-cost ratio based on the next 20 years of operation? Also, check whether broadening the river is justified.

**Solution Cost to the state**

Life of the project = 20 years

Total cost of the project,  $P$  = Rs. 2,50,00,00,000

Annual goods transported = 1,00,00,000 ton km

Current average transport cost = Rs. 30/ton/km

Annual increase in goods transported = 10,00,000 ton km

Compensation for employees = Rs. 48,000/year/employee for 300 employees.

$$\text{Annual compensation (C1)} = 48,000 \times 300 = \text{Rs. } 1,44,00,000$$

$$\begin{aligned}\text{Annual equivalent initial cost (C2)} &= P \times (A/P, 10\%, 20) \\ &= 2,50,00,00,000 \times 0.1175 \\ &= \text{Rs. } 29,37,50,000\end{aligned}$$

$$\begin{aligned}\text{Total annual equivalent cost of the project, } C_A &= \text{Annual equivalent initial cost} \\ &+ \text{Annual compensation} \\ &= \text{Rs. } 29,37,50,000 + \text{Rs. } 1,44,00,000 \\ &= \text{Rs. } 30,81,50,000\end{aligned}$$

### ***Benefit to the state***

$$\begin{aligned}\text{Worth of the right of the way of the railroad to the state} &= \text{Rs. } 3,00,00,000\end{aligned}$$

$$\begin{aligned}\text{Annual equivalent of the above amount (A1)} &= 3,00,00,000 \times (A/P, 10\%, 20) \\ &= 3,00,00,000 \times 0.1175 \\ &= \text{Rs. } 35,25,000\end{aligned}$$

$$\text{Average goods transported/year} = 1,00,00,000 \text{ ton/year}$$

$$\begin{aligned}\text{Average transport cost savings for year 1, (A2)} &= 1,00,00,000(30 - 10) \\ &= \text{Rs. } 20,00,00,000\end{aligned}$$

$$\text{Average annual increase in goods transport} = 10,00,000 \text{ tons km.}$$

$$\begin{aligned}\text{Average increase in transportation cost savings/year (G)} &= \text{Rs. } 2,00,00,000\end{aligned}$$

$$\begin{aligned}\text{Equivalent annual average transport cost savings (A3)} &= A2 + G(A/G, 10\%, 20) \\ &= 20,00,00,000 + 2,00,00,000 \times 6.5081 \\ &= \text{Rs. } 33,01,62,000\end{aligned}$$

$$\begin{aligned}\text{Total annual equivalent benefits to the state, } B_A &= A1 + A3 \\ &= \text{Rs. } 35,25,000 + \text{Rs. } 33,01,62,000 \\ &= \text{Rs. } 33,36,87,000\end{aligned}$$

$$\begin{aligned}
 \text{BC ratio} &= \frac{\text{Total annual equivalent benefit}}{\text{Total annual equivalent cost}} \\
 &= \frac{B_A}{C_A} \\
 &= \text{Rs. } 33,36,87,000 / \text{Rs. } 30,81,50,000 \\
 &= 1.08287 = 1.1 \text{ (approx.)}
 \end{aligned}$$

Since the BC ratio is more than 1, the project is justified.

**EXAMPLE 10.5** A government is planning a hydroelectric project for a river basin. In addition to the production of electric power, this project will provide flood control, irrigation and recreation benefits. The estimated benefits and costs that are expected from the three alternatives under consideration are given in the following table.

|                                      | A<br>(Rs.)   | B<br>(Rs.)   | C<br>(Rs.)   |
|--------------------------------------|--------------|--------------|--------------|
| Initial cost ( <i>P</i> )            | 15,00,00,000 | 25,00,00,000 | 40,00,00,000 |
| Annual equivalent<br>benefits & cost |              |              |              |
| (a) Operating & maintenance<br>cost  | 20,00,000    | 25,00,000    | 35,00,000    |
| (b) Power sales/year                 | 1,00,00,000  | 1,20,00,000  | 1,80,00,000  |
| (c) Flood control savings            | 25,00,000    | 35,00,000    | 50,00,000    |
| (d) Irrigation benefits              | 35,00,000    | 45,00,000    | 60,00,000    |
| (e) Recreation benefits              | 10,00,000    | 20,00,000    | 35,00,000    |

If the interest rate is 9% and the life of projects is estimated to be 50 years, by comparing the BC ratios, determine which project should be selected.

**Solution**

Benefits/year = Flood control savings + irrigation benefits  
+ Recreation benefits.

Costs/year = Annual equivalent cost of the initial cost  
+ Operating and maintenance cost/year  
– Power sales/year

Based on these guidelines, the computation of benefits, costs and BC ratio for each of the projects are summarized in Table 10.1.

**Table 10.1** Illustration of Example 10.5 (Interest rate = 9%,  $n = 50$  years)

|                                                                                                          | A<br>(Rs.)   | B<br>(Rs.)   | C<br>(Rs.)   |
|----------------------------------------------------------------------------------------------------------|--------------|--------------|--------------|
| 1. Initial cost ( $P$ )                                                                                  | 15,00,00,000 | 25,00,00,000 | 40,00,00,000 |
| 2. Annual equivalent of the<br>initial cost<br>$[P \times (A/P, 9\%, 50)]$<br>i.e. ( $P \times 0.0912$ ) | 1,36,80,000  | 2,28,00,000  | 3,64,80,000  |
| 3. Operating & maintenance cost                                                                          | 20,00,000    | 25,00,000    | 35,00,000    |
| 4. Power sales/year                                                                                      | 1,00,00,000  | 1,20,00,000  | 1,80,00,000  |
| 5. Cost/year ( $2 + 3 - 4$ )                                                                             | 56,80,000    | 1,33,00,000  | 2,19,80,000  |
| 6. Flood control savings                                                                                 | 25,00,000    | 35,00,000    | 50,00,000    |
| 7. Irrigation benefits                                                                                   | 35,00,000    | 45,00,000    | 60,00,000    |
| 8. Recreation benefits                                                                                   | 10,00,000    | 20,00,000    | 35,00,000    |
| 9. Benefit/year ( $6 + 7 + 8$ )                                                                          | 70,00,000    | 1,00,00,000  | 1,45,00,000  |
| 10. Benefit-cost ratio                                                                                   | 1.2324       | 0.7519       | 0.6597       |

From the last row of Table 10.1, it is clear that alternative  $A$  is the only eligible alternative because the BC ratio of each of the other two alternatives is less than one. Since  $A$  is the only eligible alternative, it is selected as the best alternative for implementation.

## QUESTIONS

1. Discuss the difference in evaluating alternatives of private and public organizations.
2. Consider the evaluation of the alternative of constructing a bridge across a river. List the different benefits and costs related to this alternative.
3. In a particular locality of a state, presently, the vehicle users take a roundabout route to reach certain places because of the presence of a river. This results in excessive time of travel and increased fuel cost. So, the state government is planning to construct a bridge across the river. The estimated initial investment for constructing the bridge is Rs. 40,00,000. The estimated life of the bridge is 15 years. The annual operation and maintenance cost is Rs. 2,50,000. The value of fuel savings due to the construction of the bridge is Rs. 6,00,000 in the first year and it increases by Rs. 50,000 every year thereafter till the end of the life of the bridge. Check whether the project is justified based on BC ratio by assuming an interest rate of 20%, compounded annually.
4. A state government is planning a hydroelectric project for a river basin. In addition to the production of electric power, this project will provide flood control, irrigation and recreation benefits. The estimated benefits and costs that are expected to be derived from this project are listed below.

Initial cost = Rs. 18,00,00,000

Annual power sales = Rs. 1,20,00,000

Annual flood control savings = Rs. 50,00,000

Annual irrigation benefits = Rs. 80,00,000

Annual recreation benefits = Rs. 40,00,000

Annual operating and maintenance costs = Rs. 50,00,000

Life of the project = 40 years

Check whether the state government should implement the project (assume  $i = 15\%$ ).

5. Two mutually exclusive projects are being considered for investment. Project A1 requires an initial outlay of Rs. 50,00,000 with net receipts estimated to be Rs. 11,00,000 per year for the next eight years. The initial outlay for the project A2 is Rs. 80,00,000, and net receipts have been estimated at Rs. 20,00,000 per year for the next eight years. There is no salvage value associated with either of the projects. Using the BC ratio, which project would you select? Assume an interest rate of 15%.

6. An inland state is presently connected to a seaport by means of a railroad system. The annual goods transported amount to 1,50,00,000 ton kilometre. The average transport charge is Rs. 40/ton/km. Within the next 25 years, the transport is likely to increase by 15,00,000 ton kilometre per year.

It is proposed to improve a river flowing from the state to the seaport at a cost of Rs. 3,00,00,00,000. This will make the river navigable to barges and will reduce the transport cost to Rs. 12.00/ton/km. The project will be financed by 12% bond at par. There would be some side effects of the change-over as follows.

- (a) The railroad would be bankrupt and would be sold for no salvage value. The right of way, worth about Rs. 4,00,00,000, will revert to the state.
- (b) 400 employees will be out of employment. The state will have to pay to each of them a welfare cheque of Rs. 60,000/year.
- (c) The reduction in the income from the taxes on the railroad will be compensated by the taxes on the barges.

What is the BC ratio based on the next 25 years of operation? Also, check whether broadening the river is justified.

7. A government is planning a hydroelectric project for a river basin. Besides the production of electric power, this project will provide flood control, irrigation and recreation benefits. The estimated benefits and costs expected from the three alternatives under consideration are listed in the following table:

|                                      | A<br>(Rs.)   | B<br>(Rs.)   | C<br>(Rs.)   |
|--------------------------------------|--------------|--------------|--------------|
| Initial cost ( <i>P</i> )            | 25,00,00,000 | 35,00,00,000 | 50,00,00,000 |
| Annual equivalent<br>benefits & cost |              |              |              |
| (a) Operating & maintenance<br>cost  | 30,00,000    | 35,00,000    | 45,00,000    |
| (b) Power sales/year                 | 2,00,00,000  | 2,20,00,000  | 2,80,00,000  |
| (c) Flood control savings            | 35,00,000    | 45,00,000    | 60,00,000    |
| (d) Irrigation benefits              | 45,00,000    | 55,00,000    | 70,00,000    |
| (e) Recreation benefits              | 20,00,000    | 30,00,000    | 45,00,000    |

If the interest rate is 10% and the life of the projects is estimated to be 40 years, by comparing the BC ratios, determine which project should be selected.

# 11

## INFLATION ADJUSTED DECISIONS

### 11.1 INTRODUCTION

A general inflationary trend in the cost of goods is common everywhere due to various interacting factors. If the rate of inflation is very high, it will produce extremely serious consequences for both individuals and institutions.

Inflation is the rate of increase in the prices of goods per period. So, it has a compounding effect. Thus, prices that are inflated at a rate of 7% per year will increase 7% in the first year, and for the next year the expected increase will be 7% of these new prices. The same is true for succeeding years and hence the rate of inflation is compounded in the same manner that an interest rate is compounded. If the average inflation over six years period is 7%, then the prices at the beginning of the seventh year would be 150% that of the first year by assuming 100% for the prices at the beginning of the first year of the six-year period.

If economic decisions are taken without considering the effect of inflation into account, most of them would become meaningless and as a result the organizations would end up with unpredictable return.

But there is always difficulty in determining the rate of inflation. The world-wide trend/wish is to curtail inflation. But due to various reasons, it is very difficult to have zero inflation. For practical decision making, an average estimate may be assumed depending on the period of the proposals under consideration. Hence, we need a procedure which will combine the effects of inflation rate and interest rate to take realistic economic decision.

### 11.2 PROCEDURE TO ADJUST INFLATION

A procedure to deal with this situation is summarized now.

1. Estimate all the costs/returns associated with an investment proposal in terms of today's rupees.
2. Modify the costs/returns estimated in step 1 using an assumed inflation rate so that at each future date they represent the costs/returns at that date in terms of the rupees that must be expended/received at that time, respectively.
3. As per our requirement, calculate either the annual equivalent amount or future amount or present amount of the cash flow resulting from step 2 by considering the time value of money.

**EXAMPLE 11.1** Suppose a 40-year old man is planning for his retirement. He plans to retire at the age of 60 and estimates that he can live comfortably on Rs. 24,000 per year in terms of today's rupee value. He can invest his savings at 15% compounded annually. Assume an average inflation rate of 9% for the next 30 years.

What equal amount should he save each year until he retires so that he can make withdrawals at the end of each year commencing from the end of the 21st year from now that will allow him to live as comfortably as he desires for 10 years beyond his retirement?

**Solution**

*Step 1.* The estimated future requirement per year in terms of today's rupees from his age 61 through 70 is Rs. 24,000.

*Step 2.* Modification of the costs estimated in step 1 is summarized in Table 11.1. The formula which is given below is used to get future equivalent of Rs. 24,000 with the inflation of 9% per year (IR=inflation rate).

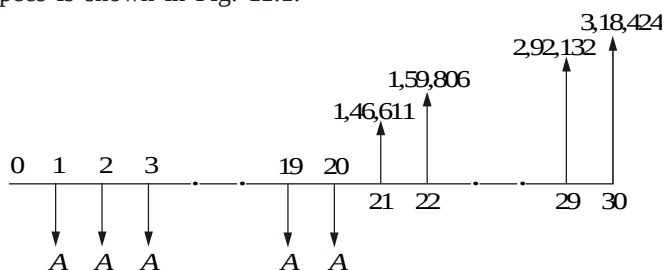
$$F = P(1 + IR)^n$$

**Table 11.1** Inflated Future Requirements

| End of year | Age (years) | Inflated value of Rs. 24,000 at each year end          |
|-------------|-------------|--------------------------------------------------------|
| 21          | 61          | $24,000 \times (1 + 0.09)^{21} = \text{Rs. } 1,46,611$ |
| 22          | 62          | $24,000 \times (1 + 0.09)^{22} = \text{Rs. } 1,59,806$ |
| 23          | 63          | $24,000 \times (1 + 0.09)^{23} = \text{Rs. } 1,74,189$ |
| 24          | 64          | $24,000 \times (1 + 0.09)^{24} = \text{Rs. } 1,89,866$ |
| 25          | 65          | $24,000 \times (1 + 0.09)^{25} = \text{Rs. } 2,06,954$ |
| 26          | 66          | $24,000 \times (1 + 0.09)^{26} = \text{Rs. } 2,25,580$ |
| 27          | 67          | $24,000 \times (1 + 0.09)^{27} = \text{Rs. } 2,45,882$ |
| 28          | 68          | $24,000 \times (1 + 0.09)^{28} = \text{Rs. } 2,68,011$ |
| 29          | 69          | $24,000 \times (1 + 0.09)^{29} = \text{Rs. } 2,92,132$ |
| 30          | 70          | $24,000 \times (1 + 0.09)^{30} = \text{Rs. } 3,18,424$ |

*Step 3.* Now, the calculation of the equivalent amount of cash flow as per the requirement is presented.

The overall cash flow diagram for the savings and withdrawal in terms of future rupees is shown in Fig. 11.1.



**Fig. 11.1** Overall cash flow diagram.

The sum of the present equivalents of the year end withdrawals from the year 21 to 30 is computed by assuming the end of the year 20 as the base (time zero) and it is shown at the end of the year 20 in Fig. 11.2. The method of computing the present equivalent of the withdrawals is as follows:

$$\begin{aligned}
 PW(i = 15\%) &= 1,46,611/(1 + 0.15)^1 + 1,59,806/(1 + 0.15)^2 \\
 &\quad + 1,74,189/(1 + 0.15)^3 + 1,89,866/(1 + 0.15)^4 \\
 &\quad + 2,06,954/(1 + 0.15)^5 + 2,25,580/(1 + 0.15)^6 \\
 &\quad + 2,45,882/(1 + 0.15)^7 + 2,68,011/(1 + 0.15)^8 \\
 &\quad + 2,92,132/(1 + 0.15)^9 + 3,18,424/(1 + 0.15)^{10} \\
 &= \text{Rs. } 10,13,631.
 \end{aligned}$$

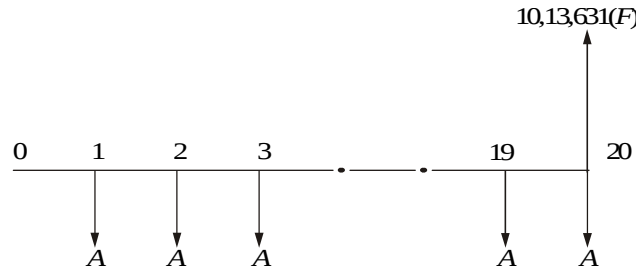


Fig. 11.2 Equivalent cash flow diagram.

The annual equivalent amount ( $A$ ), which should be invested from the end of year 1 (age 41) to year 20 (age 60), is computed using the following formula.

$$\begin{aligned}
 A &= F(A/F, 15\%, 20) \\
 &= 10,13,631 \times (0.0098) \\
 &= \text{Rs. } 9,934
 \end{aligned}$$

**Recommendation:** The person has to invest an amount of Rs. 9,934 at the end of every year starting from his age 41 (year 1) through 60 (year 20) which will enable him to make withdrawals at the end of every year starting from his age 61 (year 21) through 70 (year 30) as shown in the Table 11.1 (also in Fig. 11.1).

### 11.3 INFLATION ADJUSTED ECONOMIC LIFE OF MACHINE

(Panneerselvam, 1998)

In any industrial/service organization, equipment/machinery forms an important element. The productivity of any organization is a function of many factors. It is largely affected by efficient and effective use of machinery and equipment. So, operations and maintenance of these equipment are very important to the organization.

A machine which is purchased today cannot be used forever. It has a definite economic lifetime. After the economic life, the machine should be replaced with a substitute machine with similar operational capabilities. This kind of analysis is called *replacement analysis*.

The elements of costs involved in the replacement analysis are as follows:

1. Purchase cost (initial cost)
2. Annual operation and maintenance cost
3. Salvage value at the end of every year, if it is significant

The trade-off between different cost elements is shown in Fig. 11.3.

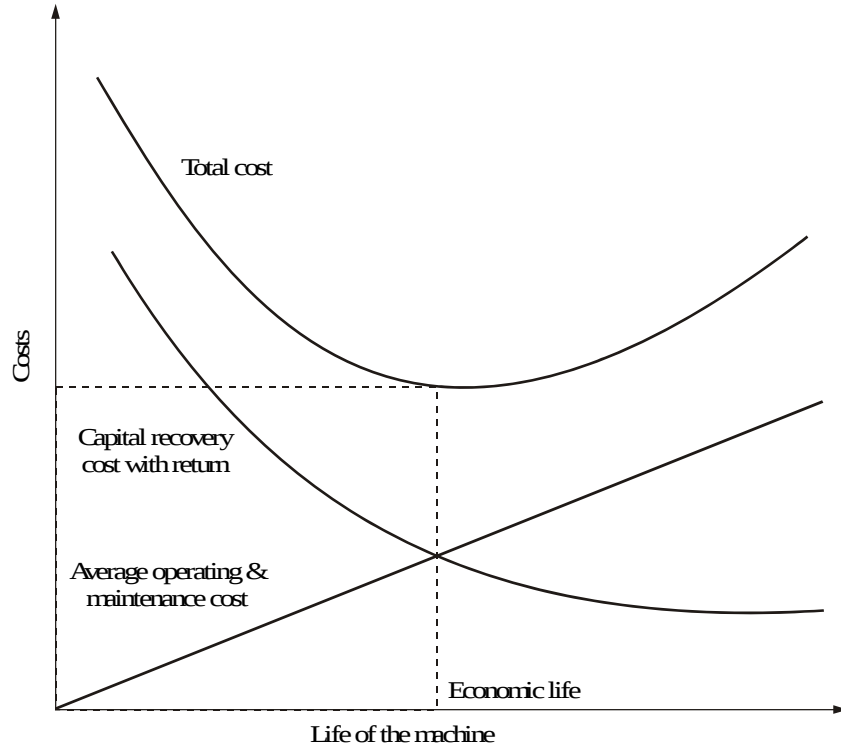


Fig. 11.3 Chart showing economic life.

From Fig. 11.3, it is clear that the sum of operation and maintenance cost increases with the life of the machine. But the capital recovery with return decreases with the life of the machine. The total cost of the machine goes on decreasing initially but it starts increasing after some years. The year with the minimum total cost is called as the economic life of the machine.

### 11.3.1 Limitation of Existing Model

In the case where the machine is replaced due to wear and tear, the following costs are considered (refer Chapter 8):

1. Initial cost
2. Operation and maintenance cost
3. Salvage value

In the existing model to deal with this type of replacement analysis, the different cost elements are estimated without taking the effect of inflation into account.

The annual cost of operation and maintenance of the machine will increase with the age of the machine due to decline in efficiency of the machine. In the existing model, this increase in the operation and maintenance cost is taken into account. But the increase in the operation and maintenance cost due to inflation is not considered. Similarly, in the existing model, the salvage value is estimated without taking into account the effect of inflation.

To highlight this particular fact on salvage value, an example is now given.

The internal combustion engines (R.A. Lister) which were made in England during pre-independence of India are still functioning well. Their resale value is going up year after year. This may be partly due to inflation and partly due to good quality of the engine parts. So, consideration of the effect of the inflation on the economic life of the machine is a realistic approach.

In replacement analysis, a discount rate is usually assumed to reflect the time value of money. First the concept of replacement analysis is demonstrated without taking the inflation into account. Then, the same is demonstrated by taking the effect of inflation into account. At the end, a comparison between the two models is presented.

### 11.3.2 Economic Life Determination without Inflationary Effect

The determination of economic life of a machine without considering the effect of inflation is demonstrated using the following example.

**EXAMPLE 11.2** A machine costs Rs. 5,00,000. Its annual operation cost during the first year is Rs. 40,000 and it increases by Rs. 5,000 every year thereafter. The maintenance cost during the first year is Rs. 60,000 and it increases by Rs. 6,000 every year thereafter. The resale value of the machine is Rs. 4,00,000 at the end of the first year and it decreases by Rs. 50,000 every year thereafter. Assume an interest rate (discounting factor) of 20%.

The method of finding the economic life of the machine with a discounting factor of 20% at zero inflation rate is summarized in Table 11.2. From the table it is clear that the total annual equivalent cost is minimum if the machine is used for 14 years. Hence, the economic life of the machine is 14 years.

### 11.3.3 Economic Life Determination with Inflationary Effect

The illustration in Section 11.3.2 is reconsidered for analyzing the effect of inflation on the economic life of the machine. An average annual inflation rate of 6% is assumed for discussion. The corresponding steps are explained in Table 11.3.

From the Table 11.3, it is clear that the total annual equivalent cost is minimum if the machine is used for three years. Thus, the economic life of the machine is three years.

Table 11.2 Determination of Economic Life of the Machine without Inflation

| End of year | Operation cost | Maintenance cost | Operation & maint. cost | $P/F, i, n$<br>( $i = 20\%$ ) | Present worth of column 4 | Cumulative of column (6) | Salvage value (S) | Present worth of salvage value | Total present worth  | $AP, i, n$<br>( $i = 20\%$ ) | Annual equivalent amount |
|-------------|----------------|------------------|-------------------------|-------------------------------|---------------------------|--------------------------|-------------------|--------------------------------|----------------------|------------------------------|--------------------------|
| (n)         | (2)            | (3)              | (2) + (3)               | (5)                           | (4) × (5)                 | (7)                      | (8)               | (8) × (5)                      | Column 7 - 9<br>(10) | (11)                         | (10) × (11)<br>(12)      |
| (Rs.)       | (Rs.)          | (Rs.)            | (Rs.)                   |                               | (Rs.)                     | (Rs.)                    | (Rs.)             | (Rs.)                          | (Rs.)                |                              | (Rs.)                    |
| 1           | 40,000         | 60,000           | 1,00,000                | 0.8333                        | 83,330.00                 | 83,330.00                | 4,00,000          | 3,33,320.00                    | 2,50,010.00          | 1.2000                       | 3,00,012.00              |
| 2           | 45,000         | 66,000           | 1,11,000                | 0.6945                        | 77,089.50                 | 1,60,419.50              | 3,50,000          | 2,43,075.00                    | 4,17,344.50          | 0.6546                       | 2,73,193.70              |
| 3           | 50,000         | 72,000           | 1,22,000                | 0.5787                        | 70,601.40                 | 2,31,020.90              | 3,00,000          | 1,73,610.00                    | 5,57,410.90          | 0.4747                       | 2,64,602.90              |
| 4           | 55,000         | 78,000           | 1,33,000                | 0.4823                        | 64,145.90                 | 2,95,166.80              | 2,50,000          | 1,20,575.00                    | 6,74,591.80          | 0.3863                       | 2,60,594.80              |
| 5           | 60,000         | 84,000           | 1,44,000                | 0.4019                        | 57,873.60                 | 3,53,040.40              | 2,00,000          | 80,380.00                      | 7,72,660.40          | 0.3344                       | 2,58,377.60              |
| 6           | 65,000         | 90,000           | 1,55,000                | 0.3349                        | 51,909.50                 | 4,04,949.90              | 1,50,000          | 50,235.00                      | 8,54,714.90          | 0.3007                       | 2,57,012.70              |
| 7           | 70,000         | 96,000           | 1,66,000                | 0.2791                        | 46,330.60                 | 4,51,280.50              | 1,00,000          | 27,910.00                      | 9,23,370.50          | 0.2774                       | 2,56,142.90              |
| 8           | 75,000         | 1,02,000         | 1,77,000                | 0.2326                        | 41,170.20                 | 4,92,450.70              | 50,000            | 11,630.00                      | 9,80,820.70          | 0.2606                       | 2,55,601.80              |
| 9           | 80,000         | 1,08,000         | 1,88,000                | 0.1938                        | 36,434.40                 | 5,28,885.10              | 0                 | 0.00                           | 10,28,885.00         | 0.2481                       | 2,55,266.30              |
| 10          | 85,000         | 1,14,000         | 1,99,000                | 0.1615                        | 32,138.50                 | 5,61,023.60              | 0                 | 0.00                           | 10,61,023.00         | 0.2385                       | 2,53,054.10              |
| 11          | 90,000         | 1,20,000         | 2,10,000                | 0.1346                        | 28,266.00                 | 5,89,289.60              | 0                 | 0.00                           | 10,89,289.00         | 0.2311                       | 2,51,734.80              |
| 12          | 95,000         | 1,26,000         | 2,21,000                | 0.1122                        | 24,796.20                 | 6,14,085.80              | 0                 | 0.00                           | 11,14,085.00         | 0.2253                       | 2,51,003.50              |
| 13          | 1,00,000       | 1,32,000         | 2,32,000                | 0.0935                        | 21,692.00                 | 6,35,777.80              | 0                 | 0.00                           | 11,35,777.00         | 0.2206                       | 2,50,552.50              |
| 14          | 1,05,000       | 1,38,000         | 2,43,000                | 0.0779                        | 18,929.70                 | 6,54,707.50              | 0                 | 0.00                           | 11,54,707.00         | 0.2169                       | 2,50,456.00***           |
| 15          | 1,10,000       | 1,44,000         | 2,54,000                | 0.0649                        | 16,484.60                 | 6,71,192.10              | 0                 | 0.00                           | 11,71,192.00         | 0.2139                       | 2,50,517.90              |

\*\*\*Total annual equivalent cost is minimum if the machine is used for 14 years.

**Table 11.3** Determination of Economic Life of the Machine with Inflationary Effect

| End of year | Operation cost | Maintenance cost | Sum of operation & maintenance cost (2) + (3) | F/P, i, n (i = 6%) | Inflated operation & maint. cost (4) × (5) | P/F, i, n (i = 20%) | Present worth of column 6 (6) × (7) | Cumulative of column (8) | Salvage value (5) | Inflated salvage value (10) × (5) | Present worth of column 11 (11) × (7) | Total present worth (12) Column 9 – 12 (13) | A/P, i, n (i = 20%) | Annual equivalent amount of total present worth (13) × (14) |
|-------------|----------------|------------------|-----------------------------------------------|--------------------|--------------------------------------------|---------------------|-------------------------------------|--------------------------|-------------------|-----------------------------------|---------------------------------------|---------------------------------------------|---------------------|-------------------------------------------------------------|
| (1)         | (2)            | (3)              | (4)                                           | (5)                | (6)                                        | (7)                 | (8)                                 | (9)                      | (10)              | (11)                              | (12)                                  | (13)                                        | (14)                | (15)                                                        |
|             | (Rs.)          | (Rs.)            | (Rs.)                                         |                    | (Rs.)                                      |                     | (Rs.)                               | (Rs.)                    | (Rs.)             | (Rs.)                             | (Rs.)                                 | (Rs.)                                       |                     | (Rs.)                                                       |
| 1           | 40,000         | 60,000           | 1,00,000                                      | 1.060              | 1,06,000                                   | 0.8333              | 88,329.80                           | 88,329.80                | 4,00,000          | 4,24,000                          | 3,53,319.20                           | 2,35,010.6                                  | 1.2000              | 2,82,012.70                                                 |
| 2           | 45,000         | 66,000           | 1,11,000                                      | 1.124              | 1,24,764                                   | 0.6945              | 86,648.59                           | 1,74,978.30              | 3,50,000          | 3,93,400                          | 2,73,216.30                           | 4,01,762.0                                  | 0.6546              | 2,62,993.40                                                 |
| 3           | 50,000         | 72,000           | 1,22,000                                      | 1.191              | 1,45,302                                   | 0.5787              | 84,086.26                           | 2,59,064.60              | 3,00,000          | 3,57,300                          | 2,06,769.50                           | 5,52,295.1                                  | 0.4747              | 2,62,174.50 ***                                             |
| 4           | 55,000         | 78,000           | 1,33,000                                      | 1.262              | 1,67,846                                   | 0.4823              | 80,952.12                           | 3,40,016.70              | 2,50,000          | 3,15,500                          | 1,52,165.60                           | 6,87,851.1                                  | 0.3863              | 2,65,716.80                                                 |
| 5           | 60,000         | 84,000           | 1,44,000                                      | 1.338              | 1,92,672                                   | 0.4019              | 77,434.87                           | 4,17,451.60              | 2,00,000          | 2,67,600                          | 1,07,548.40                           | 8,09,903.2                                  | 0.3344              | 2,70,831.60                                                 |
| 6           | 65,000         | 90,000           | 1,55,000                                      | 1.419              | 2,19,945                                   | 0.3349              | 73,659.58                           | 4,91,111.20              | 1,50,000          | 2,12,850                          | 71,283.46                             | 9,19,827.7                                  | 0.3007              | 2,76,592.20                                                 |
| 7           | 70,000         | 96,000           | 1,66,000                                      | 1.504              | 2,49,664                                   | 0.2791              | 69,681.22                           | 5,60,792.40              | 1,00,000          | 1,50,400                          | 41,976.64                             | 10,18,815.0                                 | 0.2774              | 2,82,619.50                                                 |
| 8           | 75,000         | 1,02,000         | 1,77,000                                      | 1.594              | 2,82,138                                   | 0.2326              | 65,625.29                           | 6,26,417.70              | 50,000            | 79,700                            | 18,538.22                             | 11,07,879.0                                 | 0.2606              | 2,88,713.40                                                 |
| 9           | 80,000         | 1,08,000         | 1,88,000                                      | 1.689              | 3,17,532                                   | 0.1938              | 61,537.70                           | 6,87,955.40              | 0                 | 0                                 | 0.00                                  | 11,87,955.0                                 | 0.2481              | 2,94,731.70                                                 |
| 10          | 85,000         | 1,14,000         | 1,99,000                                      | 1.791              | 3,56,409                                   | 0.1615              | 57,560.05                           | 7,45,515.50              | 0                 | 0                                 | 0.00                                  | 12,45,515.0                                 | 0.2385              | 2,97,055.40                                                 |

\*\*\*Total annual equivalent cost is minimum if machine is used for three years.

### Comparison of results

The results of the two approaches are summarized in Table 11.4 . From the table, it is clear that the inflation has an effect on the economic life of the machine. Since it is meaningful and realistic to analyze this type of problem by considering the effect of inflation, the second approach should be used for such analysis.

**Table 11.4** Results of the Two Approaches

| <i>Approach</i>                                  | <i>Minimum annual<br/>equivalent cost<br/>(Rs.)</i> | <i>Corresponding<br/>life<br/>(years)</i> |
|--------------------------------------------------|-----------------------------------------------------|-------------------------------------------|
| Replacement analysis without<br>inflation effect | 2,50,456.00                                         | 14                                        |
| Replacement analysis with<br>inflation effect    | 2,62,174.50                                         | 3                                         |

**EXAMPLE 11.3** A company has received quotes for its recent advertisement for the purchase of a sophisticated milling machine. The data are as per the estimate in today's rupee value.

|                                                   | <i>Machine X</i> | <i>Machine Y</i> |
|---------------------------------------------------|------------------|------------------|
| Purchase price (Rs.)                              | 15,00,000        | 20,00,000        |
| Machine life (years)                              | 7                | 7                |
| Salvage value at the end of<br>machine life (Rs.) | 2,00,000         | 3,00,000         |
| Annual operating &<br>maintenance cost (Rs.)      | 3,00,000         | 2,50,000         |

Assuming an average annual inflation of 5% for the next five years, determine the best machine based on the present worth method. Interest rate is 15%, compounded annually.

### Solution

Average annual inflation rate = 5%

Interest rate = 15% compounded annually

### Machine X

Purchase price = Rs.15,00,000

Machine life = 7 years

Salvage value at the end of machine life = Rs. 2,00,000

Annual operating & maintenance cost = Rs. 3,00,000

The computation of the present worth of the annual operating and maintenance costs of the machine X is summarized in Table 11.5.

**Table 11.5** Computation of the Present Worth of the Annual Operating and Maintenance Costs of Machine X

| End of year (n) | Annual operating & maintenance cost (Rs.) | Inflation factor (F/P, 5%, n) | Inflated annual operating & maintenance cost (Rs.) | P/F, 15%, n | Present worth of inflated annual operating & maintenance cost |
|-----------------|-------------------------------------------|-------------------------------|----------------------------------------------------|-------------|---------------------------------------------------------------|
| A               | B                                         | C                             | D                                                  | E           | F (Rs.)                                                       |
|                 |                                           |                               | B × C                                              |             | D × E                                                         |
| 1               | 3,00,000                                  | 1.050                         | 3,15,000                                           | 0.8696      | 2,73,924                                                      |
| 2               | 3,00,000                                  | 1.102                         | 3,30,600                                           | 0.7561      | 2,49,967                                                      |
| 3               | 3,00,000                                  | 1.158                         | 3,47,400                                           | 0.6575      | 2,28,416                                                      |
| 4               | 3,00,000                                  | 1.216                         | 3,64,800                                           | 0.5718      | 2,08,593                                                      |
| 5               | 3,00,000                                  | 1.276                         | 3,82,800                                           | 0.4972      | 1,90,328                                                      |
| 6               | 3,00,000                                  | 1.340                         | 4,02,000                                           | 0.4323      | 1,73,785                                                      |
| 7               | 3,00,000                                  | 1.407                         | 4,22,100                                           | 0.3759      | 1,58,667                                                      |
|                 |                                           |                               |                                                    |             | Rs. 14,83,680                                                 |

The equation for the present worth of the machine X is

$$\begin{aligned}
 PW_X(15\%) &= \text{Purchase price} \\
 &\quad + \text{Present worth of inflated annual and operating cost} \\
 &\quad - \text{Present worth of the salvage value} \\
 &= 15,00,000 + 14,83,680 - 2,00,000 \times (\text{inflation factor}) \\
 &\quad \times (P/F, 15\%, 7) \\
 &= 15,00,000 + 14,83,680 - 2,00,000(F/P, 5\%, 7) (P/F, 15\%, 7) \\
 &= 15,00,000 + 14,83,680 - 2,00,000 \times 1.407 \times 0.3759 \\
 &= \text{Rs. } 28,77,901.74
 \end{aligned}$$

#### Machine Y

Purchase price = Rs. 20,00,000

Machine life = 7 years

Salvage value at the end of machine life = Rs. 3,00,000

Annual operating & maintenance cost = Rs. 2,50,000

The computation of the present worth of the annual operating and maintenance costs of the machine Y is summarized in Table 11.6.

**Table 11.6** Computation of the Present Worth of the Annual Operating and Maintenance Costs of Machine Y

| End of year (n) | Annual operating & maintenance cost (Rs.) | Inflation factor (F/P, 5%, n) | Inflated annual operating & maintenance cost (Rs.) | P/F, 15%, n | Present worth of inflated annual operating & maintenance cost |
|-----------------|-------------------------------------------|-------------------------------|----------------------------------------------------|-------------|---------------------------------------------------------------|
| A               | B                                         | C                             | D                                                  | E           | F (Rs.)                                                       |
|                 |                                           |                               | B × C                                              |             | D × E                                                         |
| 1               | 2,50,000                                  | 1.050                         | 2,62,500                                           | 0.8696      | 2,28,270                                                      |
| 2               | 2,50,000                                  | 1.102                         | 2,75,500                                           | 0.7561      | 2,08,306                                                      |
| 3               | 2,50,000                                  | 1.158                         | 2,89,500                                           | 0.6575      | 1,90,346                                                      |
| 4               | 2,50,000                                  | 1.216                         | 3,04,000                                           | 0.5718      | 1,73,827                                                      |
| 5               | 2,50,000                                  | 1.276                         | 3,19,000                                           | 0.4972      | 1,58,607                                                      |
| 6               | 2,50,000                                  | 1.340                         | 3,35,000                                           | 0.4323      | 1,44,821                                                      |
| 7               | 2,50,000                                  | 1.407                         | 3,51,750                                           | 0.3759      | 1,32,223                                                      |
|                 |                                           |                               |                                                    |             | Rs. 12,36,400                                                 |

The expression for the present worth of machine Y is

$$\begin{aligned}
 PW_Y(15\%) &= \text{Purchase price} \\
 &\quad + \text{present worth of inflated annual and operating cost} \\
 &\quad - \text{present worth of the salvage value} \\
 &= 20,00,000 + 12,36,400 - 3,00,000 \times \\
 &\quad (\text{inflation factor}) \times (P/F, 15\%, 7) \\
 &= 20,00,000 + 12,36,400 - 3,00,000 (F/P, 5\%, 7) (P/F, 15\%, 7) \\
 &= 20,00,000 + 12,36,400 - 3,00,000 \times 1.407 \times 0.3759 \\
 &= \text{Rs. } 30,77,732.61
 \end{aligned}$$

**Remark.** Since the present worth cost of machine X is less than that of machine Y, select machine X.

**EXAMPLE 11.4** A company is planning to start an employee welfare fund. It needs Rs. 50,00,000 during the first year and it increases by Rs. 5,00,000 every year thereafter up to the end of the 5th year. The above figures are in terms of today's rupee value. The annual average rate of inflation is 6% for the next five years. The interest rate is 18%, compounded annually. Find the single deposit which will provide the required series of fund towards employees welfare scheme after taking the inflation rate into account.

**Solution**

Fund requirement during the first year = Rs. 50,00,000  
 Annual increase in the fund requirement = Rs. 5,00,000  
 Annual inflation rate = 6%  
 Interest rate = 18%, compounded annually

The computation of the present worth of the annual fund requirements is summarized in Table 11.7.

**Table 11.7** Computation of the Present Worth of the Annual Requirements

| End of year (n) | Annual fund requirements (Rs.) | Inflation factor (F/P, 6%, n) | Inflated annual fund requirements (Rs.) | P/F, 18%, n | Present worth of inflated annual fund requirements |
|-----------------|--------------------------------|-------------------------------|-----------------------------------------|-------------|----------------------------------------------------|
| A               | B                              | C                             | D                                       | E           | F (Rs.)                                            |
|                 |                                |                               | B × C                                   |             | D × E                                              |
| 1               | 50,00,000                      | 1.060                         | 53,00,000                               | 0.8475      | 44,91,750                                          |
| 2               | 55,00,000                      | 1.124                         | 61,82,000                               | 0.7182      | 44,39,912                                          |
| 3               | 60,00,000                      | 1.191                         | 71,46,000                               | 0.6086      | 43,49,056                                          |
| 4               | 65,00,000                      | 1.262                         | 82,03,000                               | 0.5158      | 42,31,107                                          |
| 5               | 70,00,000                      | 1.338                         | 93,66,000                               | 0.4371      | 40,93,879                                          |
|                 |                                |                               |                                         |             | Rs. 2,16,05,704                                    |

The value of the single deposit to be made now to receive the specified series for the next five years is Rs. 2,16,05,704.

**QUESTIONS**

1. Define inflation.
2. Discuss the impact of inflation on investment decision.
3. Suppose a 50-year old man is planning for his retirement. He plans to retire at the age of 60 and estimates that he can live comfortably on Rs. 40,000 per year in terms of today's rupee value. Let us assume the average inflation rate for the next 20 years is 7% per year. This is only an assumption. He can invest his savings at 20%, compounded annually.  
What equal amount should he save each year until he retires so that he can make withdrawals that will allow him to live as comfortably as he desires for 10 years beyond his retirement?
4. A machine costs Rs. 7,00,000. Its annual operation cost during the first year is Rs. 60,000 and it increases by Rs. 7,000 every year thereafter. The maintenance cost during the first year is Rs. 80,000 and it increases by Rs. 10,000 every year thereafter. The resale value of the machine is Rs. 3,00,000 at the end of the first year and it decreases by Rs. 75,000 every year thereafter. Assume an interest rate (discounting factor) of 20% and inflation of 5%, compounded yearly. Find the inflation adjusted economic life of the machine.

# 12

## INVENTORY CONTROL

### 12.1 INTRODUCTION

Inventory is essential to provide flexibility in operating a system. Inventory can be classified into raw materials inventory, in-process inventory, and finished goods inventory. The raw material inventories obviate the need for dependency between suppliers and plants. The work-in-process inventories eliminate dependency between machines of a product line. The finished goods inventory remove dependency between plant and its customers/market.

The main functions of inventory are:

1. Smoothing out irregularities in supply
2. Minimizing the production cost
3. Allowing organizations to cope with perishable materials

#### Inventory decisions

Two basic inventory decisions are generally taken by managers:

1. When should the inventory of an item be replaced?
2. How much of an item is to be ordered when the inventory of that item has to be replenished?

#### Costs trade-off

If we place frequent orders, the cost of order will be more, but the inventory carrying cost will be less. On the other hand, if we place less frequent orders, the ordering cost will be less, but the carrying cost will be more. These are shown in Fig. 12.1. In the figure, for an increase in  $Q$  (order size), the carrying cost increases and the ordering cost decreases.

In Fig. 12.1, the total cost curve represents the sum of the ordering cost and carrying cost for each order size. The order size at which the total cost is minimum is called the *Economic Order Quantity (EOQ)* or  $Q^*$  (optimal order size).

There are different models of inventory. The inventory models can be classified into deterministic models and probabilistic models. The various deterministic models are as follows:

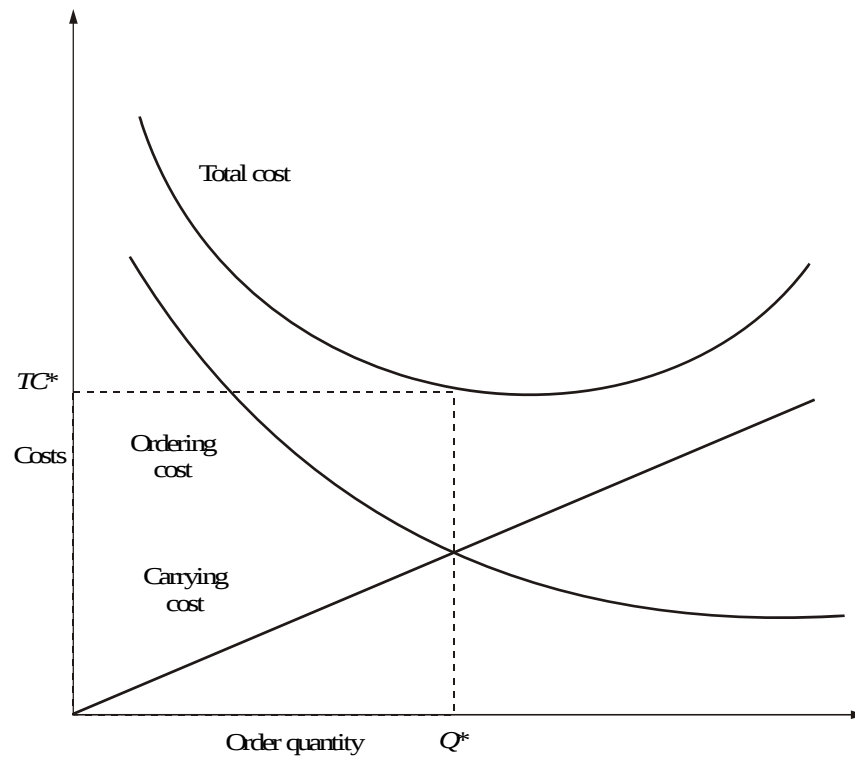


Fig. 12.1 Trade-off between costs.

1. Purchase model with instantaneous replenishment and without shortages
2. Manufacturing model without shortages
3. Purchase model with instantaneous replenishment and with shortages
4. Manufacturing model with shortages

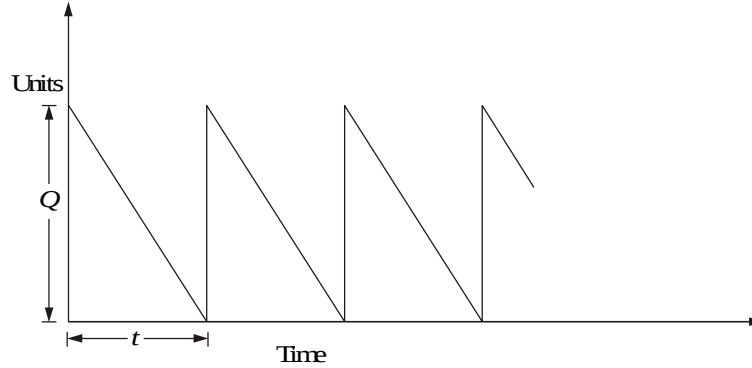
These models are explained in the following sections.

## 12.2 PURCHASE MODEL WITH INSTANTANEOUS REPLENISHMENT AND WITHOUT SHORTAGES

In this model of inventory, orders of equal size are placed at periodical intervals. The items against an order are replenished instantaneously and they are consumed at a constant rate. The purchase price per unit is same irrespective of order size. Let

- $D$  = annual demand in units
- $C_o$  = ordering cost/order
- $C_c$  = carrying cost/unit/year
- $P$  = purchase price/unit
- $Q$  = order size

Then, the corresponding model is shown in Fig. 12.2.



**Fig. 12.2** Purchase model without shortages.

For this model, the formula for the EOQ is as follows:

$$Q^* = \sqrt{\frac{2C_o D}{C_c}}$$

$$\text{No. of orders} = \frac{D}{Q^*}$$

$$\text{Time between orders} = \frac{Q^*}{D}$$

**EXAMPLE 12.1** Krishna Industry needs 24,000 units/year of a bought-out component which will be used in its main product. The ordering cost is Rs.150 per order and the carrying cost per unit per year is 18% of the purchase price per unit. The purchase price per unit is Rs. 75. Find

- Economic order quantity
- No. of orders per year
- Time between successive orders

**Solution**

(a)  $D = 24,000$  units/year

$C_o = \text{Rs. } 150/\text{order}$

Purchase price/unit = Rs. 75.

$C_c = \text{Rs. } 75 \times 0.18 = \text{Rs. } 13.5/\text{unit/year}$

Therefore,

$$\begin{aligned} \text{EOQ} &= \sqrt{\frac{2C_o D}{C_c}} = \sqrt{\frac{2 \times 150 \times 24,000}{13.50}} \\ &= 730 \text{ units (approx.)} \end{aligned}$$

$$\text{No. of orders/year} = \frac{D}{Q^*} = \frac{24,000}{730} = 32.88$$

$$\begin{aligned} \text{Time between successive orders} &= \frac{Q^*}{D} = 730/24,000 \\ &= 0.0304 \text{ year} \\ &= 0.37 \text{ month} \\ &= 11 \text{ days} \end{aligned}$$

### 12.3 MANUFACTURING MODEL WITHOUT SHORTAGES

If a company manufactures its component which is required for its main product, then the corresponding model of inventory is called the “manufacturing model”. This model will be with or without shortages. The rate of consumption of items is uniform throughout the year. The cost of production per unit is same irrespective of production lot size. Let

- $r$  = annual demand of an item
- $k$  = production rate of the item (No. of units produced per year)
- $C_o$  = cost per set-up.
- $C_c$  = carrying cost per unit per year.
- $p$  = cost of production per unit
- $t_1$  = period of production as well as consumption satisfying requirement for the period
- $t_2$  = period of consumption only.
- $Q$  = production size

The operation of the manufacturing model without shortages is shown in Fig. 12.3.

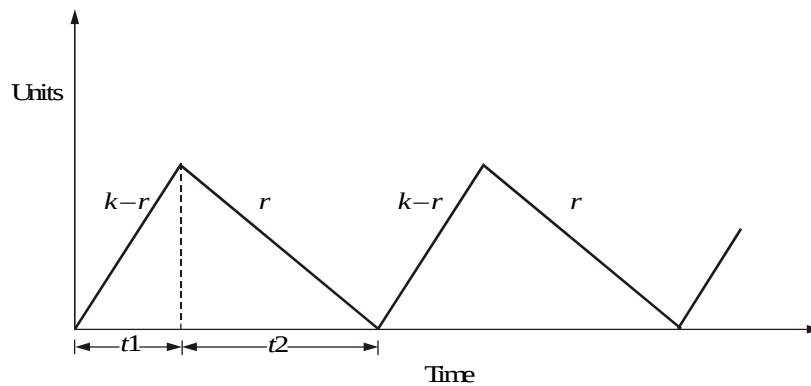


Fig. 12.3 Manufacturing model without shortages.

During the period  $t_1$ , the item is produced at the rate of  $k$  units per period and simultaneously it is consumed at the rate of  $r$  units per period. So, during

this period, the inventory is built at the rate of  $k - r$  units per period. During the period  $t_2$ , the production of the item is discontinued but the consumption of that item is continued. Hence, the inventory is decreased at the rate of  $r$  units per period during this period.

The various formulae for this situation are as follows:

$$Q^* = \text{EOQ} = \sqrt{\frac{2C_o r}{C_c(1 - r/k)}}$$

$$t_1^* = Q^*/k$$

$$t_2^* = \frac{Q^*[1 - r/k]}{r} = \frac{(k - r)t_1}{r}$$

$$\text{Cycle time} = t_1^* + t_2^*$$

$$\text{No. of set-ups/year} = r/Q^*$$

**EXAMPLE 12.2** If a product is to be manufactured within the company, the particulars are:

$$r = 12,000 \text{ units/year}$$

$$k = 24,000 \text{ units/year}$$

$$C_o = \text{Rs. } 175/\text{set-up}$$

$$C_c = \text{Rs. } 15/\text{unit/year}$$

Find the EOQ and cycle time.

**Solution**

$$\begin{aligned} Q^* = \text{EOQ} &= \sqrt{\frac{2C_o r}{C_c(1 - r/k)}} \\ &= \sqrt{\frac{2 \times 175 \times 12,000}{15(1 - 12,000/24,000)}} \\ &= 749 \text{ units (approx.)} \end{aligned}$$

$$\begin{aligned} t_1^* &= \frac{Q^*}{k} = 749/24,000 \\ &= 0.031 \text{ year} = 0.372 \text{ month} = 11 \text{ days (approx.)} \end{aligned}$$

$$\begin{aligned} t_2^* &= \frac{Q^*}{r} \left(1 - \frac{r}{k}\right) = \frac{749}{12,000} \left(1 - \frac{12,000}{24,000}\right) \\ &= 0.031 \text{ year} = 0.372 \text{ month} = 11 \text{ days} \end{aligned}$$

Therefore,

$$\begin{aligned} \text{Cycle time} &= t_1^* + t_2^* \\ &= 11 + 11 = 22 \text{ days} \end{aligned}$$

$$\text{No. of set-ups/year} = r/Q^* = 12,000/749 = 16.02$$

## 12.4 PURCHASE MODEL WITH SHORTAGES (Instantaneous Supply)

In this model, the items on order will be received instantaneously and they are consumed at a constant rate. The purchase price per unit is same irrespective of order size. If there is no stock at the time of receiving a request for the items, it is assumed that it will be satisfied at a later date with a penalty. This is called *backordering*. The operation of this model is illustrated in Fig. 12.4.

The variables which are used in this model are

$D$  = demand/period

$C_c$  = carrying cost/unit/period

$C_o$  = ordering cost/order

$C_s$  = shortage cost/unit/period

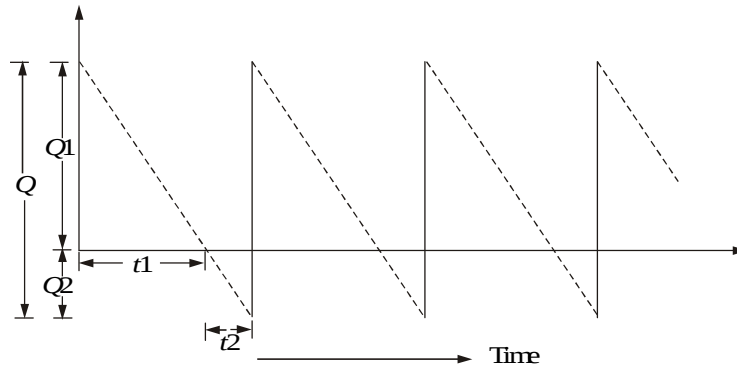


Fig. 12.4 Purchase model of inventory with shortages.

In the above model,

$Q$  = order size,

$Q1$  = maximum inventory,

$Q2$  = maximum stock-out,

$t1$  = period of positive stock,

$t2$  = period of shortage.

$$Q^* = \text{EOQ} = \sqrt{\frac{2C_o D}{C_c} \frac{(C_s + C_c)}{C_s}}$$

$$Q1^* = \sqrt{\frac{2C_o D}{C_c} \frac{C_s}{C_s + C_c}}$$

$$Q2^* = Q^* - Q1^*$$

$$t^* = Q^*/D$$

$$t1^* = Q1^*/D$$

$$t2^* = Q2^*/D$$

$$\text{No. of orders/year} = D/Q^*$$

**EXAMPLE 12.3** The annual demand for a component is 30,000 units. The carrying cost is Rs. 2.00/unit/year, the ordering cost is Rs. 100.00/order, and the shortage cost is Rs. 12.00/unit/year. Find the optimal values of the following:

- (a) Ordering quantity
- (b) Maximum inventory
- (c) Maximum shortage quantity
- (d) Cycle time
- (e) Inventory period ( $t_1$ )
- (f) Shortage period ( $t_2$ )

**Solution**

$$D = 30,000 \text{ units/year}$$

$$C_c = \text{Rs. } 2.00/\text{unit/year}$$

$$C_o = \text{Rs. } 100/\text{order}$$

$$C_s = \text{Rs. } 12/\text{unit/year}$$

$$\begin{aligned} Q^* = \text{EOQ} &= \sqrt{\frac{2C_o D}{C_c} \frac{(C_s + C_c)}{C_s}} \\ &= \sqrt{\frac{2 \times 100 \times 30,000}{2} \frac{(12 + 2)}{12}} \\ &= 1,871 \text{ units} \end{aligned}$$

$$\begin{aligned} Q_1^* &= \sqrt{\frac{2C_o D}{C_c} \frac{C_s}{C_s + C_c}} \\ &= \sqrt{\frac{2 \times 100 \times 30,000}{2} \frac{12}{(12 + 2)}} \\ &= 1,604 \text{ units} \end{aligned}$$

$$Q_2^* = 1,871 - 1,604 = 267 \text{ units}$$

$$t^* = Q^*/D = (1,871/30,000) \times 365 = 23 \text{ days (approx.)}$$

$$t_1^* = Q_1^*/D = (1,604/30,000) \times 365 = 20 \text{ days}$$

$$t_2^* = t^* - t_1^* = 23 - 20 = 3 \text{ days}$$

$$\text{No. of orders/year} = D/Q^* = 30,000/1,871 = 16.03$$

## 12.5 MANUFACTURING MODEL WITH SHORTAGES

In this model, the items are produced and consumed simultaneously for a portion of the cycle time. During the remaining cycle time, the items are only

consumed. The cost of production per unit is same irrespective of production lot size. In this model, stock-out is permitted. It is assumed that the stock-out units will be satisfied from the units which will be produced at a later date with a penalty. This is called backordering. The operation of this model is shown in Fig. 12.5. The variables which are used in this model are given now.

Let

$r$  = annual demand of an item per period

$k$  = production rate of the item (No. of units produced per period)

$C_o$  = cost per set-up.

$C_c$  = carrying cost per unit per period

$C_s$  = shortage cost per unit per period

$p$  = cost of production per unit

$t_1$  = period of production as well as consumption satisfying period's requirement

$t_2$  = period of consumption only

$t_3$  = period of shortage

$t_4$  = period of production as well as consumption satisfying back order

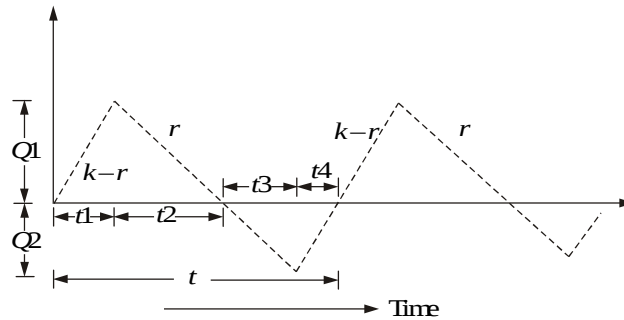


Fig. 12.5 Manufacturing model of inventory with shortages.

In the above model,

$Q$  = production size

$Q_1$  = maximum inventory

$Q_2$  = maximum stock-out

The formulae for the optimal values of the above variables are presented now.

$$Q^* = \text{EOQ} = \sqrt{\frac{2C_o}{C_c} \frac{kr}{k-r} \frac{C_c + C_s}{C_s}}$$

$$Q_1^* = \sqrt{\frac{2C_o}{C_c} \frac{r(k-r)}{k} \frac{C_s}{C_c + C_s}}$$

$$Q2^* = \sqrt{\frac{2C_o C_c}{C_s(C_c + C_s)} \frac{r(k-r)}{k}}$$

$$Q1^* = \frac{k-r}{k} Q^* - Q2^*$$

$$t^* = Q^*/r$$

$$t1^* = Q1^*/(k-r)$$

$$t2^* = Q1^*/r$$

$$t3^* = Q2^*/r$$

$$t4^* = Q2^*/(k-r)$$

**EXAMPLE 12.4** The demand for an item is 12,000/year. Its production rate is 2,000/month. The carrying cost is Re. 0.20/unit/month and the set-up cost is Rs. 400.00/set-up. The shortage cost is Rs. 15.00/unit/year. Find the various parameters of the inventory system.

**Solution**

$$r = 12,000 \text{ units/year}$$

$$k = 2,000 \times 12 = 24,000 \text{ units/year}$$

$$C_o = \text{Rs. } 400/\text{set-up.}$$

$$C_c = \text{Rs. } (0.20 \times 12) = \text{Rs. } 2.40/\text{year}$$

$$C_s = \text{Rs. } 15.00/\text{unit/year}$$

$$Q^* = \text{EOQ} = \sqrt{\frac{2 \times 400}{2.40} \frac{24,000 \times 12,000}{(24,000 - 12,000)} \frac{(2.40 + 15)}{15}}$$

$$= 3,046 \text{ units (approx.)}$$

$$Q2^* = \sqrt{\frac{2 \times 400 \times 2.40}{15(2.40 + 15)} \frac{12,000(24,000 - 12,000)}{24,000}}$$

$$= 210$$

$$Q1^* = \frac{24,000 - 12,000}{24,000} 3,046 - 210$$

$$= 1,313$$

$$t^* = [Q^*/r]365 \text{ days} = [3,046/12,000]365 \text{ days} = 93 \text{ days}$$

$$t1^* = [Q1^*/(k-r)]365 \text{ days} = [1,313/(24,000 - 12,000)]365 \text{ days}$$

$$= 40 \text{ days}$$

$$t2^* = [Q1^*/r]365 \text{ days} = [1,313/12,000]365 \text{ days} = 40 \text{ days}$$

$$t3^* = [Q2^*/r]365 \text{ days} = [210/12,000]365 \text{ days} = 7 \text{ days}$$

$$t4^* = [Q2^*/(k-r)]365 \text{ days} = [210/(24,000 - 12,000)]365 \text{ days}$$

$$= 7 \text{ days}$$

**QUESTIONS**

1. What are the reasons for stocking items in inventory?
2. List and explain different types of costs in the inventory system.
3. What are the types of model of inventory system? Explain them in brief.
4. Explain the cost trade-off of the purchase model of inventory system without shortage.
5. Alpha Industry estimates that it will sell 12,000 units of its product for the forthcoming year. The ordering cost is Rs. 100 per order and the carrying cost per unit per year is 20% of the purchase price per unit. The purchase price per unit is Rs. 50. Find:
  - (a) Economic order size
  - (b) No. of orders per year
  - (c) Time between successive orders
6. A product is to be manufactured within the company whose details are as follows:  
 $r = 24,000$  units/year  
 $k = 48,000$  units/year  
 $C_o = \text{Rs. } 200/\text{set-up}$   
 $C_c = \text{Rs. } 20/\text{unit/year}$   
Find the EOQ and cycle time.
7. The annual demand for an automobile component is 24,000 units. The carrying cost is Rs. 0.40/unit/year, the ordering cost is Rs. 20.00/order, and the shortage cost is Rs. 10.00/unit/year. Find the optimal values of the following:
  - (a) Economic order quantity
  - (b) Maximum inventory
  - (c) Maximum shortage quantity
  - (d) Cycle time
  - (e) Inventory period ( $t_1$ )
  - (f) Shortage period ( $t_2$ ).
8. The demand for an item is 18,000 per year. Its production rate is 3,000/month. The carrying cost is Re. 0.15/unit/month, and the set-up cost is Rs. 500.00/set-up. The shortage cost is Rs. 20.00/unit/year. Find various parameters of the inventory system.

# 13

## MAKE OR BUY DECISION

### 13.1 INTRODUCTION

In the process of carrying out business activities of an organization, a component/product can be made within the organization or bought from a subcontractor. Each decision involves its own costs. So, in a given situation, the organization should evaluate each of the above make or buy alternatives and then select the alternative which results in the lowest cost. This is an important decision since it affects the productivity of the organization. In the long run, the make or buy decision is not static. The make option of a component/product may be economical today; but after some time, it may turn out to be uneconomical to make the same.

Thus, the make or buy decision should be reviewed periodically, say, every 1 to 3 years. This is mainly to cope with the changes in the level of competition and various other environmental factors.

### 13.2 CRITERIA FOR MAKE OR BUY

In this section the criteria for make or buy are discussed.

#### Criteria for make

The following are the criteria for make:

1. The finished product can be made cheaper by the firm than by outside suppliers.
2. The finished product is being manufactured only by a limited number of outside firms which are unable to meet the demand.
3. The part has an importance for the firm and requires extremely close quality control.
4. The part can be manufactured with the firm's existing facilities and similar to other items in which the company has manufacturing experience.

**Criteria for buy**

The following are the criteria for buy:

1. Requires high investments on facilities which are already available at suppliers plant.
2. The company does not have facilities to make it and there are more profitable opportunities for investing company's capital.
3. Existing facilities of the company can be used more economically to make other parts.
4. The skill of personnel employed by the company is not readily adaptable to make the part.
5. Patent or other legal barriers prevent the company for making the part.
6. Demand for the part is either temporary or seasonal.

**13.3 APPROACHES FOR MAKE OR BUY DECISION**

Types of analysis followed in make or buy decision are as follows:

1. Simple cost analysis
2. Economic analysis
3. Break-even analysis

**13.3.1 Simple Cost Analysis**

The concept is illustrated using an example problem.

**EXAMPLE 13.1** A company has extra capacity that can be used to produce a sophisticated fixture which it has been buying for Rs. 900 each. If the company makes the fixtures, it will incur materials cost of Rs. 300 per unit, labour costs of Rs. 250 per unit, and variable overhead costs of Rs. 100 per unit. The annual fixed cost associated with the unused capacity is Rs. 10,00,000. Demand over the next year is estimated at 5,000 units. Would it be profitable for the company to make the fixtures?

**Solution** We assume that the unused capacity has alternative use.

**Cost to make**

$$\begin{aligned}\text{Variable cost/unit} &= \text{Material} + \text{labour} + \text{overheads} \\ &= \text{Rs. } 300 + \text{Rs. } 250 + \text{Rs. } 100 \\ &= \text{Rs. } 650\end{aligned}$$

$$\begin{aligned}\text{Total variable cost} &= (5,000 \text{ units}) (\text{Rs. } 650/\text{unit}) \\ &= \text{Rs. } 32,50,000\end{aligned}$$

|                                                   |                 |
|---------------------------------------------------|-----------------|
| Add fixed cost associated<br>with unused capacity | + Rs. 10,00,000 |
| Total cost                                        | = Rs. 42,50,000 |

**Cost to buy**

|                                                   |                                |
|---------------------------------------------------|--------------------------------|
| Purchase cost                                     | = (5,000 units) (Rs. 900/unit) |
|                                                   | = Rs. 45,00,000                |
| Add fixed cost associated<br>with unused capacity | + Rs. 10,00,000                |
| Total cost                                        | = Rs. 55,00,000                |

The cost of making fixtures is less than the cost of buying fixtures from outside. Therefore, the organization should make the fixtures.

**13.3.2 Economic Analysis**

The following inventory models are considered to illustrate this concept:

- Purchase model
- Manufacturing model

The formulae for EOQ and total cost (TC) for each model are given in the following table:

| <i>Purchase model</i>                                         | <i>Manufacturing model</i>                                           |
|---------------------------------------------------------------|----------------------------------------------------------------------|
| $Q1 = \sqrt{\frac{2C_oD}{C_c}}$                               | $Q2 = \sqrt{\frac{2C_oD}{C_c(1-r/k)}}$                               |
| $TC = D \times P + \frac{DC_o}{Q1} + \frac{Q1 \times C_c}{2}$ | $TC = D \times P + \frac{DC_o}{Q2}$<br>$+ C_c(k-r) \frac{Q2}{2 * k}$ |

where

$D$  = demand/year

$P$  = purchase price/unit

$C_c$  = carrying cost/unit/year

$C_o$  = ordering cost/order or set-up cost/set-up

$k$  = production rate (No. of units/year)

$r$  = demand/year

$Q1$  = economic order size

$Q2$  = economic production size

$TC$  = total cost per year

**EXAMPLE 13.2** An item has a yearly demand of 2,000 units. The different costs in respect of make and buy are as follows. Determine the best option.

|                                    | Buy        | Make        |
|------------------------------------|------------|-------------|
| Item cost/unit                     | Rs. 8.00   | Rs. 5.00    |
| Procurement cost/order             | Rs. 120.00 |             |
| Set-up cost/set-up                 |            | Rs. 60.00   |
| Annual carrying cost/<br>item/year | Rs. 1.60   | Rs. 1.00    |
| Production rate/year               |            | 8,000 units |

**Solution**

**Buy option**

$D = 2,000$  units/year

$C_o = \text{Rs. } 120/\text{order}$

$C_c = \text{Rs. } 1.60/\text{unit/year}$

$$Q1 = \sqrt{\frac{2C_o D}{C_c}} = \sqrt{\frac{2 \times 2,000 \times 120}{1.60}}$$

$$= 548 \text{ units (approx.)}$$

$$TC = DP + \frac{DC_o}{Q1} + \frac{Q1C_c}{2}$$

$$= 2,000 \times 8 + \frac{2,000 \times 120}{548} + \frac{548 \times 1.60}{2}$$

$$= \text{Rs. } 16,876.36$$

**Make option**

$C_o = \text{Rs. } 60/\text{set-up}$

$r = 2,000$  units/year

$C_c = \text{Rs. } 1/\text{unit/year}$

$k = 8,000$  units/year

$$Q2 = \sqrt{\frac{2C_o r}{C_c[1 - (r/k)]}}$$

$$= \sqrt{\frac{2 \times 60 \times 2,000}{10(1 - 2,000/8,000)}} = 566 \text{ units (approx.)}$$

$$\begin{aligned}
 TC &= DP + \frac{D \times C_o}{Q2} + C_c (k - r) \frac{Q2}{2 \times k} \\
 &= 2,000 \times 5.00 + \frac{2,000 \times 60}{566} + 1.0 (8,000 - 2,000) \frac{566}{2 \times 8,000} \\
 &= \text{Rs. } 10,424.26
 \end{aligned}$$

**Result:** The cost of making is less than the cost of buying. Therefore, the firm should go in for the making option.

### 13.3.3 Break-even Analysis

The break-even analysis chart is shown in Fig. 13.1. In the figure

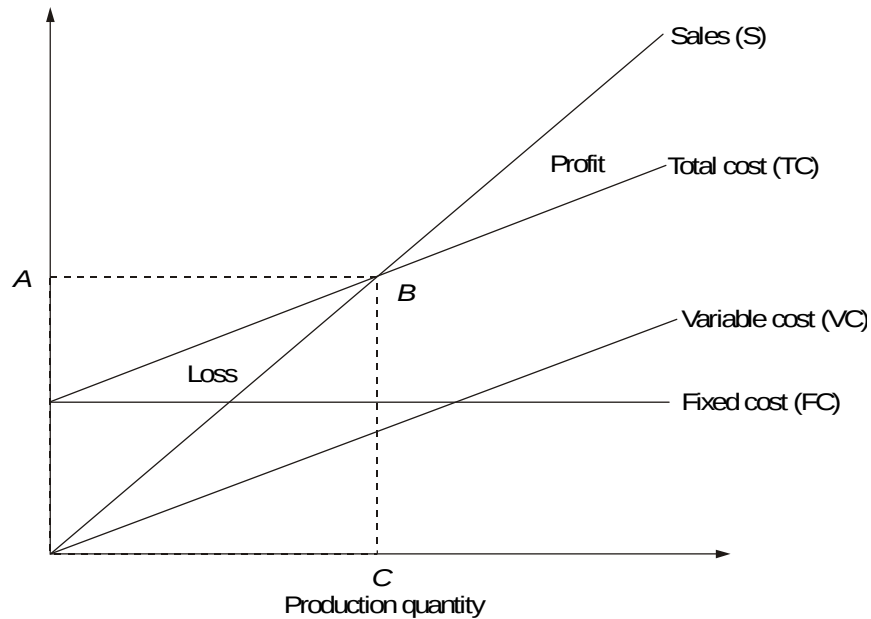


Fig. 13.1 Break-even chart.

$TC$  = total cost

$FC$  = fixed cost

$TC = FC + \text{variable cost}$

$B$  = the intersection of  $TC$  and sales (no loss or no gain situation)

$A$  = break-even sales

$C$  = break-even quantity/break-even point (BEP)

The formula for the break-even point (BEP) is

$$\text{BEP} = \frac{FC}{\text{Selling price/unit} - \text{Variable cost/unit}}$$

**EXAMPLE 13.3** A manufacturer of TV buys TV cabinet at Rs. 500 each. In case the company makes it within the factory, the fixed and variable costs would be Rs. 4,00,000 and Rs. 300 per cabinet respectively. Should the manufacturer make or buy the cabinet if the demand is 1,500 TV cabinets?.

**Solution**

$$\begin{aligned}
 \text{Selling price/unit (SP)} &= \text{Rs. 500} \\
 \text{Variable cost/unit (VC)} &= \text{Rs. 300} \\
 \text{Fixed cost (FC)} &= \text{Rs. 4,00,000} \\
 \text{BEP} &= \frac{4,00,000}{500 - 300} = 2,000 \text{ units}
 \end{aligned}$$

Since the demand (1,500 units) is less than the break-even quantity, the company should buy the cabinets for its TV production.

**EXAMPLE 13.4** There are three alternatives available to meet the demand of a particular product. They are as follows:

- (a) Manufacturing the product by using process A
- (b) Manufacturing the product by using process B
- (c) Buying the product

The details are as given in the following table:

| <i>Cost elements</i>      | <i>Manufacturing<br/>the product by<br/>process A</i> | <i>Manufacturing<br/>the product by<br/>process B</i> | <i>Buy</i> |
|---------------------------|-------------------------------------------------------|-------------------------------------------------------|------------|
| Fixed cost/year (Rs.)     | 5,00,000                                              | 6,00,000                                              |            |
| Variable/unit (Rs.)       | 175                                                   | 150                                                   |            |
| Purchase price/unit (Rs.) |                                                       |                                                       | 125        |

The annual demand of the product is 8,000 units. Should the company make the product using process A or process B or buy it?

**Solution**

$$\begin{aligned}
 \text{Annual cost of process A} &= \text{FC} + \text{VC} \times \text{Volume} \\
 &= 5,00,000 + 175 \times 8,000 \\
 &= \text{Rs. 19,00,000}
 \end{aligned}$$

$$\begin{aligned}
 \text{Annual cost of process B} &= \text{FC} + \text{VC} \times \text{Volume} \\
 &= 6,00,000 + 150 \times 8,000 \\
 &= \text{Rs. 18,00,000}
 \end{aligned}$$

$$\begin{aligned}
 \text{Annual cost of buy} &= \text{Purchase price/unit} \times \text{Volume} \\
 &= 125 \times 8,000 \\
 &= \text{Rs. 10,00,000}
 \end{aligned}$$

Since the annual cost of buy option is the minimum among all the alternatives, the company should buy the product.

### QUESTIONS

1. Briefly explain the various criteria for make or buy decisions.
2. What are the approaches available for make or buy decisions? Explain any one of them with a suitable example.
3. An automobile company has extra capacity that can be used to produce gears that the company has been buying for Rs. 300 each. If the company makes gears, it will incur materials cost of Rs. 90 per unit, labour costs of Rs. 120 per unit, and variable overhead costs of Rs. 30 per unit. The annual fixed cost associated with the unused capacity is Rs. 2,40,000. Demand for next year is estimated at 4,000 units.
  - (a) Would it be profitable for the company to make the gears?
  - (b) Suppose the capacity could be used by another department for the production of some agricultural equipment that would cover its fixed and variable cost and contribute Rs. 90,000 to profit which would be more advantageous, gear production or agricultural equipment production?
4. An item has an yearly demand of 1,000 units. The different costs with regard to make and buy are as follows. Determine the best option.

|                                | <i>Buy</i> | <i>Make</i> |
|--------------------------------|------------|-------------|
| Item cost/unit                 | Rs. 6.00   | Rs. 5.90    |
| Procurement cost/order         | Rs. 10.00  |             |
| Set-up cost/set-up             |            | Rs. 50.00   |
| Annual carrying cost/item/year | Rs. 1.32   | Rs. 1.30    |
| Production rate/year           |            | 6,000 units |

5. A manufacturer of motor cycles buys side boxes at Rs. 240 each. In case he makes it himself, the fixed and variable costs would be Rs. 30,00,000 and Rs. 90 per side box respectively. Should the manufacturer make or buy the side boxes if the demand is 2,500 side boxes?
6. There are three alternatives available to meet the demand of a particular product. They are as follows:
  - (a) Manufacturing the product by using process A
  - (b) Manufacturing the product by using process B
  - (c) Buying the product

The details are as follows:

| <i>Cost elements</i> | <i>Manufacturing<br/>the product by<br/>using process A<br/>(Rs.)</i> | <i>Manufacturing<br/>the product by<br/>using process B<br/>(Rs.)</i> | <i>Buy<br/>(Rs.)</i> |
|----------------------|-----------------------------------------------------------------------|-----------------------------------------------------------------------|----------------------|
| Fixed cost/year      | 1,00,000                                                              | 3,00,000                                                              |                      |
| Variable cost/unit   | 75                                                                    | 70                                                                    |                      |
| Purchase price/unit  |                                                                       |                                                                       | 80                   |

The annual demand of the product is 10,000 units.

- Should the company make the product using process A or process B or buy it?
- At what annual volume should the company switch from buying to manufacturing the product by using process A?
- At what annual volume, should the company switch from process A to B?

# 14

## PROJECT MANAGEMENT

### 14.1 INTRODUCTION

A project consists of interrelated activities which are to be executed in a certain order before the entire task is completed. The activities are interrelated in a logical sequence, which is known as *precedence relationship*. An activity of a project cannot be started until all its immediate preceding activities are completed. Some of the typical projects are as follows:

- Construction of a house
- Commissioning of a factory
- Construction of a ship
- Fabrication of a steam boiler
- Construction of a bridge
- Construction of a dam
- Commissioning of a power plant
- Shutdown maintenance of major equipment/plants
- State level professional course admission process
- New product launching
- Launching a new weapon system
- Conducting national elections
- Research to develop a new technology
- Construction of railway coaches

Project management is generally applied for constructing public utilities, large industrial projects, and organizing mega events. Project management is considered to be an important field in production scheduling mainly because many of the industrial activities can also be viewed as project management problems. For example, fabrication of boilers, construction of railway coaches, launching satellites, product launching, organizing R&D activities, etc. can be viewed as project management problems.

From the examples, one can recognize the fact that many of the projects are repeated either by the same organization or by different organizations. Though they are repeatable by nature, each project is unique in itself. In the case of the new product launching of an organization, the first launching will be done at some key city. Subsequent launchings will be done at other cities as per priorities fixed by the marketing department of the organization. The project

schedule which is prepared for the first city cannot be applied to other cities without any modification because the time estimates of different activities of the project (product launching) will be different for each city due to environmental conditions. In some cases, addition or deletion of some activities will take place. So, if a project is repeated at a different place/different time, then a detailed planning effort is required.

Project is represented in the form of a network for the purpose of analytical treatment to get solutions for scheduling and controlling its activities. A network consists of a set of arcs which are connected meaningfully through a set of nodes. The precedence relationship among various activities of a project can be conveniently represented using a network. So, the collection of precedence relationships among various activities of a project is known as project network.

There are two methods of representing any project in the network form. There are: (i) Activities on Arrows Diagram (AOA diagram), and (ii) Activities on Nodes Diagram (AON Diagram).

The AOA diagram is commonly used in project management. This concept is demonstrated through two examples.

**EXAMPLE 14.1** A construction company has listed various activities that are involved in constructing a community hall. These are summarized along with predecessor(s) details in Table 14.1.

**Table 14.1** Details of Activities and Predecessor(s) for Constructing Community Hall

| Activity | Description                                               | Immediate predecessor(s) |
|----------|-----------------------------------------------------------|--------------------------|
| A        | Plan approval                                             |                          |
| B        | Site preparation                                          |                          |
| C        | Arranging foundation materials                            | A                        |
| D        | Excavation for foundation                                 | B                        |
| E        | Carpentry work for door and window main supporting frames | A                        |
| F        | Laying foundation                                         | C,D                      |
| G        | Raising wall from foundation to window base               | F                        |
| H        | Raising wall from window base to lintel level             | E,G                      |
| I        | Roofing                                                   | H                        |
| J        | Electrical wiring & fitting                               | I                        |
| K        | Plastering                                                | J,L                      |
| L        | Making doors and windows & fitting them                   | A                        |
| M        | White washing                                             | K                        |
| N        | Clearing the site before handing over                     | M                        |

Draw a project network for the above project.

**Solution** The project network summarizing the precedence relationships of various activities of constructing the community hall is shown in Fig. 14.1. In this figure, activities A and B are concurrent activities. Activities C, E and L can be started only after completing activity A. Activity D follows activity B.

Activity *F* can be started only when activities *C* and *D* are completed. One can use similar logic to infer the remaining precedence relationships in the Fig. 14.1.

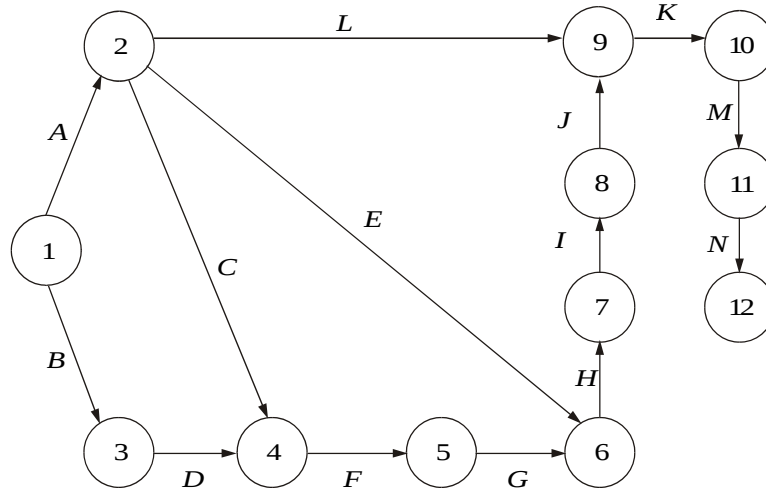


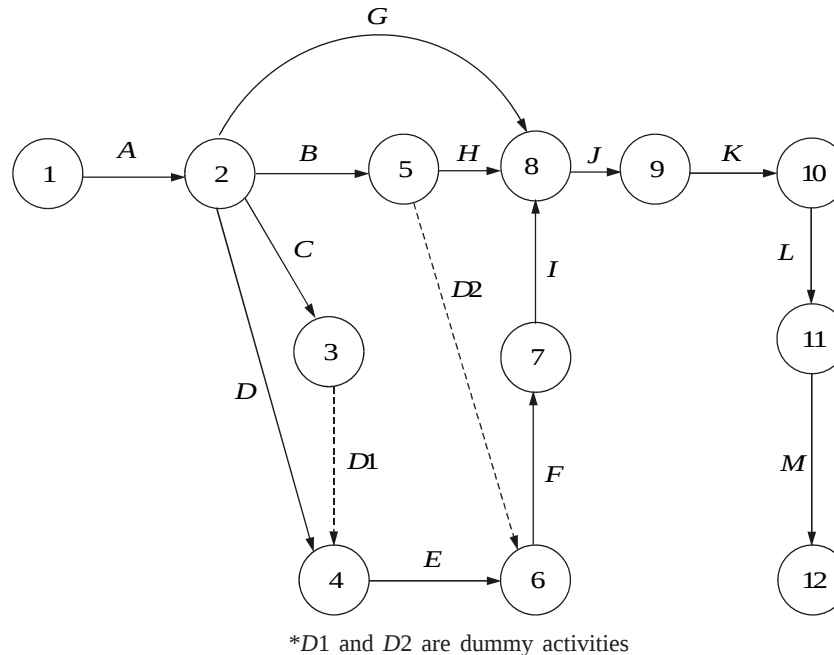
Fig. 14.1 Project network for construction of a community hall.

**EXAMPLE 14.2** Nowadays, in all professional institutes, students selection is done through a written test, group discussion and interview. Prior to the written test, there are so many activities, which are to be performed right from deciding the date of conducting all other events. Though the system looks simple, the precedence relationships among the activities necessitate coordination of these activities in completing the whole task of admitting students to any professional course. A delay at any stage would lead to ineffective operation of the system. Hence, this problem may be treated as a project consisting of activities with precedence relationships as shown in Table 14.2 (Rajkumar and Panneerselvam 1992). Construct a CPM network for this problem.

Table 14.2 Precedence Relationships for Professional Course Admission Process

| Activity | Description                                                                | Immediate predecessors |
|----------|----------------------------------------------------------------------------|------------------------|
| A        | Decide the date, time of the entrance test, group discussion and interview |                        |
| B        | Decide the examination venue(s)                                            | A                      |
| C        | Call for application through advertisement                                 | A                      |
| D        | Print application form                                                     | A                      |
| E        | Despatch applications                                                      | D, C                   |
| F        | Receive and process applications and send hall tickets                     | E, B                   |
| G        | Set question paper                                                         | A                      |
| H        | Arrange invigilators, and examiner                                         | B                      |
| I        | Make seating arrangements                                                  | F                      |
| J        | Conduct examination                                                        | G, H, I                |
| K        | Valuation and announcement of entrance test result                         | J                      |
| L        | Conduct group discussion and interview                                     | K                      |
| M        | Publish admission result                                                   | L                      |

**Solution** The CPM network for the above problem is shown in Fig. 14.2.



**Fig. 14.2** Network for professional course admission process.

Figure 14.2 shows how the precedence relations between various activities of the admission project are defined. In this figure, activity A is the starting activity. Activities B, C and D can be started only after activity A is completed. Activity E can be started after completing activities D and C. The precedence relationship between all other activities are defined in this manner. Activity M is the final activity representing the end of the selection process.

There are two main basic techniques in project management, namely, Critical Path Method (CPM), and Project Evaluation and Review Technique (PERT). CPM was developed by E.I. Du Pont de Nemours & Company as an application to construction projects and was later extended by Mauchly Associates. PERT was developed by a consulting firm for scheduling the research and development activities for the polaris missile program of US Navy. In CPM, the activities timings are deterministic in nature. But in PERT, each activity will have three time estimates: optimistic time, most likely time, and pessimistic time. There are some more advanced topics in project management which are as follows:

- Crashing of a project network
- Resource levelling
- Resource allocation

These advanced topics are beyond the scope of this book.

## 14.2 PHASES OF PROJECT MANAGEMENT

Project management has three phases: planning, scheduling and controlling.

The planning phase involves the following steps:

- (a) Dividing the project into distinct activities.
- (b) Estimating time requirement for each activity.
- (c) Establishing precedence relationships among the activities.
- (d) Construction of the arrow diagram (network).

The scheduling phase determines the start and end times of each and every activity. These can be summarized in the form of a time chart/Gantt chart. For each non-critical activity, the amount of slack time must be shown on the same time chart. This will be useful at the time of adjusting non-critical activities for resource levelling/resource allocation.

The control phase uses the arrow diagram and time chart for continuous monitoring and progress reporting. In this phase, the network will be updated if there is any variation in the proposed schedule.

## 14.3 GUIDELINES FOR NETWORK CONSTRUCTION

The terminologies which are used in network construction are explained as follows:

**Note:** Generally, a node represents the starting or ending of an activity.

**Branch/Arc:** A branch represents the actual activity which consumes some kind of resource.

*Precedence relations of activities:* For any activity, the precedence relations provide the following information:

1. The activities that precede it
2. The activities that follow it
3. The activities that may be concurrent with it.

Network construction requires a detailed list of individual activities of a project, estimates of activity durations, and specifications of precedence relationships among various activities of the project.

### Rules for network construction

The following are the primary rules for constructing an AOA diagram:

1. The starting event and ending event of an activity are called *tail event* and *head event*, respectively.
2. The network should have a unique starting node.

3. The network should have a unique completion node.
4. No activity has to be represented by more than one arc in the network.
5. No two activities need to have the same starting node and the same ending node.
6. Dummy activity is an imaginary activity indicating precedence relationship only. Duration of a dummy activity is zero.

#### 14.4 CRITICAL PATH METHOD (CPM)

As stated earlier, CPM deals with project management involving deterministic time estimates. In this section, the concept of CPM is demonstrated through an example.

**EXAMPLE 14.3** Consider Table 14.3 summarizing the details of a project involving 16 activities.

**Table 14.3** Project Details

| <i>Activity</i> | <i>Immediate predecessor</i> | <i>Duration (months)</i> |
|-----------------|------------------------------|--------------------------|
| A               |                              | 1                        |
| B               |                              | 4                        |
| C               |                              | 2                        |
| D               | B                            | 2                        |
| E               | B                            | 3                        |
| F               | A, D                         | 3                        |
| G               | A, D                         | 2                        |
| H               | B                            | 1                        |
| I               | C, E                         | 3                        |
| J               | C, E                         | 2                        |
| K               | F                            | 3                        |
| L               | G, H, I                      | 6                        |
| M               | G, H, I                      | 5                        |
| N               | J                            | 2                        |
| P               | K, L                         | 4                        |
| Q               | M, N                         | 7                        |

- (a) Construct the CPM network.
- (b) Determine the critical path.
- (c) Compute total floats and free floats for non-critical activities.

**Solution** (a) The CPM network is shown in Fig. 14.3.

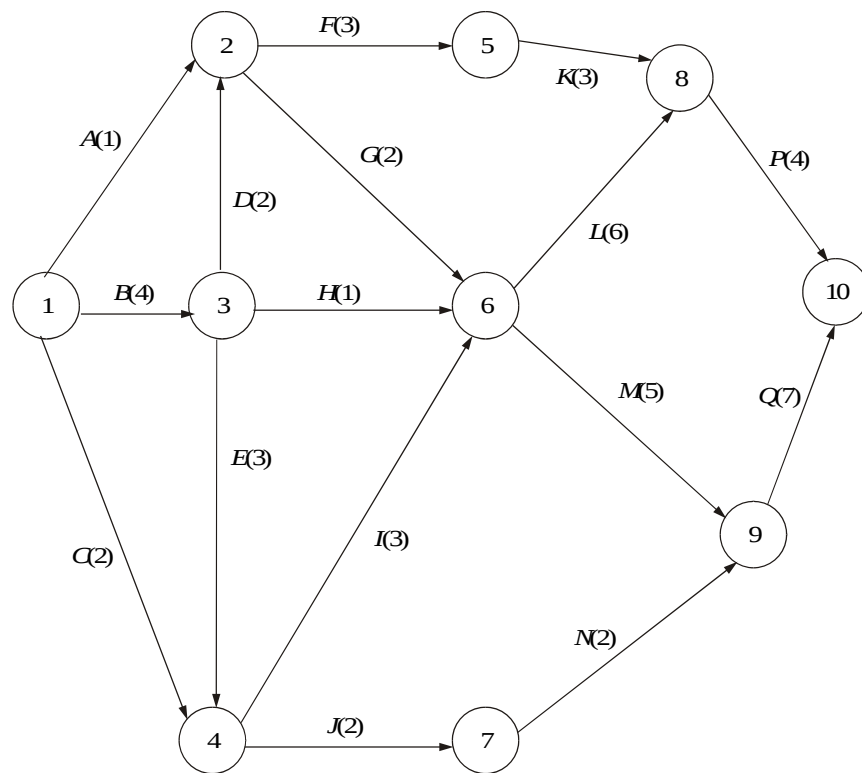


Fig. 14.3 CPM network for Example 14.3.

**(b) Determination of critical path.** The critical path of a project network is the longest path in the network. This can be identified by simply listing out all the possible paths from the start node of the project (node 1) to the end node of the project (node 10), and then selecting the path with the maximum sum of activity times on that path.

This method has several drawbacks. In a large network, one may commit mistake in listing all the paths. Moreover, this method will not provide necessary details like total floats and free floats for further analysis.

A different approach is therefore to be used to identify the critical path. This consists of two phases: *Phase 1* determines the earliest start times (ES) of all the nodes. This is called the *forward pass*; *phase 2* determines the latest completion times (LC) of various nodes. This is called the *backward pass*. Let

$D_{ij}$  = duration of the activity  $(i, j)$ ,

$ES_j$  = earliest start time of all the activities which emanate from node  $j$ ,

$LC_j$  = latest completion time of all the activities that end at node  $j$ .

*(i) Determination of earliest start times ( $ES_j$ ):* During forward pass, use the following formula to compute the earliest start time for all nodes:

$$ES_j = \max_i [ES_i + D_{ij}]$$

The calculations of  $ES_j$  are as follows:

NODE 1: For node 1,  $ES_1 = 0$

$$\begin{aligned} \text{NODE 3: } ES_3 &= \max_{i=1} [ES_i + D_{i3}] \\ &= \max [ES_1 + D_{1,3}] = 0 + 4 = 4 \end{aligned}$$

$$\begin{aligned} \text{NODE 2: } ES_2 &= \max_{i=1,3} [ES_i + D_{i2}] \\ &= \max [(ES_1 + D_{1,2}), (ES_3 + D_{3,2})] \\ &= \max (0 + 1, 4 + 2) = 6 \end{aligned}$$

$$\begin{aligned} \text{NODE 4: } ES_4 &= \max_{i=1,3} [ES_i + D_{i4}] \\ &= \max [ES_1 + D_{1,4}, ES_3 + D_{3,4}] \\ &= \max (0 + 2, 4 + 3) = 7 \end{aligned}$$

Similarly, the  $ES_j$  values for all other nodes are computed and summarized in Fig. 14.4.

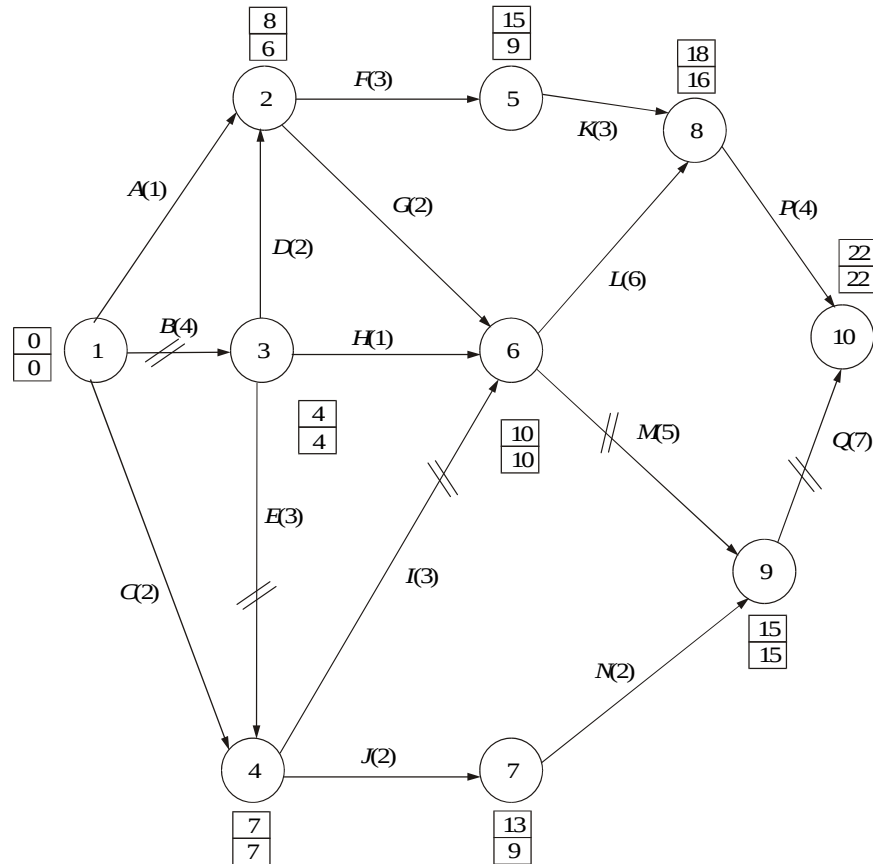


Fig. 14.4 Network with critical path calculations.

(ii) *Determination of latest completion times ( $LC_i$ )*. During backward pass, the following formula is used to compute the latest completion times ( $LC_i$ ):

$$LC_i = \min_j [LC_j - D_{ij}]$$

NODE 10: For the last node 10,  $LC_{10} = ES_{10} = 22$ .

$$\begin{aligned} \text{NODE 9: } LC_9 &= \min_{j=10} [LC_j - D_{9,j}] \\ &= \min [LC_{10} - D_{9,10}] = 22 - 7 = 15. \end{aligned}$$

$$\begin{aligned} \text{NODE 8: } LC_8 &= \min_{j=10} [LC_j - D_{8,j}] \\ &= \min [LC_{10} - D_{8,10}] = 22 - 4 = 18. \end{aligned}$$

$$\begin{aligned} \text{NODE 7: } LC_7 &= \min_{j=9} [LC_j - D_{7,j}] \\ &= \min [LC_9 - D_{7,9}] = 15 - 2 = 13. \end{aligned}$$

$$\begin{aligned} \text{NODE 6: } LC_6 &= \min_{j=8,9} [LC_j - D_{6,j}] \\ &= \min [LC_8 - D_{6,8}, LC_9 - D_{6,9}] \\ &= \min [18 - 6, 15 - 5] = 10 \end{aligned}$$

Similarly, the  $LC_j$  values for all other nodes are summarized in Fig. 14.4.

An activity ( $i, j$ ) is said to be critical if all the following conditions are satisfied:

- (i)  $ES_i = LC_i$
- (ii)  $ES_j = LC_j$
- (iii)  $ES_j - ES_i = LC_j - LC_i = D_{ij}$

By applying the above conditions to the activities in Fig. 14.4, the critical activities are identified and are shown in the same figure with cross lines on them. The corresponding critical path is 1–3–4–6–9–10 ( $B-E-I-M-Q$ ).

### (c) Determination of Total Floats and Free Floats

**TOTAL FLOATS.** It is the amount of time that the completion time of an activity can be delayed without affecting the project completion time.

$$\begin{aligned} TF_{ij} &= LC_j - ES_i - D_{ij} \\ &= LC_j - (ES_i + D_{ij}) = LC_j - EC_{ij} \end{aligned}$$

where  $EC_{ij}$  is the earliest completion of the activity ( $i, j$ ).

Also,  $TF_{ij} = LS_{ij} - ES_i$ , where  $LS_{ij}$  is the latest start of the activity ( $i, j$ ).

$$LS_{ij} = LC_j - D_{ij}$$

**FREE FLOATS.** It is the amount of time that the activity completion time can be delayed without affecting the earliest start time of immediate successor activities in the network.

$$\begin{aligned}
 FF_{ij} &= ES_j - ES_i - D_{ij} \\
 &= ES_j - (ES_i + D_{ij}) = ES_j - EC_{ij}
 \end{aligned}$$

A summary of calculations of total floats and free floats of the activities is given in Table 14.4.

**Table 14.4** Total Floats and Free Floats

| Activity (i, j) | Duration ( $D_{ij}$ ) | Total float ( $TF_{ij}$ ) | Free float ( $FF_{ij}$ ) |
|-----------------|-----------------------|---------------------------|--------------------------|
| 1-2             | 1                     | 7                         | 5                        |
| 1-3             | 4                     | 0                         | 0                        |
| 1-4             | 2                     | 5                         | 5                        |
| 2-5             | 3                     | 6                         | 0                        |
| 2-6             | 2                     | 2                         | 2                        |
| 3-2             | 2                     | 2                         | 0                        |
| 3-4             | 3                     | 0                         | 0                        |
| 3-6             | 1                     | 5                         | 5                        |
| 4-6             | 3                     | 0                         | 0                        |
| 4-7             | 2                     | 4                         | 0                        |
| 5-8             | 3                     | 6                         | 4                        |
| 6-8             | 6                     | 2                         | 0                        |
| 6-9             | 5                     | 0                         | 0                        |
| 7-9             | 2                     | 4                         | 4                        |
| 8-10            | 4                     | 2                         | 2                        |
| 9-10            | 7                     | 0                         | 0                        |

Any critical activity will have zero total float ( $TF_{ij}$ ) and zero free float ( $FF_{ij}$ ). Based on this property, one can determine the critical activities. From Table 14.4, one can check that the total floats and free floats for the activities (1,3), (3,4), (4,6), (6,9) and (9,10) are zero. Therefore, they are critical activities.

The corresponding critical path is 1-3-4-6-9-10 (*B-E-I-M-Q*).

## 14.5 GANTT CHART/TIME CHART

The next stage after completing network calculations is to draw a Gantt chart/time chart. The start time and completion time of each and every activity will be represented on this chart. This chart gives a clear calendar schedule for the whole project.

This chart is also used for resource levelling purpose. When there are limitations on the available resource(s) (manpower, equipment, money, etc.), using this chart, one can adjust the schedule of non-critical activities, depending upon their total floats to minimize the peak requirement of resource(s). This helps to level the resource requirements smoothly throughout the project execution. Even if there is no restriction on any resource, it is the usual practice to smooth out the resource requirements by resource levelling technique.

The time chart for the example problem is given in Fig. 14.5.

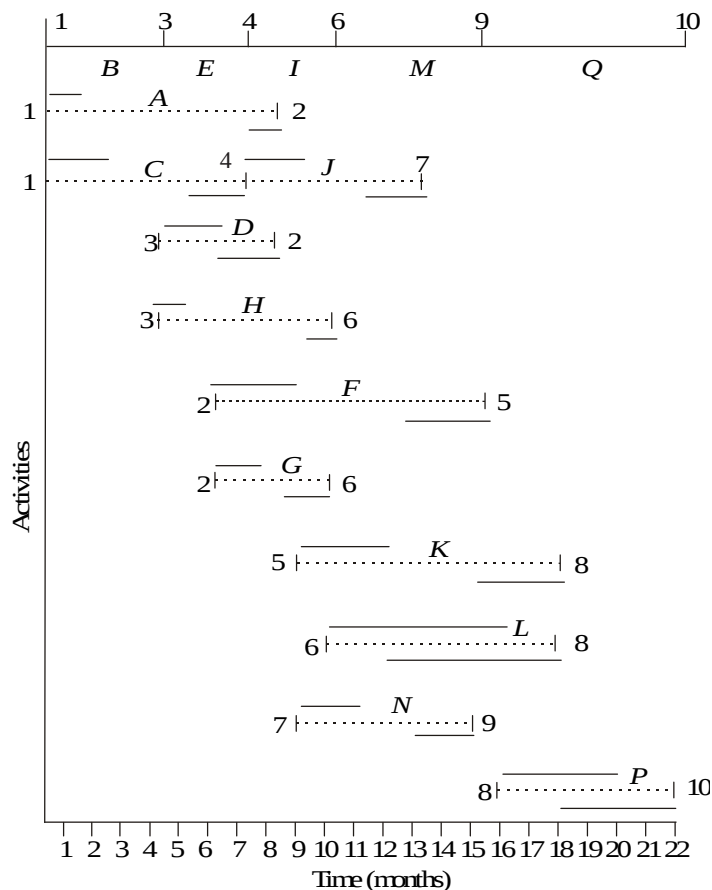


Fig. 14.5 Time chart/Gantt chart.

In Fig. 14.5, the thick line represents the schedule for various critical activities [(1,3), (3,4), (4,6), (6,9) and (9,10)]. The horizontal dashed lines represent the total time span over which a non-critical activity can be performed. As stated earlier, a non-critical activity will have float in excess of its time duration. So, it is possible to adjust the start and completion time of any non-critical activity over its entire range shown by the dashed lines without delaying the project completion time. Infinite number of schedules are possible for the non-critical activities. But to give a finite number of schedules, we consider only two types of schedules: 1) earliest start schedule, and 2) latest start schedule. These are shown by continuous horizontal lines over and below the dashed line showing the time range for execution of a given non-critical activity, respectively.

## 14.6 PERT (PROJECT EVALUATION AND REVIEW TECHNIQUE)

Earlier, we assumed deterministic estimates for time durations of various activities in a project. But, in reality, activity durations may be probabilistic. Hence, probabilistic considerations are incorporated while obtaining time durations of the activities in a project.

The following three estimates are used:

$a$  = optimistic time

$b$  = pessimistic time

$m$  = most likely time

The *optimistic time* is a time estimate if the execution is very good. The *pessimistic time* is a time estimate if the execution is very badly done. The *most likely time* is a time estimate if execution is normal. The probabilistic data for project activities generally follow beta distribution. The formula for mean ( $\mu$ ) and variance ( $\sigma^2$ ) of the beta distribution are as follows:

$$\mu = (a + 4m + b)/6$$

$$\sigma^2 = [(b - a)/6]^2$$

The range for the time estimates is from  $a$  to  $b$ . The most likely time will be anywhere in between the range from  $a$  to  $b$ .

The expected project completion time =  $\sum \mu_i$

where  $\mu_i$  is the expected duration of the  $i$ th critical activity.

The variance of the project completion time =  $\sum \sigma_i^2$

where  $\sigma_i^2$  is the variance of the  $i$ th critical activity in the critical path.

As a part of statistical analysis, we may be interested in knowing the probability of completing the project on or before a given due date ( $C$ ), or we may be interested in knowing the expected project completion time if the probability of completing the project is given.

For the purpose of this analysis, the Beta distribution is approximated to standard normal distribution whose statistic is given by

$$z = (x - \mu)/\sigma$$

where  $x$  is the actual project completion time.

$\mu$  = expected project completion time (sum of the expected durations of the critical activities).

$\sigma$  = standard deviation of the expected project completion time (square root of the sum of the variances of all the critical activities).

Therefore,  $P(x \leq C)$  represents the probability that the project will be completed on or before the  $C$  time units. This can be converted into the standard normal statistic  $z$  as follows:

$$P[(x - \mu)/\sigma] \leq (C - \mu)/\sigma = P[z \leq (C - \mu)/\sigma]$$

**EXAMPLE 14.4** Consider the following (Table 14.5) summarizing the details of a project involving nine activities.

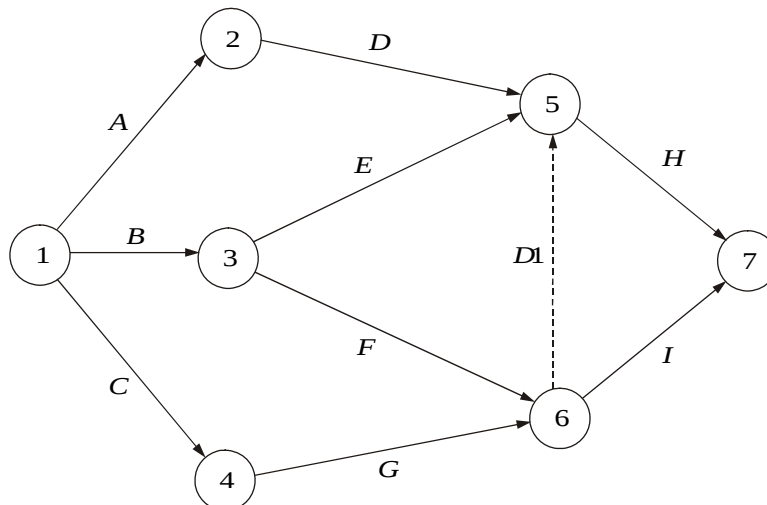
**Table 14.5** Details of Project with Nine Activities

| Activity | Predecessor(s)    | Duration (weeks) |          |          |
|----------|-------------------|------------------|----------|----------|
|          |                   | <i>a</i>         | <i>m</i> | <i>b</i> |
| <i>A</i> |                   | 2                | 2        | 8        |
| <i>B</i> |                   | 1                | 2        | 3        |
| <i>C</i> |                   | 1                | 5        | 9        |
| <i>D</i> | <i>A</i>          | 1                | 2        | 9        |
| <i>E</i> | <i>B</i>          | 1                | 2        | 3        |
| <i>F</i> | <i>B</i>          | 1                | 2        | 3        |
| <i>G</i> | <i>C</i>          | 1                | 4        | 7        |
| <i>H</i> | <i>D, E, F, G</i> | 6                | 7        | 8        |
| <i>I</i> | <i>F, G</i>       | 1                | 2        | 9        |

The project comprises the following steps:

- Construct the project network.
- Find the expected duration and variance of each activity.
- Find the critical path and the expected project completion time.
- What is the probability of completing the project on or before 19 weeks?
- If the probability of completing the project is 0.85, find the expected project completion time.

**Solution** (a) The project network is shown in Fig. 14.6.



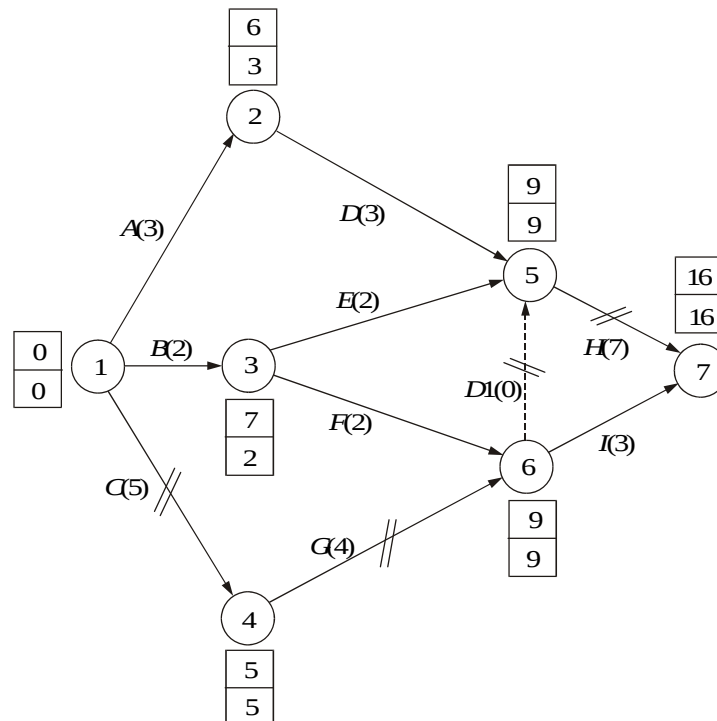
**Fig. 14.6** Network for Example 14.4.

(b) The expected duration and variance of each activity are as shown in Table 14.6.

**Table 14.6** Computations of Expected Duration and Variance

| Activity | Duration (weeks) |          |          | Expected duration | Variance |
|----------|------------------|----------|----------|-------------------|----------|
|          | <i>a</i>         | <i>m</i> | <i>b</i> |                   |          |
| <i>A</i> | 2                | 2        | 8        | 3                 | 1.00     |
| <i>B</i> | 1                | 2        | 3        | 2                 | 0.11     |
| <i>C</i> | 1                | 5        | 9        | 5                 | 1.78     |
| <i>D</i> | 1                | 2        | 9        | 3                 | 1.78     |
| <i>E</i> | 1                | 2        | 3        | 2                 | 0.11     |
| <i>F</i> | 1                | 2        | 3        | 2                 | 0.11     |
| <i>G</i> | 1                | 4        | 7        | 4                 | 1.00     |
| <i>H</i> | 6                | 7        | 8        | 7                 | 0.11     |
| <i>I</i> | 1                | 2        | 9        | 3                 | 1.78     |

(c) The calculations of critical path based on expected durations are summarized in Fig. 14.7.



**Fig. 14.7** Network for Example 14.4 with PERT calculations.

The critical path is  $C-G-D1-H$ , and the corresponding project completion time is 16 weeks.

(d) The sum of the variances of all the activities on the critical path is 2.89 weeks  $(1.78 + 1.00 + 0.00 + 0.11)$ . Therefore,

$$\sigma = \sqrt{2.89} = 1.7 \text{ weeks.}$$

$$P(x \leq 19) = P[(x - \mu)/\sigma \leq (19 - 16)/1.7]$$

$$= P(z \leq 1.77)$$

$$= 0.9616 \text{ (value obtained from standard normal table).}$$

Thus, the probability of completing the project on or before 19 weeks is 0.9616.

$$(e) P(x \leq C) = 0.85$$

$$P[(x - \mu)/\sigma \leq (C - \mu)/\sigma] = 0.85$$

$$P[z \leq (C - 16)/1.7] = 0.85$$

From the standard normal table, the value of  $z$  is 1.04 when the cumulative probability is 0.85. Therefore,

$$(C - 16)/1.7 = 1.04$$

$$C = 1.04 \times 1.7 + 16 = 17.768 \text{ weeks}$$

$$= 18 \text{ weeks (approx.)}$$

The project will be completed in 17.768 weeks (approx. 18 weeks) if the probability of completing the project is 0.85.

## QUESTIONS

1. Explain different phases of project management.
2. Distinguish between CPM and PERT.
3. Discuss the guidelines for network construction.
4. The activities involved in Alpha Garment Manufacturing Company are given in the following table, with their time estimates. Draw the network for the given activities and carry out the critical path calculations.

| Activity | Description                 | Immediate predecessor(s) | Duration (days) |
|----------|-----------------------------|--------------------------|-----------------|
| A        | Forecast sales volume       |                          | 10              |
| B        | Study competitive market    |                          | 7               |
| C        | Design item and facilities  | A                        | 5               |
| D        | Prepare production plan     | C                        | 3               |
| E        | Estimate cost of production | D                        | 2               |
| F        | Set sales price             | B, E                     | 1               |
| G        | Prepare budget              | E, F                     | 14              |

5. Consider the following problem involving activities from A to J.

| Activity | Immediate predecessor | Duration (months) |
|----------|-----------------------|-------------------|
| A        |                       | 1                 |
| B        | A                     | 4                 |
| C        | A                     | 2                 |
| D        | A                     | 2                 |
| E        | D                     | 3                 |
| F        | D                     | 3                 |
| G        | E                     | 2                 |
| H        | F, G                  | 1                 |
| I        | C, H                  | 3                 |
| J        | B                     | 2                 |

- Construct the CPM network.
- Determine the critical path.
- Compute the total floats and free floats for non-critical activities.

6. Consider the following data of a project:

| Activity | Predecessor(s) | Duration (weeks) |          |          |
|----------|----------------|------------------|----------|----------|
|          |                | <i>a</i>         | <i>m</i> | <i>b</i> |
| A        |                | 1                | 2        | 3        |
| B        |                | 2                | 2        | 8        |
| C        | A              | 6                | 7        | 8        |
| D        | B              | 1                | 2        | 3        |
| E        | A              | 1                | 4        | 7        |
| F        | C, D           | 1                | 5        | 9        |
| G        | C, D, E        | 1                | 2        | 3        |
| H        | F              | 1                | 2        | 9        |

- Construct the project network.
- Find the expected duration and variance of each activity.
- Find the critical path and the expected project completion time.
- What is the probability of completing the project on or before 20 weeks?
- If the probability of completing the project is 0.8, find the expected project completion time.

# 15

## VALUE ANALYSIS/VALUE ENGINEERING

### 15.1 INTRODUCTION

Value analysis is one of the major techniques of cost reduction and cost prevention. It is a disciplined approach that ensures necessary functions for minimum cost without sacrificing quality, reliability, performance, and appearance. According to the Society of American Value Engineers (SAVE),

Value Analysis is the systematic application of recognized techniques which identify the function of a product or service, establish a monetary value for the function and provide the necessary function reliably at the lowest overall cost.

It is an organized approach to identify unnecessary costs associated with any product, material part, component, system or service by analysing the function and eliminating such costs without impairing the quality, functional reliability, or the capacity of the product to give service.

### 15.2 WHEN TO APPLY VALUE ANALYSIS

One can definitely expect very good results by initiating a VA programme if one or more of the following symptoms are present:

1. Company's products show decline in sales.
2. Company's prices are higher than those of its competitors.
3. Raw materials cost has grown disproportionate to the volume of production.
4. New designs are being introduced.
5. The cost of manufacture is rising disproportionate to the volume of production.
6. Rate of return on investment has a falling trend.
7. Inability of the firm to meet its delivery commitments.

#### 15.2.1 Value Analysis vs. Value Engineering

Often the terms *value analysis* and *value engineering* are used synonymously.

Though the philosophy underlying the two is same, i.e. identification of unnecessary cost, yet they are different. The difference lies in the time and the stage at which the techniques are applied.

*Value analysis* is the application of a set of techniques to an existing product with a view to improve its value. It is thus a remedial process. *Value engineering* is the application of exactly the same set of techniques to a new product at the design stage, project concept or preliminary design when no hardware exists to ensure that bad features are not added. *Value engineering*, therefore, is a preventive process.

### Value

The term 'value' is used in different ways and, consequently, has different meanings. The designer equates the value with reliability; a purchase person with price paid for the item; a production person with what it costs to manufacture, and a sales person with what the customer is willing to pay. Value, in value investigation, refers to "economic value", which itself can be divided into four types: cost value, exchange value, use value, and esteem value. These are now briefly described.

*Cost value.* It is the summation of the labour, material, overhead and all other elements of cost required to produce an item or provide a service compared to a base.

*Exchange value.* It is the measure of all the properties, qualities and features of the product, which make the product possible of being traded for another product or for money. In a conventional sense, *exchange value* refers to the price that a purchaser will offer for the product, the price being dependent upon satisfaction (value) which he derives from the product. Value derived from the product consists of two parts "use value" and "esteem value", which are now described.

*Use value.* It is known as the function value. The *use value* is equal to the value of the functions performed. Therefore, it is the price paid by the buyer (buyer's view), or the cost incurred by the manufacturer (manufacturer's view) in order to ensure that the product performs its intended functions efficiently. The use value is the fundamental form of economic value. An item without "use value" can have neither "exchange value" nor "esteem value".

*Esteem value.* It involves the qualities and appearance of a product (like a TV set), which attract persons and create in them a desire to possess the product. Therefore, *esteem value* is the price paid by the buyer or the cost incurred by the manufacturer beyond the use value.

### Performance

The performance of a product is the measure of functional features and properties that make it suitable for a specific purpose. Appropriate performance requires that (a) the product reliably accomplish the intended use of work or

service requirement (functional requirements), (b) the product provide protection against accident, harmful effects on body and danger to human life (safety requirements), (c) the product give trouble-free service cover during its specified life span (reliability requirements), (d) service and maintenance work can be carried out on the product with ease and with simple tools (maintainability requirements), and (e) appearance of the product creates an impression on the buyer and induces in him or her the desire to own the product (appearance requirements).

Performance and cost must be interwoven. Desired performance at the least cost should be achieved by selecting appropriate materials and manufacturing operations, which is the measure of value. Therefore, the value of the product is the ratio of performance (utility) to cost. Thus,

$$\text{Value} = \frac{\text{Performance (utility)}}{\text{Cost}}$$

Value can be increased by increasing the utility for the same cost or by decreasing the cost for the same utility. Satisfactory performance at lesser cost through identification and development of low cost alternatives is the philosophy of Value analysis.

### 15.3 FUNCTION

Function is the purpose for which the product is made. Identification of the basic functions and determination of the cost currently being spent on them are the two major considerations of value analysis.

Function identifies the characteristics which make the product/component/part/item/device to work or sell. “Work functions” lend performance value while “sell functions” provide esteem value. Verbs like “support”, “hold”, “transmit”, “prevent”, “protect”, “exhibits”, “control”, etc., are used to describe work functions, while “attract”, enhance”, “improve”, “create”, etc., are used to describe “sell” functions. For example, in a “bus driver cabin”, the functional analysis of some of the parts are given in Table 15.1.

**Table 15.1** Functional Analysis of Some Parts of a Bus Driver Cabin

| <i>Component of study</i> | <i>Functional analysis</i> |              |
|---------------------------|----------------------------|--------------|
|                           | <i>Verb</i>                | <i>Noun</i>  |
| Steering wheel            | Control                    | Direction    |
| Gear box                  | Change                     | Speed        |
| Brake system              | Stop                       | Vehicle      |
| Wiper                     | Clear                      | Water        |
| Horn                      | Make                       | Sound        |
| Side mirror               | Show                       | Side traffic |

### Classification of the functions

Rarely do all functions assume equal importance. Usually, some functions are more important than others. Functions can be classified into the following three categories:

1. Primary function
2. Secondary function
3. Tertiary function

*Primary functions* are the basic functions for which the product is specially designed to achieve. Primary functions, therefore, are the most essential functions whose non-performance would make the product worthless, e.g. a photo frame exhibits photographs, a chair supports weight, a fluorescent tube gives light.

*Secondary functions* are those which, if not in-built, would not prevent the device from performing its primary functions, e.g., arms of a chair provide support for hands. Secondary functions are usually related to convenience. The product can still work and fulfill its intended objective even if these functions are not in-built and yet they may be necessary to sell the product.

*Tertiary functions* are usually related to esteem appearance. For example, Sunmica top of a table gives esteem appearance for the table.

Let us consider a single example of painting a company bus to explain all the above three functions. Here, the primary function of painting is to avoid corrosion. The secondary function is to identify the company to which the bus belongs by the colour of the paint (e.g. blue colour for Ashok Leyland Ltd.). The tertiary function is to impart a very good appearance to the bus by using brilliant colours.

## 15.4 AIMS

The aims of value engineering are as follows:

1. Simplify the product.
2. Use (new) cheaper and better materials.
3. Modify and improve product design.
4. Use efficient processes.
5. Reduce the product cost.
6. Increase the utility of the product by economical means.
7. Save money or increase the profits.

The value content of each piece of a product is assessed using the following questions:

1. Does its use contribute to value?
2. Is its cost proportionate to its usefulness?
3. Does it need all its features?

These three questions pertain to the function of the part which may decide the elimination of parts.

- Is there anything better for the intended use?
- Can company or vendor standard be used?
- Can a usable part be made by a lower-cost method?
- Is it made with the proper tooling, considering volume?
- Does the part yield suitable profit?
- Can another vendor furnish the same at a lower cost?

## 15.5 VALUE ENGINEERING PROCEDURE

The basic steps of value engineering are as follows:

- (a) Blast
  - (i) Identify the product.
  - (ii) Collect relevant information.
  - (iii) Define different functions.
- (b) Create
  - (iv) Different alternatives.
  - (v) Critically evaluate the alternatives.
- (c) Refine
  - (vi) Develop the best alternative.
  - (vii) Implement the alternative.

*Step 1: Identify the product.* First, identify the component for study. In future, any design change should add value and it should not make the product as obsolete one. Value engineering can be applied to a product as a whole or to sub-units.

*Step 2: Collect relevant information.* Information relevant to the following must be collected:

- Technical specifications with drawings
- Production processes, machine layout and instruction sheet
- Time study details and manufacturing capacity
- Complete cost data and marketing details
- Latest development in related products

*Step 3: Define different functions.* Identify and define the primary, secondary and tertiary functions of the product or parts of interest. Also, specify the value content of each function and identify the high cost areas.

*Step 4: Different alternatives.* Knowing the functions of each component part and its manufacturing details, generate the ideas and create different alternatives so as to increase the value of the product. Value engineering should be done after a **brain storming** session. All feasible or non-feasible suggestions are recorded without any criticism; rather, persons are encouraged to express their views freely.

### Basic principles of brain storming

Some of the important principles of brain storming which are useful in value analysis are now listed.

(i) *A quality idea comes from quantity of ideas.* If the number of ideas generated is more, the more good solutions do turn up.

(ii) *Creative ideas emerge from unconventional thinking.* This is possible when members of the group “talk off the top of their heads” and voice weird ideas as they flash through their minds, regardless of how stupid or impractical they may appear. Often, non-technical personnel can prove to be the greatest innovators in technical areas since their viewpoints are objective and they do not know that some of their ideas are technically not feasible at all. So it is preferable to include one or two non-technical persons in the study team. Members are to be told by the team leader in the beginning of the session itself, not to breathe a word of criticism of even the most weirdest idea.

(iii) *Spontaneous evaluation of ideas curbs imaginative thinking and retards the flow of creative ideas.* The group should not evaluate the alternatives suggested by its member immediately since immediate evaluation may curb imaginative thinking and slow down the flow of creative ideas.

(iv) *Hitch-hiking on the ideas often lead to better ideas.* Participants have to improve upon ideas of other members either directly or by combining more ideas in addition to contributing ideas of their own. A brilliant idea may not be a practical one initially, or it may look to be silly or useless but discussions can convert it into a valuable one.

(v) *Creativity is a regenerative process and the recording of ideas as they emerge helps serve as a catalyst to generate more ideas.* Memory may not retain all ideas or recall them when they are needed. So, a stenographer may be asked to record ideas simultaneously. A tape recorder can also be used for this purpose or even ideas can be written on a blackboard. These recorded ideas can be reviewed at some later date.

(vi) *When ideas cease to flow, short diversions enable the mind to rebound with new ideas after recuperation.* Members of the syndicate may reach a stage where new ideas do not come. At such a stage, short diversions—rest, favourite sport, hobby, lunch or tea break, etc.—may be taken during which members are advised to sleep over the ideas and report fresh after the break. Such short diversions enable mind to recoup and rebound with new ideas.

*Step 5: Critically evaluate the alternatives.* Different ideas recorded under step 4 are compared, evaluated and critically assessed for their virtues, validity and feasibility as regards their financial and technical requirements. The ideas technically found and involving lower costs are further developed.

*Step 6: Develop the best alternative.* Detailed development plans are made for those ideas which emerged during step 5 and appear most suitable and promising. Development plans comprise drawing the sketches, building of

models, conducting discussions with the purchase section, finance section, marketing division, etc.

*Step 7: Implement the alternative.* The best alternative is converted into a proto-type manufacturing model which ultimately goes into operation and its results are recorded.

## 15.6 ADVANTAGES AND APPLICATION AREAS

### Advantages

The advantages of value engineering are as follows:

1. It is a much faster cost reduction technique.
2. It is a less expensive technique.
3. It reduces production costs and adds value to sales income of the product.

### Applications

The various application areas of value engineering are machine tool industries, industries making accessories for machine tools, auto industries, import substitutes, etc.

## QUESTIONS

1. Define value analysis(VA)/ value engineering (VE).
2. Discuss the symptoms favouring the applications of VA/VE.
3. Define value. What are the types of value?
4. What are the types of functions? Explain them with examples.
5. What are the aims of value engineering?
6. Briefly explain the steps of value engineering.
7. Discuss the advantages and application areas of value engineering.

# 16

## LINEAR PROGRAMMING

### 16.1 INTRODUCTION

A generalized format of the linear programming (LP) problem is

$$\text{Maximize } Z = C_1X_1 + C_2X_2 + C_3X_3 + \dots + C_nX_n$$

subject to

$$a_{11} X_1 + a_{12} X_2 + \dots + a_{1n} X_n \leq b_1$$

$$a_{21} X_1 + a_{22} X_2 + \dots + a_{2n} X_n \leq b_2$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$a_{i1} X_1 + a_{i2} X_2 + \dots + a_{in} X_n \leq b_i$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$a_{m1} X_1 + a_{m2} X_2 + \dots + a_{mn} X_n \leq b_m$$

where,  $X_1, X_2, X_3, \dots, X_n \geq 0$ .

If the number of variables in the problem is two, then one can use the graphical method to solve the problem. If the number of variables in the problem is more than two, one should use the simplex method to solve the problem.

### 16.2 DEVELOPMENT OF LP MODELS

Modelling is an art. One can develop this expertise only by seeing more and more models. In this section, the concept of model building is demonstrated using two example problems.

**EXAMPLE 16.1** A company manufactures two types of products,  $P_1$  and  $P_2$ . Each product uses a lathe and milling machine. The processing time for  $P_1$  on the lathe is five hours and on the milling machine it is four hours. The processing time for  $P_2$  on the lathe is 10 hours and on the milling machine it is four hours. The maximum number of hours per week available on the lathe and the milling machine are 60 hours and 40 hours, respectively. The profit per unit of selling  $P_1$  and  $P_2$  are Rs. 6.00 and Rs. 8.00, respectively. Formulate a linear programming model to determine the production volume of each product such that the total profit is maximized.

**Solution** The data of the problem are summarized in Table 16.1.

**Table 16.1** Details of Products

| Machine           | Product |    | Limit on machine hours |
|-------------------|---------|----|------------------------|
|                   | P1      | P2 |                        |
| Lathe             | 5       | 10 | 60                     |
| Milling machine   | 4       | 4  | 40                     |
| Profit/unit (Rs.) | 6       | 8  |                        |

Let  $X_1$  be the production volume of the product,  $P_1$ , and  
 $X_2$  be the production volume of the product,  $P_2$ .

The corresponding linear programming model to determine the production volume of each product such that the total profit is maximized is as shown below:

$$\begin{aligned}
 &\text{Maximize } Z = 6X_1 + 8X_2 \\
 &\text{subject to} \\
 &\quad 5X_1 + 10X_2 \leq 60 \\
 &\quad 4X_1 + 4X_2 \leq 40 \\
 &\quad X_1, X_2 \geq 0
 \end{aligned}$$

**EXAMPLE 16.2** A nutrition scheme for babies is proposed by a committee of doctors. Babies can be given two types of food (Type I and Type II), which are available in standard packets weighing 50 grams. The cost per packet of these foods is Rs. 2.00 and Rs. 3.00 respectively. The vitamin availability in each type of food per packet and the minimum vitamin requirement in each type of vitamin is summarized in Table 16.2. Develop a linear programming model to determine the optimal combination of food types with the minimum cost such that the minimum required vitamin in each type is satisfied.

**Table 16.2** Details of Food Types

| Vitamin           | Vitamin availability per packet |             | Minimum daily required vitamin |
|-------------------|---------------------------------|-------------|--------------------------------|
|                   | Food type 1                     | Food type 2 |                                |
| 1                 | 1                               | 1           | 6                              |
| 2                 | 7                               | 1           | 14                             |
| Cost/packet (Rs.) | 2                               | 3           |                                |

**Solution** Let,  $X_1$  be the No. of packets of food type 1 suggested for babies, and  $X_2$  be the No. of packets of food type 2 suggested for babies.

A linear programming model to determine the number of packets of each food type to be suggested for babies with the minimum cost such that the minimum daily required vitamin in each food type is satisfied is as shown below.

Minimize  $Z = 2X_1 + 3X_2$   
subject to

$$X_1 + X_2 \geq 6$$

$$7X_1 + X_2 \geq 14$$

$$X_1, X_2 \geq 0$$

### 16.3 GRAPHICAL METHOD

As already stated, if the number of variables in any linear programming problem is two, one can use the graphical methods to solve it. In this section, the same is demonstrated with two example problems.

**EXAMPLE 16.3** Solve the following LP problem using the graphical method.

Maximize  $Z = 6X_1 + 8X_2$   
subject to

$$5X_1 + 10X_2 \leq 60$$

$$4X_1 + 4X_2 \leq 40$$

$$X_1, X_2 \geq 0$$

**Solution** Given the following LP model:

Maximize  $Z = 6X_1 + 8X_2$   
subject to

$$5X_1 + 10X_2 \leq 60$$

$$4X_1 + 4X_2 \leq 40$$

$$X_1, X_2 \geq 0$$

The introduction of the non-negative constraints  $X_1 \geq 0$  and  $X_2 \geq 0$  will eliminate the second, third and fourth quadrants of the  $XY$  plane as shown in Fig. 16.1.

Compute the coordinates to plot equations relating to the constraints on the  $XY$  plane as shown below.

Consider the first constraint in the form

$$5X_1 + 10X_2 = 60$$

When  $X_1 = 0$ ,

$$10X_2 = 60, \quad X_2 = 6$$

When  $X_2 = 0$ ,

$$5X_1 = 60, \quad X_1 = 12$$

Now, plot constraint 1 as shown in the Fig. 16.1.

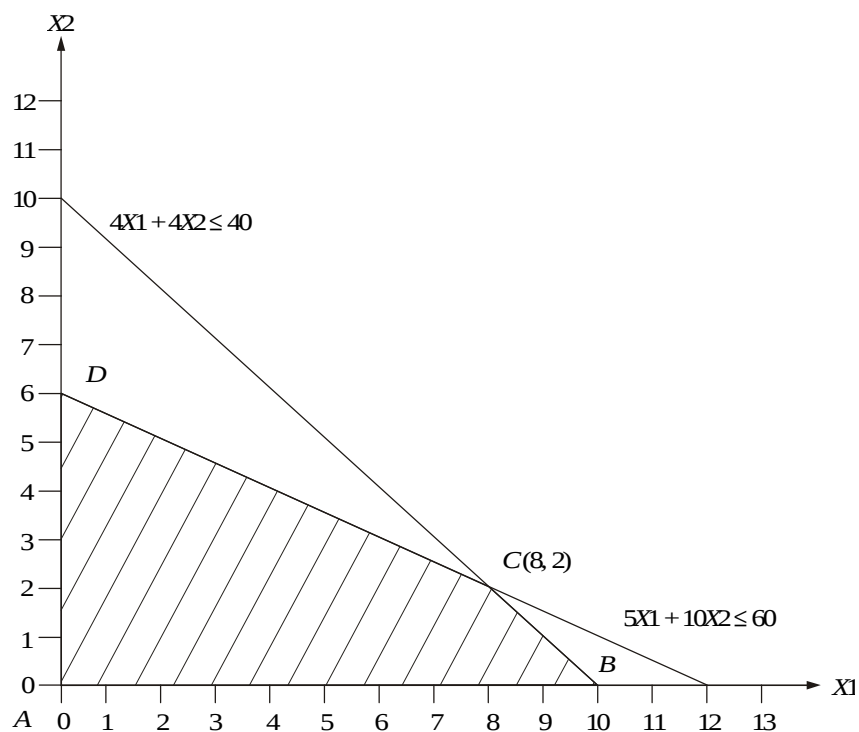


Fig. 16.1 Graphical plot of Example 16.3.

Consider the second constraint in the form

$$4X_1 + 4X_2 = 40$$

When  $X_1 = 0$ ,

$$4X_2 = 40, \quad X_2 = 10$$

When  $X_2 = 0$ ,

$$4X_1 = 40, \quad X_1 = 10$$

Now, plot constraint 2 as shown in Fig. 16.1.

The closed polygon  $A-B-C-D$  is the feasible region.

The objective function value at each of the corner points of the closed polygon is computed as follows by substituting its coordinates in the objective function:

$$Z(A) = 6 \times 0 + 8 \times 0 = 0$$

$$Z(B) = 6 \times 10 + 8 \times 0 = 60$$

$$Z(C) = 6 \times 8 + 8 \times 2 = 48 + 16 = 64$$

$$Z(D) = 6 \times 0 + 8 \times 6 = 48$$

Since the type of the objective function is maximization, the solution corresponding to the maximum  $Z$  value should be selected as the optimum solution. The  $Z$  value is maximum for the corner point  $C$ . Hence, the corresponding solution is

$$X_1^* = 8, \quad X_2^* = 2, \quad Z(\text{optimum}) = 64$$

**EXAMPLE 16.4** Solve the following LP problem using the graphical method:

Minimize  $Z = 2X_1 + 3X_2$

subject to

$$X_1 + X_2 \geq 6$$

$$7X_1 + X_2 \geq 14$$

$$X_1, X_2 \geq 0$$

**Solution** Consider the following LP problem:

Minimize  $Z = 2X_1 + 3X_2$

subject to

$$X_1 + X_2 \geq 6$$

$$7X_1 + X_2 \geq 14$$

$$X_1, X_2 \geq 0$$

The introduction of the non-negative constraints  $X_1 \geq 0$  and  $X_2 \geq 0$  will eliminate the second, third and fourth quadrants of the  $XY$ -plane as shown in Fig. 16.2.

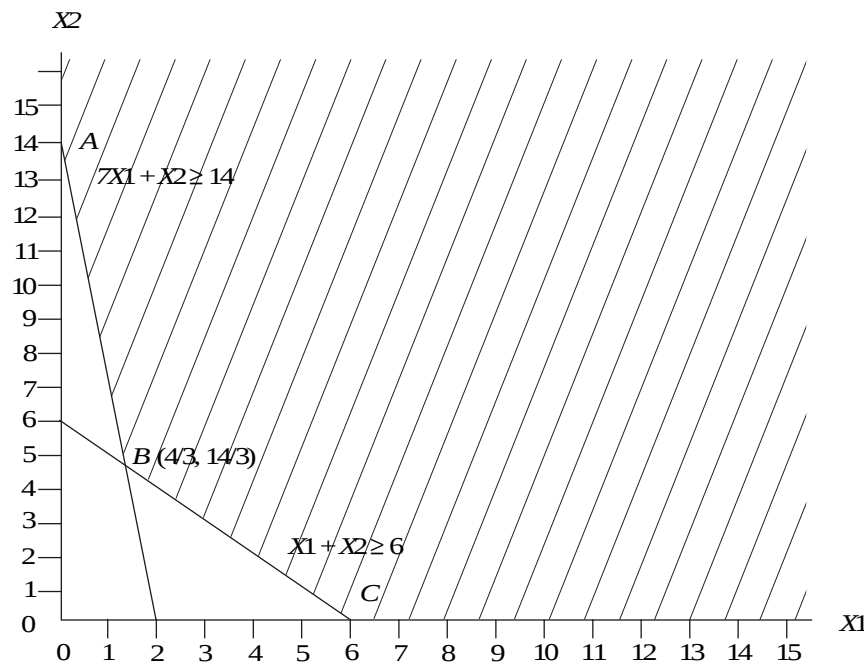


Fig. 16.2 Graphical plot of Example 16.4.

Compute the coordinates to plot the equations relating to the constraints on the  $XY$ -plane as shown below.

Consider the first constraint in the form

$$X_1 + X_2 = 6$$

$$\text{When } X_1 = 0, X_2 = 6$$

$$\text{When } X_2 = 0, X_1 = 6$$

Now, plot constraint 1 as shown in Fig. 16.2.

Consider the second constraint in the form

$$7X_1 + X_2 = 14$$

$$\text{When } X_1 = 0, \quad X_2 = 14$$

$$\text{When } X_2 = 0,$$

$$7X_1 = 14, \quad X_1 = 2$$

Now, plot constraint 2 as shown in Fig. 16.2.

The optimum solution will be in any one of the corners A, B and C.

The objective function value at each of these corner points of the feasible solution space is computed as follows by substituting its coordinates in the objective function:

$$Z(A) = 2 \times 0 + 3 \times 14 = 42$$

$$Z(B) = 2(4/3) + 3(14/3) = 50/3 = 16.67$$

$$Z(C) = 2 \times 6 + 3 \times 0 = 12$$

Since the type of the objective function is minimization, the solution corresponding to the minimum Z-value is to be selected as the optimum solution. The Z-value is minimum for the corner point C. Accordingly, the corresponding solution is presented as

$$X_1^* = 6, \quad X_2^* = 0, \quad Z(\text{optimum}) = 12.$$

## 16.4 SIMPLEX METHOD

This method is demonstrated using a numerical problem which is given below.

**EXAMPLE 16.5** Consider the linear programming model of Example 16.1 as shown below and solve it by the simplex method.

Maximize  $Z = 6X_1 + 8X_2$   
subject to

$$5X_1 + 10X_2 \leq 60$$

$$4X_1 + 4X_2 \leq 40$$

$$X_1, X_2 \geq 0$$

**Solution** The general form of the linear programming problem is

Maximize  $Z = 6X_1 + 8X_2$   
subject to

$$5X_1 + 10X_2 \leq 60$$

$$4X_1 + 4X_2 \leq 40$$

$$X_1, X_2 \geq 0$$

The canonical form of the above LP problem is

Maximize  $Z = 6X_1 + 8X_2 + 0S_1 + 0S_2$   
subject to

$$5X_1 + 10X_2 + S_1 = 60$$

$$4X_1 + 4X_2 + S_2 = 40$$

$$X_1, X_2, S_1, S_2 \geq 0$$

( $S_1, S_2$  are slack variables).

The initial Simplex table of the above problem is shown in Table 16.3.

**Table 16.3** Initial Simplex Table

| $CB_i$ | $C_j$          | 6     | 8     | 0     | 0     |          |                  |
|--------|----------------|-------|-------|-------|-------|----------|------------------|
|        | Basic variable | $X_1$ | $X_2$ | $S_1$ | $S_2$ | Solution | Ratio            |
| 0      | $S_1$          | 5     | 10    | 1     | 0     | 60       | $60/10 = 6^{**}$ |
| 0      | $S_2$          | 4     | 4     | 0     | 1     | 40       | $40/4 = 10$      |
|        | $Z_j$          | 0     | 0     | 0     | 0     | 0        |                  |
|        | $C_j - Z_j$    | 6     | $8^*$ | 0     | 0     |          |                  |

\* Key column,      \*\* Key row.

The value at the intersection of the key row and the key column is called the *key element*.

$$Z_j = \sum_{i=1}^2 CB_i * T_{ij}$$

where  $T_{ij}$  is the technological coefficient for the  $i$ th row and the  $j$ th column of the table.

$C_j - Z_j$  is the relative contribution. Here,  $C_j$  is the objective coefficient for the  $j$ th variable. The objective function value of the problem at this iteration is zero, i.e. the value of  $Z_j$  against the solution column is the value of the objective function.

### Optimality condition

For maximization problem, if all  $C_j - Z_j \leq 0$ , then optimality is reached. Otherwise, select the entering variable with the maximum  $C_j - Z_j$  value.

[For minimization problem, if all  $C_j - Z_j$  are greater than or equal to zero, the optimality is reached; otherwise, select the entering variable with the most negative value.]

In Table 16.3, all the values for  $C_j - Z_j$  are not less than or equal to zero.

Hence, the solution can be improved further.  $C_j - Z_j$  is maximum for the variable  $X_2$ . Therefore,  $X_2$  enters the basis. This is known as the *entering variable*, and the corresponding column is called the *key column*.

### Feasibility condition

To maintain the feasibility of the solution in each iteration, do the following:

1. Find the ratio between the solution value and the respective value in the key column value in each row.
2. Then, select the variable from the present set of basic variables w.r.t. the minimum ratio (break tie randomly). This is the leaving variable and the corresponding row is called the *key row*. The value at the intersection of the key row and key column is called the *key element* or *pivot element*.

In Table 16.3, the leaving variable is  $S_1$  and row 1 is the key row. The key element is 10.

The next iteration is shown in Table 16.4. In this table, the basic variable  $S_1$  of Table 16.3 is replaced by  $X_2$ .

$$\text{New value} = \text{Old value} - \frac{\text{Key column value} \times \text{key row value}}{\text{Key value}}$$

*Sample calculation.* New value for row 2 and column  $X_1$

$$\text{New value} = 4 - \frac{4 \times 5}{10} = 4 - \frac{20}{10} = 4 - 2 = 2$$

**Table 16.4** Iteration 1

| $CB_i$      | $C_j$          | 6     | 8     | 0     | 0     | Solution | Ratio           |
|-------------|----------------|-------|-------|-------|-------|----------|-----------------|
|             | Basic variable | $X_1$ | $X_2$ | $S_1$ | $S_2$ |          |                 |
| 8           | $X_2$          | 1/2   | 1     | 1/10  | 0     | 6        | $6/(1/2) = 12$  |
| 0           | $S_2$          | 2     | 0     | -2/5  | 1     | 16       | $16/2 = 8^{**}$ |
| $Z_j$       |                | 4     | 8     | 4/5   | 0     | 48       |                 |
| $C_j - Z_j$ |                | 2*    | 0     | -4/5  | 0     |          |                 |

\* Key column,      \*\* Key row.

The solution in Table 16.4 is not optimal. Hence, the variable  $X_1$  is selected as the entering variable and after computing the ratios,  $S_2$  is selected as the leaving variable. The next iteration is shown in Table 16.5.

**Table 16.5** Iteration 2

|        | $C_j$          | 6  | 8  | 0    | 0    |          |
|--------|----------------|----|----|------|------|----------|
| $CB_i$ | Basic variable | X1 | X2 | S1   | S2   | Solution |
| 8      | X2             | 0  | 1  | 1/5  | -1/4 | 2        |
| 6      | X1             | 1  | 0  | -1/5 | 1/2  | 8        |
|        | $Z_j$          | 6  | 8  | 2/5  | 1    | 64       |
|        | $C_j - Z_j$    | 0  | 0  | -2/5 | -1   |          |

In Table 16.5, all the values for  $C_j - Z_j$  are 0 and negative. Therefore, the optimality is reached.

The corresponding optimal solution is

X1 (production volume of P1) = 8 units

X2 (production volume of P2) = 2 units

Optimal objective value Z (total profit) = Rs. 64.00.

**EXAMPLE 16.6** Consider the linear programming model (shown below) of Example 16.2, and solve it by using the simplex method.

Minimize  $Z = 2X_1 + 3X_2$   
subject to

$$X_1 + X_2 \geq 6$$

$$7X_1 + X_2 \geq 14$$

$$X_1, X_2 \geq 0$$

**Solution** The general form of the linear programming problem is as follows:

Minimize  $Z = 2X_1 + 3X_2$   
subject to

$$X_1 + X_2 \geq 6$$

$$7X_1 + X_2 \geq 14$$

$$X_1, X_2 \geq 0$$

**Standard form.** The standard form of the above model is as follows:

Minimize  $Z = 2X_1 + 3X_2$   
subject to

$$X_1 + X_2 - S_1 = 6$$

$$7X_1 + X_2 - S_2 = 14$$

$$X_1, X_2, S_1 \text{ and } S_2 \geq 0$$

$S_1$  and  $S_2$  are called *surplus variables* which are introduced to balance the constraints.

**Canonical form.** The canonical form of the above model is

Minimize  $Z = 2X_1 + 3X_2 + M_1 \times R_1 + M_2 \times R_2$   
subject to

$$X_1 + X_2 - S_1 + R_1 = 6$$

$$7X_1 + X_2 - S_2 + R_2 = 14$$

$$X_1, X_2, S_1, S_2; R_1 \text{ and } R_2 \geq 0$$

Here,  $R_1$  and  $R_2$  are called *artificial variables* which are introduced to have basic variables in each of the constraints. These artificial variables are included in the model for its solution. So, in the final solution, these variables should not be available. This is achieved by including these variables in the objective function with a very high positive coefficient,  $M$ , since the objective function is minimization type (a very high negative coefficient,  $M$  for maximization objective type). Even, a small value of  $R_1$  or  $R_2$  will increase the value of the objective function infinitely which is against the objective of minimization. So, the solution procedure should necessarily assign zero value to each of the artificial variables in the final solution.

The initial table of the model is shown in Table 16.6.

**Table 16.6** Initial Table

|        | $C_j$          | 2        | 3      | 0     | 0     | $M$   | $M$   |       |       |
|--------|----------------|----------|--------|-------|-------|-------|-------|-------|-------|
| $CB_i$ | Basic variable | $X_1$    | $X_2$  | $S_1$ | $S_2$ | $R_1$ | $R_2$ | Sol.  | Ratio |
| $M$    | $R_1$          | 1        | 1      | -1    | 0     | 1     | 0     | 6     | 6     |
| $M$    | $R_2$          | 7        | 1      | 0     | -1    | 0     | 1     | 14    | 2**   |
|        | $Z_j$          | $8M$     | $2M$   | $-M$  | $-M$  | $M$   | $M$   | $20M$ |       |
|        | $C_j - Z_j$    | $2-8M^*$ | $3-2M$ | $M$   | $M$   | 0     | 0     |       |       |

\* Key column,      \*\*Key row.

**Optimality condition.** For the minimization problem, if all  $C_j - Z_j$  are greater than or equal to zero, optimality is reached; otherwise, select the entering variable with the most negative value.

In Table 16.6, all the values for  $C_j - Z_j$  are not greater than or equal to zero. Hence, the solution can be improved further.  $C_j - Z_j$  has maximum negative value for the variable  $X_1$ . So,  $X_1$  enters the basis. This is known as the *entering variable*, and the corresponding column is called the *key column*. If  $C_j - Z_j$  is a function of  $M$ , then ignore the constant numeric terms in it while making comparison with another  $C_j - Z_j$ . The feasibility condition is same as that for the maximization problem. The ratio is minimum for  $R_2$ . Therefore, it is called the *key row* and it leaves the basis. The modified table after applying the pivot operation is shown in Table 16.7.

**Table 16.7** Iteration 1

| $CB_i$      | $C_j$          | 2  | 3                              | 0    | 0                            | $M$ | $M$                          | Solution | Ratio  |
|-------------|----------------|----|--------------------------------|------|------------------------------|-----|------------------------------|----------|--------|
|             | Basic variable | X1 | X2                             | S1   | S2                           | R1  | R2                           |          |        |
| $M$         | R1             | 0  | 6/7                            | -1   | 1/7                          | 1   | -1/7                         | 4        | 14/3** |
| 2           | X1             | 1  | 1/7                            | 0    | -1/7                         | 0   | 1/7                          | 2        | 14     |
| $Z_j$       |                | 2  | $\frac{6}{7}M + \frac{2}{7}$   | $-M$ | $\frac{M}{7} - \frac{2}{7}$  | $M$ | $-\frac{M}{7} + \frac{2}{7}$ | $4M + 4$ |        |
| $C_j - Z_j$ |                | 0  | $-\frac{6}{7}M + \frac{19}{7}$ | $M$  | $-\frac{M}{7} + \frac{2}{7}$ | 0   | $\frac{8}{7}M - \frac{2}{7}$ |          |        |

\*Key column,      \*\*Key row.

In Table 16.7, the column X2 becomes the key column and the row R1 becomes the key row. So, R1 is replaced by X2 in the following (Table 16.8) with corresponding pivot operations.

**Table 16.8** Iteration 2

| $CB_i$      | $C_j$          | 2  | 3  | 0     | 0     | $M$                | $M$       | Solution | Ratio |
|-------------|----------------|----|----|-------|-------|--------------------|-----------|----------|-------|
|             | Basic variable | X1 | X2 | S1    | S2    | R1                 | R2        |          |       |
| 3           | X2             | 0  | 1  | -7/6  | 1/6   | 7/6                | -1/6      | 14/3     | 28**  |
| 2           | X1             | 1  | 0  | 1/6   | -1/6  | -1/6               | 1/6       | 4/3      |       |
| $Z_j$       |                | 2  | 3  | -19/6 | 1/6   | 19/6               | -1/6      | 50/3     |       |
| $C_j - Z_j$ |                | 0  | 0  | 19/6  | -1/6* | $M - \frac{19}{6}$ | $M + 1/6$ |          |       |

\*Key column,      \*\*Key row.

In Table 16.8, column S2 becomes the key column and the row X2 becomes the key row. So, X2 is replaced by S2 as shown in Table 16.9 with the corresponding pivot operations.

**Table 16.9** Iteration 3

| $CB_i$      | $C_j$          | 2  | 3  | 0  | 0  | $M$     | $M$ | Solution |
|-------------|----------------|----|----|----|----|---------|-----|----------|
|             | Basic variable | X1 | X2 | S1 | S2 | R1      | R2  |          |
| 0           | S2             | 0  | 6  | -7 | 1  | 7       | -1  | 28       |
| 2           | X1             | 1  | 1  | -1 | 0  | 1       | 0   | 6        |
| $Z_j$       |                | 2  | 2  | -2 | 0  | 2       | 0   | 12       |
| $C_j - Z_j$ |                | 0  | 1  | 2  | 0  | $M - 2$ | $M$ |          |

Since all the  $C_j - Z_j$  are equal to 0 and above, optimality is reached.  
The corresponding optimal solutions are

$$X_1 = 6, \quad X_2 = 0, \quad Z (\text{opt.}) = 12$$

Interpretation of the results:

$X_1$  (No. of packets of type 1 food) = 6

$X_2$  (No. of packets of type 2 food) = 0

Corresponding daily total cost of food = Rs. 12.00

Thus, the babies are to be given six packets of type 1 food daily.

## QUESTIONS

1. A manufacturer of leather belts makes three types of belts A, B and C which are processed on three machines M1, M2 and M3. Belt A requires two hours on machine M1 and three hours on machine M2. Belt B requires three hours on machine M1, two hours on machine M2 and two hours on machine M3, and belt C requires five hours on machine M2, and four hours on machine M3. There are 80 hours of time per week available on machine M1, 100 hours of time available on machine M2, and 150 hours of time on machine M3. The profit gained from belt A is Rs. 5.00 per unit, from belt B is Rs. 7.00 per unit, and from belt C is Rs. 6.00 per unit. What should be the weekly production of each type of belts so that the profit is maximum?

2. Solve the following linear programming problem:

$$\text{Maximize } Z = X_1 + 2X_2 + 3X_3 + X_4$$

subject to

$$X_1 + 2X_2 + 3X_3 \leq 15$$

$$2X_1 + X_2 + 5X_3 \leq 20$$

$$X_1 + 2X_2 + X_3 + X_4 \leq 10$$

$$X_1, X_2, X_3, X_4 \geq 0.$$

3. Consider the following linear programming problem:

(a) Solve it graphically.

(b) Solve it using the simplex method.

$$\text{Maximize } Z = 8X_1 + 18X_2$$

subject to

$$10X_1 + 24X_2 \leq 120$$

$$15X_1 + 25X_2 \leq 140$$

$$20X_1 + 80X_2 \leq 200$$

$$X_1, X_2 \geq 0.$$

4. Consider the following linear programming problem:

(a) Solve it graphically.

(b) Solve it using the simplex method.

$$\text{Minimize } Z = 20X_1 + 8X_2$$

subject to

$$15X_1 + 25X_2 \geq 180$$

$$10X_1 + 15X_2 \geq 160$$

$$12X_1 + 16X_2 \geq 120$$

$$X_1, X_2 \geq 0.$$

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## APPENDIX—INTEREST TABLES

Interest Table for Annual Compounding with  $i = 0.25\%$

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1   | 1.003       | 0.9975      | 1.000       | 1.0000      | 0.9975      | 1.0025      | 0.0000      |
| 2   | 1.005       | 0.9950      | 2.003       | 0.4994      | 1.9926      | 0.5019      | 0.5048      |
| 3   | 1.008       | 0.9925      | 3.008       | 0.3325      | 2.9851      | 0.3350      | 1.0020      |
| 4   | 1.010       | 0.9901      | 4.015       | 0.2491      | 3.9752      | 0.2516      | 1.5012      |
| 5   | 1.013       | 0.9876      | 5.025       | 0.1990      | 4.9628      | 0.2015      | 1.9965      |
| 6   | 1.015       | 0.9851      | 6.038       | 0.1656      | 5.9479      | 0.1681      | 2.4988      |
| 7   | 1.018       | 0.9827      | 7.053       | 0.1418      | 6.9306      | 0.1443      | 2.9938      |
| 8   | 1.020       | 0.9802      | 8.070       | 0.1239      | 7.9108      | 0.1264      | 3.4907      |
| 9   | 1.023       | 0.9778      | 9.091       | 0.1100      | 8.8886      | 0.1125      | 3.9865      |
| 10  | 1.025       | 0.9753      | 10.113      | 0.0989      | 9.8639      | 0.1014      | 4.4830      |
| 11  | 1.028       | 0.9729      | 11.139      | 0.0898      | 10.8369     | 0.0923      | 4.9779      |
| 12  | 1.030       | 0.9705      | 12.167      | 0.0822      | 11.8074     | 0.0847      | 5.4744      |
| 13  | 1.033       | 0.9681      | 13.197      | 0.0758      | 12.7754     | 0.0783      | 5.9676      |
| 14  | 1.036       | 0.9656      | 14.230      | 0.0703      | 13.7411     | 0.0728      | 6.4633      |
| 15  | 1.038       | 0.9632      | 15.266      | 0.0655      | 14.7043     | 0.0680      | 6.9569      |
| 16  | 1.041       | 0.9608      | 16.304      | 0.0613      | 15.6652     | 0.0638      | 7.4498      |
| 17  | 1.043       | 0.9584      | 17.344      | 0.0577      | 16.6237     | 0.0602      | 7.9441      |
| 18  | 1.046       | 0.9561      | 18.388      | 0.0544      | 17.5797     | 0.0569      | 8.4373      |
| 19  | 1.049       | 0.9537      | 19.434      | 0.0515      | 18.5334     | 0.0540      | 8.9293      |
| 20  | 1.051       | 0.9513      | 20.482      | 0.0488      | 19.4847     | 0.0513      | 9.4211      |
| 21  | 1.054       | 0.9489      | 21.534      | 0.0464      | 20.4336     | 0.0489      | 9.9116      |
| 22  | 1.056       | 0.9465      | 22.587      | 0.0443      | 21.3802     | 0.0468      | 10.4033     |
| 23  | 1.059       | 0.9442      | 23.644      | 0.0423      | 22.3244     | 0.0448      | 10.8944     |
| 24  | 1.062       | 0.9418      | 24.703      | 0.0405      | 23.2662     | 0.0430      | 11.3840     |
| 25  | 1.064       | 0.9395      | 25.765      | 0.0388      | 24.2057     | 0.0413      | 11.8736     |
| 26  | 1.067       | 0.9371      | 26.829      | 0.0373      | 25.1428     | 0.0398      | 12.3629     |
| 27  | 1.070       | 0.9348      | 27.896      | 0.0358      | 26.0776     | 0.0383      | 12.8520     |
| 28  | 1.072       | 0.9325      | 28.966      | 0.0345      | 27.0102     | 0.0370      | 13.3407     |
| 29  | 1.075       | 0.9301      | 30.038      | 0.0333      | 27.9403     | 0.0358      | 13.8284     |
| 30  | 1.078       | 0.9278      | 31.114      | 0.0321      | 28.8681     | 0.0346      | 14.3166     |
| 31  | 1.080       | 0.9255      | 32.191      | 0.0311      | 29.7937     | 0.0336      | 14.8043     |
| 32  | 1.083       | 0.9232      | 33.272      | 0.0301      | 30.7169     | 0.0326      | 15.2906     |
| 33  | 1.086       | 0.9209      | 34.355      | 0.0291      | 31.6378     | 0.0316      | 15.7773     |
| 34  | 1.089       | 0.9186      | 35.441      | 0.0282      | 32.5564     | 0.0307      | 16.2632     |
| 35  | 1.091       | 0.9163      | 36.530      | 0.0274      | 33.4727     | 0.0299      | 16.7491     |
| 36  | 1.094       | 0.9140      | 37.621      | 0.0266      | 34.3868     | 0.0291      | 17.2342     |
| 37  | 1.097       | 0.9118      | 38.715      | 0.0258      | 35.2985     | 0.0283      | 17.7192     |
| 38  | 1.100       | 0.9095      | 39.812      | 0.0251      | 36.2080     | 0.0276      | 18.2036     |
| 39  | 1.102       | 0.9072      | 40.911      | 0.0244      | 37.1153     | 0.0269      | 18.6878     |
| 40  | 1.105       | 0.9049      | 42.014      | 0.0238      | 38.0202     | 0.0263      | 19.1708     |
| 41  | 1.108       | 0.9027      | 43.119      | 0.0232      | 38.9229     | 0.0257      | 19.6544     |
| 42  | 1.111       | 0.9004      | 44.226      | 0.0226      | 39.8233     | 0.0251      | 20.1368     |
| 43  | 1.113       | 0.8982      | 45.337      | 0.0221      | 40.7215     | 0.0246      | 20.6187     |

Interest Table for Annual Compounding with  $i = 0.25\%$  (Cont.)

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 44  | 1.116       | 0.8960      | 46.450      | 0.0215      | 41.6175     | 0.0240      | 21.1010     |
| 45  | 1.119       | 0.8937      | 47.567      | 0.0210      | 42.5112     | 0.0235      | 21.5826     |
| 46  | 1.122       | 0.8915      | 48.686      | 0.0205      | 43.4028     | 0.0230      | 22.0641     |
| 47  | 1.125       | 0.8893      | 49.807      | 0.0201      | 44.2920     | 0.0226      | 22.5441     |
| 48  | 1.127       | 0.8871      | 50.932      | 0.0196      | 45.1791     | 0.0221      | 23.0245     |
| 49  | 1.130       | 0.8848      | 52.059      | 0.0192      | 46.0640     | 0.0217      | 23.5048     |
| 50  | 1.133       | 0.8826      | 53.189      | 0.0188      | 46.9466     | 0.0213      | 23.9840     |
| 51  | 1.136       | 0.8804      | 54.322      | 0.0184      | 47.8270     | 0.0209      | 24.4631     |
| 52  | 1.139       | 0.8782      | 55.458      | 0.0180      | 48.7052     | 0.0205      | 24.9412     |
| 53  | 1.141       | 0.8760      | 56.597      | 0.0177      | 49.5813     | 0.0202      | 25.4191     |
| 54  | 1.144       | 0.8739      | 57.738      | 0.0173      | 50.4552     | 0.0198      | 25.8977     |
| 55  | 1.147       | 0.8717      | 58.883      | 0.0170      | 51.3269     | 0.0195      | 26.3752     |
| 56  | 1.150       | 0.8695      | 60.030      | 0.0167      | 52.1964     | 0.0192      | 26.8514     |
| 57  | 1.153       | 0.8673      | 61.180      | 0.0163      | 53.0637     | 0.0188      | 27.3278     |
| 58  | 1.156       | 0.8652      | 62.333      | 0.0160      | 53.9289     | 0.0185      | 27.8042     |
| 59  | 1.159       | 0.8630      | 63.489      | 0.0158      | 54.7919     | 0.0183      | 28.2795     |
| 60  | 1.162       | 0.8609      | 64.647      | 0.0155      | 55.6529     | 0.0180      | 28.7554     |
| 61  | 1.165       | 0.8587      | 65.809      | 0.0152      | 56.5115     | 0.0177      | 29.2295     |
| 62  | 1.167       | 0.8566      | 66.974      | 0.0149      | 57.3682     | 0.0174      | 29.7046     |
| 63  | 1.170       | 0.8544      | 68.141      | 0.0147      | 58.2226     | 0.0172      | 30.1784     |
| 64  | 1.173       | 0.8523      | 69.311      | 0.0144      | 59.0749     | 0.0169      | 30.6521     |
| 65  | 1.176       | 0.8502      | 70.485      | 0.0142      | 59.9251     | 0.0167      | 31.1252     |
| 66  | 1.179       | 0.8481      | 71.661      | 0.0140      | 60.7732     | 0.0165      | 31.5981     |
| 67  | 1.182       | 0.8460      | 72.840      | 0.0137      | 61.6192     | 0.0162      | 32.0704     |
| 68  | 1.185       | 0.8438      | 74.022      | 0.0135      | 62.4630     | 0.0160      | 32.5423     |
| 69  | 1.188       | 0.8417      | 75.207      | 0.0133      | 63.3048     | 0.0158      | 33.0138     |
| 70  | 1.191       | 0.8396      | 76.395      | 0.0131      | 64.1444     | 0.0156      | 33.4850     |
| 71  | 1.194       | 0.8375      | 77.586      | 0.0129      | 64.9820     | 0.0154      | 33.9556     |
| 72  | 1.197       | 0.8355      | 78.780      | 0.0127      | 65.8174     | 0.0152      | 34.4259     |
| 73  | 1.200       | 0.8334      | 79.977      | 0.0125      | 66.6508     | 0.0150      | 34.8956     |
| 74  | 1.203       | 0.8313      | 81.177      | 0.0123      | 67.4821     | 0.0148      | 35.3650     |
| 75  | 1.206       | 0.8292      | 82.380      | 0.0121      | 68.3113     | 0.0146      | 35.8342     |
| 76  | 1.209       | 0.8272      | 83.586      | 0.0120      | 69.1385     | 0.0145      | 36.3027     |
| 77  | 1.212       | 0.8251      | 84.795      | 0.0118      | 69.9636     | 0.0143      | 36.7708     |
| 78  | 1.215       | 0.8230      | 86.007      | 0.0116      | 70.7867     | 0.0141      | 37.2391     |
| 79  | 1.218       | 0.8210      | 87.222      | 0.0115      | 71.6076     | 0.0140      | 37.7061     |
| 80  | 1.221       | 0.8189      | 88.440      | 0.0113      | 72.4266     | 0.0138      | 38.1733     |
| 81  | 1.224       | 0.8169      | 89.661      | 0.0112      | 73.2435     | 0.0137      | 38.6398     |
| 82  | 1.227       | 0.8149      | 90.885      | 0.0110      | 74.0584     | 0.0135      | 39.1059     |
| 83  | 1.230       | 0.8128      | 92.113      | 0.0109      | 74.8712     | 0.0134      | 39.5717     |
| 84  | 1.233       | 0.8108      | 93.343      | 0.0107      | 75.6819     | 0.0132      | 40.0367     |
| 85  | 1.236       | 0.8088      | 94.576      | 0.0106      | 76.4907     | 0.0131      | 40.5016     |
| 86  | 1.240       | 0.8068      | 95.813      | 0.0104      | 77.2975     | 0.0129      | 40.9663     |
| 87  | 1.243       | 0.8047      | 97.052      | 0.0103      | 78.1023     | 0.0128      | 41.4305     |
| 88  | 1.246       | 0.8027      | 98.295      | 0.0102      | 78.9050     | 0.0127      | 41.8936     |
| 89  | 1.249       | 0.8007      | 99.541      | 0.0100      | 79.7057     | 0.0125      | 42.3571     |
| 90  | 1.252       | 0.7987      | 100.790     | 0.0099      | 80.5045     | 0.0124      | 42.8199     |
| 91  | 1.255       | 0.7967      | 102.041     | 0.0098      | 81.3012     | 0.0123      | 43.2824     |
| 92  | 1.258       | 0.7948      | 103.297     | 0.0097      | 82.0960     | 0.0122      | 43.7443     |
| 93  | 1.261       | 0.7928      | 104.555     | 0.0096      | 82.8888     | 0.0121      | 44.2058     |
| 94  | 1.265       | 0.7908      | 105.816     | 0.0095      | 83.6796     | 0.0120      | 44.6671     |
| 95  | 1.268       | 0.7888      | 107.081     | 0.0093      | 84.4685     | 0.0118      | 45.1280     |
| 96  | 1.271       | 0.7869      | 108.349     | 0.0092      | 85.2553     | 0.0117      | 45.5881     |
| 97  | 1.274       | 0.7849      | 109.619     | 0.0091      | 86.0402     | 0.0116      | 46.0481     |
| 98  | 1.277       | 0.7829      | 110.893     | 0.0090      | 86.8232     | 0.0115      | 46.5075     |
| 99  | 1.280       | 0.7810      | 112.171     | 0.0089      | 87.6042     | 0.0114      | 46.9667     |
| 100 | 1.284       | 0.7790      | 113.451     | 0.0088      | 88.3832     | 0.0113      | 47.4253     |

Interest Table for Annual Compounding with  $i = 0.5\%$ 

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1   | 1.005       | 0.9950      | 1.000       | 1.0000      | 0.9950      | 1.0050      | 0.0000      |
| 2   | 1.010       | 0.9901      | 2.005       | 0.4988      | 1.9851      | 0.5038      | 0.4994      |
| 3   | 1.015       | 0.9851      | 3.015       | 0.3317      | 2.9702      | 0.3367      | 0.9964      |
| 4   | 1.020       | 0.9802      | 4.030       | 0.2481      | 3.9505      | 0.2531      | 1.4944      |
| 5   | 1.025       | 0.9754      | 5.050       | 0.1980      | 4.9259      | 0.2030      | 1.9903      |
| 6   | 1.030       | 0.9705      | 6.076       | 0.1646      | 5.8964      | 0.1696      | 2.4864      |
| 7   | 1.036       | 0.9657      | 7.106       | 0.1407      | 6.8621      | 0.1457      | 2.9794      |
| 8   | 1.041       | 0.9609      | 8.141       | 0.1228      | 7.8230      | 0.1278      | 3.4743      |
| 9   | 1.046       | 0.9561      | 9.182       | 0.1089      | 8.7791      | 0.1139      | 3.9665      |
| 10  | 1.051       | 0.9513      | 10.228      | 0.0978      | 9.7304      | 0.1028      | 4.4593      |
| 11  | 1.056       | 0.9466      | 11.279      | 0.0887      | 10.6770     | 0.0937      | 4.9505      |
| 12  | 1.062       | 0.9419      | 12.336      | 0.0811      | 11.6190     | 0.0861      | 5.4411      |
| 13  | 1.067       | 0.9372      | 13.397      | 0.0746      | 12.5562     | 0.0796      | 5.9304      |
| 14  | 1.072       | 0.9326      | 14.464      | 0.0691      | 13.4887     | 0.0741      | 6.4193      |
| 15  | 1.078       | 0.9279      | 15.537      | 0.0644      | 14.4166     | 0.0694      | 6.9068      |
| 16  | 1.083       | 0.9233      | 16.614      | 0.0602      | 15.3399     | 0.0652      | 7.3941      |
| 17  | 1.088       | 0.9187      | 17.697      | 0.0565      | 16.2586     | 0.0615      | 7.8803      |
| 18  | 1.094       | 0.9141      | 18.786      | 0.0532      | 17.1728     | 0.0582      | 8.3659      |
| 19  | 1.099       | 0.9096      | 19.880      | 0.0503      | 18.0824     | 0.0553      | 8.8504      |
| 20  | 1.105       | 0.9051      | 20.979      | 0.0477      | 18.9874     | 0.0527      | 9.3344      |
| 21  | 1.110       | 0.9006      | 22.084      | 0.0453      | 19.8880     | 0.0503      | 9.8172      |
| 22  | 1.116       | 0.8961      | 23.194      | 0.0431      | 20.7841     | 0.0481      | 10.2997     |
| 23  | 1.122       | 0.8916      | 24.310      | 0.0411      | 21.6757     | 0.0461      | 10.7807     |
| 24  | 1.127       | 0.8872      | 25.432      | 0.0393      | 22.5629     | 0.0443      | 11.2613     |
| 25  | 1.133       | 0.8828      | 26.559      | 0.0377      | 23.4457     | 0.0427      | 11.7409     |
| 26  | 1.138       | 0.8784      | 27.692      | 0.0361      | 24.3240     | 0.0411      | 12.2197     |
| 27  | 1.144       | 0.8740      | 28.830      | 0.0347      | 25.1980     | 0.0397      | 12.6976     |
| 28  | 1.150       | 0.8697      | 29.975      | 0.0334      | 26.0677     | 0.0384      | 13.1749     |
| 29  | 1.156       | 0.8653      | 31.124      | 0.0321      | 26.9331     | 0.0371      | 13.6513     |
| 30  | 1.161       | 0.8610      | 32.280      | 0.0310      | 27.7941     | 0.0360      | 14.1268     |
| 31  | 1.167       | 0.8567      | 33.441      | 0.0299      | 28.6508     | 0.0349      | 14.6012     |
| 32  | 1.173       | 0.8525      | 34.609      | 0.0289      | 29.5033     | 0.0339      | 15.0752     |
| 33  | 1.179       | 0.8482      | 35.782      | 0.0279      | 30.3515     | 0.0329      | 15.5482     |
| 34  | 1.185       | 0.8440      | 36.961      | 0.0271      | 31.1956     | 0.0321      | 16.0204     |
| 35  | 1.191       | 0.8398      | 38.145      | 0.0262      | 32.0354     | 0.0312      | 16.4917     |
| 36  | 1.197       | 0.8356      | 39.336      | 0.0254      | 32.8710     | 0.0304      | 16.9622     |
| 37  | 1.203       | 0.8315      | 40.533      | 0.0247      | 33.7025     | 0.0297      | 17.4320     |
| 38  | 1.209       | 0.8274      | 41.736      | 0.0240      | 34.5299     | 0.0290      | 17.9008     |
| 39  | 1.215       | 0.8232      | 42.944      | 0.0233      | 35.3531     | 0.0283      | 18.3688     |
| 40  | 1.221       | 0.8191      | 44.159      | 0.0226      | 36.1723     | 0.0276      | 18.8360     |
| 41  | 1.227       | 0.8151      | 45.380      | 0.0220      | 36.9873     | 0.0270      | 19.3023     |
| 42  | 1.233       | 0.8110      | 46.607      | 0.0215      | 37.7983     | 0.0265      | 19.7679     |
| 43  | 1.239       | 0.8070      | 47.840      | 0.0209      | 38.6053     | 0.0259      | 20.2326     |
| 44  | 1.245       | 0.8030      | 49.079      | 0.0204      | 39.4083     | 0.0254      | 20.6966     |
| 45  | 1.252       | 0.7990      | 50.324      | 0.0199      | 40.2072     | 0.0249      | 21.1597     |
| 46  | 1.258       | 0.7950      | 51.576      | 0.0194      | 41.0022     | 0.0244      | 21.6219     |
| 47  | 1.264       | 0.7910      | 52.834      | 0.0189      | 41.7932     | 0.0239      | 22.0832     |
| 48  | 1.270       | 0.7871      | 54.098      | 0.0185      | 42.5804     | 0.0235      | 22.5439     |
| 49  | 1.277       | 0.7832      | 55.368      | 0.0181      | 43.3635     | 0.0231      | 23.0036     |
| 50  | 1.283       | 0.7793      | 56.645      | 0.0177      | 44.1428     | 0.0227      | 23.4626     |

Interest Table for Annual Compounding with  $i = 0.5\%$  (Cont.)

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 51  | 1.290       | 0.7754      | 57.928      | 0.0173      | 44.9182     | 0.0223      | 23.9206     |
| 52  | 1.296       | 0.7716      | 59.218      | 0.0169      | 45.6898     | 0.0219      | 24.3780     |
| 53  | 1.303       | 0.7677      | 60.514      | 0.0165      | 46.4575     | 0.0215      | 24.8344     |
| 54  | 1.309       | 0.7639      | 61.817      | 0.0162      | 47.2214     | 0.0212      | 25.2902     |
| 55  | 1.316       | 0.7601      | 63.126      | 0.0158      | 47.9815     | 0.0208      | 25.7449     |
| 56  | 1.322       | 0.7563      | 64.441      | 0.0155      | 48.7378     | 0.0205      | 26.1989     |
| 57  | 1.329       | 0.7525      | 65.764      | 0.0152      | 49.4903     | 0.0202      | 26.6520     |
| 58  | 1.335       | 0.7488      | 67.092      | 0.0149      | 50.2391     | 0.0199      | 27.1044     |
| 59  | 1.342       | 0.7451      | 68.428      | 0.0146      | 50.9842     | 0.0196      | 27.5559     |
| 60  | 1.349       | 0.7414      | 69.770      | 0.0143      | 51.7256     | 0.0193      | 28.0066     |
| 61  | 1.356       | 0.7377      | 71.119      | 0.0141      | 52.4633     | 0.0191      | 28.4564     |
| 62  | 1.362       | 0.7340      | 72.475      | 0.0138      | 53.1973     | 0.0188      | 28.9054     |
| 63  | 1.369       | 0.7304      | 73.837      | 0.0135      | 53.9277     | 0.0185      | 29.3536     |
| 64  | 1.376       | 0.7267      | 75.206      | 0.0133      | 54.6544     | 0.0183      | 29.8011     |
| 65  | 1.383       | 0.7231      | 76.582      | 0.0131      | 55.3775     | 0.0181      | 30.2476     |
| 66  | 1.390       | 0.7195      | 77.965      | 0.0128      | 56.0970     | 0.0178      | 30.6933     |
| 67  | 1.397       | 0.7159      | 79.355      | 0.0126      | 56.8129     | 0.0176      | 31.1382     |
| 68  | 1.404       | 0.7124      | 80.752      | 0.0124      | 57.5253     | 0.0174      | 31.5823     |
| 69  | 1.411       | 0.7088      | 82.155      | 0.0122      | 58.2342     | 0.0172      | 32.0256     |
| 70  | 1.418       | 0.7053      | 83.566      | 0.0120      | 58.9395     | 0.0170      | 32.4681     |
| 71  | 1.425       | 0.7018      | 84.984      | 0.0118      | 59.6412     | 0.0168      | 32.9097     |
| 72  | 1.432       | 0.6983      | 86.409      | 0.0116      | 60.3395     | 0.0166      | 33.3505     |
| 73  | 1.439       | 0.6948      | 87.841      | 0.0114      | 61.0344     | 0.0164      | 33.7905     |
| 74  | 1.446       | 0.6914      | 89.280      | 0.0112      | 61.7258     | 0.0162      | 34.2297     |
| 75  | 1.454       | 0.6879      | 90.727      | 0.0110      | 62.4137     | 0.0160      | 34.6681     |
| 76  | 1.461       | 0.6845      | 92.180      | 0.0108      | 63.0982     | 0.0158      | 35.1056     |
| 77  | 1.468       | 0.6811      | 93.641      | 0.0107      | 63.7793     | 0.0157      | 35.5424     |
| 78  | 1.476       | 0.6777      | 95.109      | 0.0105      | 64.4570     | 0.0155      | 35.9783     |
| 79  | 1.483       | 0.6743      | 96.585      | 0.0104      | 65.1314     | 0.0154      | 36.4133     |
| 80  | 1.490       | 0.6710      | 98.068      | 0.0102      | 65.8024     | 0.0152      | 36.8476     |
| 81  | 1.498       | 0.6677      | 99.558      | 0.0100      | 66.4700     | 0.0150      | 37.2810     |
| 82  | 1.505       | 0.6643      | 101.056     | 0.0099      | 67.1343     | 0.0149      | 37.7136     |
| 83  | 1.513       | 0.6610      | 102.561     | 0.0098      | 67.7953     | 0.0148      | 38.1454     |
| 84  | 1.520       | 0.6577      | 104.074     | 0.0096      | 68.4531     | 0.0146      | 38.5765     |
| 85  | 1.528       | 0.6545      | 105.594     | 0.0095      | 69.1075     | 0.0145      | 39.0066     |
| 86  | 1.536       | 0.6512      | 107.122     | 0.0093      | 69.7588     | 0.0143      | 39.4360     |
| 87  | 1.543       | 0.6480      | 108.658     | 0.0092      | 70.4067     | 0.0142      | 39.8645     |
| 88  | 1.551       | 0.6447      | 110.201     | 0.0091      | 71.0515     | 0.0141      | 40.2923     |
| 89  | 1.559       | 0.6415      | 111.752     | 0.0089      | 71.6930     | 0.0139      | 40.7191     |
| 90  | 1.567       | 0.6383      | 113.311     | 0.0088      | 72.3313     | 0.0138      | 41.1452     |
| 91  | 1.574       | 0.6352      | 114.878     | 0.0087      | 72.9665     | 0.0137      | 41.5705     |
| 92  | 1.582       | 0.6320      | 116.452     | 0.0086      | 73.5985     | 0.0136      | 41.9949     |
| 93  | 1.590       | 0.6289      | 118.034     | 0.0085      | 74.2274     | 0.0135      | 42.4187     |
| 94  | 1.598       | 0.6257      | 119.624     | 0.0084      | 74.8531     | 0.0134      | 42.8415     |
| 95  | 1.606       | 0.6226      | 121.223     | 0.0082      | 75.4757     | 0.0132      | 43.2634     |
| 96  | 1.614       | 0.6195      | 122.829     | 0.0081      | 76.0953     | 0.0131      | 43.6847     |
| 97  | 1.622       | 0.6164      | 124.443     | 0.0080      | 76.7117     | 0.0130      | 44.1051     |
| 98  | 1.630       | 0.6134      | 126.065     | 0.0079      | 77.3251     | 0.0129      | 44.5246     |
| 99  | 1.638       | 0.6103      | 127.695     | 0.0078      | 77.9354     | 0.0128      | 44.9434     |
| 100 | 1.647       | 0.6073      | 129.334     | 0.0077      | 78.5427     | 0.0127      | 45.3614     |

Interest Table for Annual Compounding with  $i = 0.75\%$ 

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1   | 1.008       | 0.9926      | 1.000       | 1.0000      | 0.9926      | 1.0075      | 0.0000      |
| 2   | 1.015       | 0.9852      | 2.008       | 0.4981      | 1.9777      | 0.5056      | 0.4993      |
| 3   | 1.023       | 0.9778      | 3.023       | 0.3308      | 2.9556      | 0.3383      | 0.9956      |
| 4   | 1.030       | 0.9706      | 4.045       | 0.2472      | 3.9261      | 0.2547      | 1.4920      |
| 5   | 1.038       | 0.9633      | 5.076       | 0.1970      | 4.8895      | 0.2045      | 1.9862      |
| 6   | 1.046       | 0.9562      | 6.114       | 0.1636      | 5.8456      | 0.1711      | 2.4792      |
| 7   | 1.054       | 0.9490      | 7.160       | 0.1397      | 6.7947      | 0.1472      | 2.9710      |
| 8   | 1.062       | 0.9420      | 8.213       | 0.1218      | 7.7367      | 0.1293      | 3.4623      |
| 9   | 1.070       | 0.9350      | 9.275       | 0.1078      | 8.6717      | 0.1153      | 3.9514      |
| 10  | 1.078       | 0.9280      | 10.344      | 0.0967      | 9.5997      | 0.1042      | 4.4396      |
| 11  | 1.086       | 0.9211      | 11.422      | 0.0876      | 10.5208     | 0.0951      | 4.9265      |
| 12  | 1.094       | 0.9142      | 12.508      | 0.0800      | 11.4350     | 0.0875      | 5.4122      |
| 13  | 1.102       | 0.9074      | 13.602      | 0.0735      | 12.3425     | 0.0810      | 5.8967      |
| 14  | 1.110       | 0.9007      | 14.704      | 0.0680      | 13.2431     | 0.0755      | 6.3799      |
| 15  | 1.119       | 0.8940      | 15.814      | 0.0632      | 14.1371     | 0.0707      | 6.8618      |
| 16  | 1.127       | 0.8873      | 16.932      | 0.0591      | 15.0244     | 0.0666      | 7.3425      |
| 17  | 1.135       | 0.8807      | 18.059      | 0.0554      | 15.9052     | 0.0629      | 7.8221      |
| 18  | 1.144       | 0.8742      | 19.195      | 0.0521      | 16.7793     | 0.0596      | 8.3003      |
| 19  | 1.153       | 0.8676      | 20.339      | 0.0492      | 17.6470     | 0.0567      | 8.7771      |
| 20  | 1.161       | 0.8612      | 21.491      | 0.0465      | 18.5082     | 0.0540      | 9.2529      |
| 21  | 1.170       | 0.8548      | 22.653      | 0.0441      | 19.3630     | 0.0516      | 9.7274      |
| 22  | 1.179       | 0.8484      | 23.823      | 0.0420      | 20.2114     | 0.0495      | 10.2007     |
| 23  | 1.188       | 0.8421      | 25.001      | 0.0400      | 21.0535     | 0.0475      | 10.6726     |
| 24  | 1.196       | 0.8358      | 26.189      | 0.0382      | 21.8893     | 0.0457      | 11.1435     |
| 25  | 1.205       | 0.8296      | 27.385      | 0.0365      | 22.7190     | 0.0440      | 11.6130     |
| 26  | 1.214       | 0.8234      | 28.591      | 0.0350      | 23.5424     | 0.0425      | 12.0812     |
| 27  | 1.224       | 0.8173      | 29.805      | 0.0336      | 24.3597     | 0.0411      | 12.5483     |
| 28  | 1.233       | 0.8112      | 31.029      | 0.0322      | 25.1709     | 0.0397      | 13.0140     |
| 29  | 1.242       | 0.8052      | 32.261      | 0.0310      | 25.9761     | 0.0385      | 13.4786     |
| 30  | 1.251       | 0.7992      | 33.503      | 0.0298      | 26.7753     | 0.0373      | 13.9420     |
| 31  | 1.261       | 0.7932      | 34.755      | 0.0288      | 27.5685     | 0.0363      | 14.4040     |
| 32  | 1.270       | 0.7873      | 36.015      | 0.0278      | 28.3559     | 0.0353      | 14.8649     |
| 33  | 1.280       | 0.7815      | 37.285      | 0.0268      | 29.1374     | 0.0343      | 15.3245     |
| 34  | 1.289       | 0.7757      | 38.565      | 0.0259      | 29.9130     | 0.0334      | 15.7829     |
| 35  | 1.299       | 0.7699      | 39.854      | 0.0251      | 30.6829     | 0.0326      | 16.2400     |
| 36  | 1.309       | 0.7641      | 41.153      | 0.0243      | 31.4471     | 0.0318      | 16.6959     |
| 37  | 1.318       | 0.7585      | 42.462      | 0.0236      | 32.2055     | 0.0311      | 17.1506     |
| 38  | 1.328       | 0.7528      | 43.780      | 0.0228      | 32.9584     | 0.0303      | 17.6040     |
| 39  | 1.338       | 0.7472      | 45.109      | 0.0222      | 33.7056     | 0.0297      | 18.0561     |
| 40  | 1.348       | 0.7416      | 46.447      | 0.0215      | 34.4472     | 0.0290      | 18.5071     |
| 41  | 1.358       | 0.7361      | 47.795      | 0.0209      | 35.1834     | 0.0284      | 18.9568     |
| 42  | 1.369       | 0.7306      | 49.154      | 0.0203      | 35.9140     | 0.0278      | 19.4054     |
| 43  | 1.379       | 0.7252      | 50.523      | 0.0198      | 36.6392     | 0.0273      | 19.8526     |
| 44  | 1.389       | 0.7198      | 51.901      | 0.0193      | 37.3590     | 0.0268      | 20.2986     |
| 45  | 1.400       | 0.7144      | 53.291      | 0.0188      | 38.0735     | 0.0263      | 20.7434     |
| 46  | 1.410       | 0.7091      | 54.690      | 0.0183      | 38.7826     | 0.0258      | 21.1869     |
| 47  | 1.421       | 0.7039      | 56.101      | 0.0178      | 39.4865     | 0.0253      | 21.6292     |
| 48  | 1.431       | 0.6986      | 57.521      | 0.0174      | 40.1851     | 0.0249      | 22.0703     |
| 49  | 1.442       | 0.6934      | 58.953      | 0.0170      | 40.8785     | 0.0245      | 22.5102     |
| 50  | 1.453       | 0.6882      | 60.395      | 0.0166      | 41.5668     | 0.0241      | 22.9489     |

Interest Table for Annual Compounding with  $i = 0.75\%$  (Cont.)

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 51  | 1.464       | 0.6831      | 61.848      | 0.0162      | 42.2499     | 0.0237      | 23.3862     |
| 52  | 1.475       | 0.6780      | 63.312      | 0.0158      | 42.9280     | 0.0233      | 23.8224     |
| 53  | 1.486       | 0.6730      | 64.787      | 0.0154      | 43.6010     | 0.0229      | 24.2574     |
| 54  | 1.497       | 0.6680      | 66.273      | 0.0151      | 44.2690     | 0.0226      | 24.6911     |
| 55  | 1.508       | 0.6630      | 67.770      | 0.0148      | 44.9320     | 0.0223      | 25.1236     |
| 56  | 1.520       | 0.6581      | 69.278      | 0.0144      | 45.5901     | 0.0219      | 25.5549     |
| 57  | 1.531       | 0.6532      | 70.798      | 0.0141      | 46.2432     | 0.0216      | 25.9849     |
| 58  | 1.542       | 0.6483      | 72.329      | 0.0138      | 46.8915     | 0.0213      | 26.4138     |
| 59  | 1.554       | 0.6435      | 73.871      | 0.0135      | 47.5350     | 0.0210      | 26.8414     |
| 60  | 1.566       | 0.6387      | 75.425      | 0.0133      | 48.1737     | 0.0208      | 27.2678     |
| 61  | 1.577       | 0.6339      | 76.991      | 0.0130      | 48.8077     | 0.0205      | 27.6929     |
| 62  | 1.589       | 0.6292      | 78.568      | 0.0127      | 49.4369     | 0.0202      | 28.1169     |
| 63  | 1.601       | 0.6245      | 80.157      | 0.0125      | 50.0615     | 0.0200      | 28.5396     |
| 64  | 1.613       | 0.6199      | 81.759      | 0.0122      | 50.6814     | 0.0197      | 28.9611     |
| 65  | 1.625       | 0.6153      | 83.372      | 0.0120      | 51.2966     | 0.0195      | 29.3814     |
| 66  | 1.637       | 0.6107      | 84.997      | 0.0118      | 51.9073     | 0.0193      | 29.8005     |
| 67  | 1.650       | 0.6061      | 86.635      | 0.0115      | 52.5135     | 0.0190      | 30.2183     |
| 68  | 1.662       | 0.6016      | 88.284      | 0.0113      | 53.1151     | 0.0188      | 30.6350     |
| 69  | 1.675       | 0.5972      | 89.947      | 0.0111      | 53.7123     | 0.0186      | 31.0504     |
| 70  | 1.687       | 0.5927      | 91.621      | 0.0109      | 54.3050     | 0.0184      | 31.4646     |
| 71  | 1.700       | 0.5883      | 93.308      | 0.0107      | 54.8933     | 0.0182      | 31.8776     |
| 72  | 1.713       | 0.5839      | 95.008      | 0.0105      | 55.4773     | 0.0180      | 32.2895     |
| 73  | 1.725       | 0.5796      | 96.721      | 0.0103      | 56.0568     | 0.0178      | 32.7000     |
| 74  | 1.738       | 0.5753      | 98.446      | 0.0102      | 56.6321     | 0.0177      | 33.1094     |
| 75  | 1.751       | 0.5710      | 100.185     | 0.0100      | 57.2031     | 0.0175      | 33.5176     |
| 76  | 1.765       | 0.5667      | 101.936     | 0.0098      | 57.7698     | 0.0173      | 33.9246     |
| 77  | 1.778       | 0.5625      | 103.701     | 0.0096      | 58.3323     | 0.0171      | 34.3303     |
| 78  | 1.791       | 0.5583      | 105.478     | 0.0095      | 58.8907     | 0.0170      | 34.7349     |
| 79  | 1.805       | 0.5542      | 107.269     | 0.0093      | 59.4448     | 0.0168      | 35.1382     |
| 80  | 1.818       | 0.5500      | 109.074     | 0.0092      | 59.9949     | 0.0167      | 35.5403     |
| 81  | 1.832       | 0.5459      | 110.892     | 0.0090      | 60.5408     | 0.0165      | 35.9413     |
| 82  | 1.845       | 0.5419      | 112.724     | 0.0089      | 61.0827     | 0.0164      | 36.3410     |
| 83  | 1.859       | 0.5378      | 114.569     | 0.0087      | 61.6206     | 0.0162      | 36.7396     |
| 84  | 1.873       | 0.5338      | 116.428     | 0.0086      | 62.1544     | 0.0161      | 37.1369     |
| 85  | 1.887       | 0.5299      | 118.302     | 0.0085      | 62.6843     | 0.0160      | 37.5331     |
| 86  | 1.901       | 0.5259      | 120.189     | 0.0083      | 63.2102     | 0.0158      | 37.9280     |
| 87  | 1.916       | 0.5220      | 122.090     | 0.0082      | 63.7322     | 0.0157      | 38.3218     |
| 88  | 1.930       | 0.5181      | 124.006     | 0.0081      | 64.2503     | 0.0156      | 38.7143     |
| 89  | 1.945       | 0.5143      | 125.936     | 0.0079      | 64.7646     | 0.0154      | 39.1057     |
| 90  | 1.959       | 0.5104      | 127.881     | 0.0078      | 65.2751     | 0.0153      | 39.4959     |
| 91  | 1.974       | 0.5066      | 129.840     | 0.0077      | 65.7817     | 0.0152      | 39.8849     |
| 92  | 1.989       | 0.5029      | 131.814     | 0.0076      | 66.2846     | 0.0151      | 40.2727     |
| 93  | 2.004       | 0.4991      | 133.802     | 0.0075      | 66.7837     | 0.0150      | 40.6593     |
| 94  | 2.019       | 0.4954      | 135.806     | 0.0074      | 67.2791     | 0.0149      | 41.0447     |
| 95  | 2.034       | 0.4917      | 137.824     | 0.0073      | 67.7708     | 0.0148      | 41.4289     |
| 96  | 2.049       | 0.4881      | 139.858     | 0.0072      | 68.2589     | 0.0147      | 41.8120     |
| 97  | 2.064       | 0.4844      | 141.907     | 0.0070      | 68.7433     | 0.0145      | 42.1939     |
| 98  | 2.080       | 0.4808      | 143.971     | 0.0069      | 69.2241     | 0.0144      | 42.5745     |
| 99  | 2.095       | 0.4772      | 146.051     | 0.0068      | 69.7014     | 0.0143      | 42.9541     |
| 100 | 2.111       | 0.4737      | 148.147     | 0.0068      | 70.1751     | 0.0143      | 43.3324     |

Interest Table for Annual Compounding with  $i = 1\%$ 

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1   | 1.010       | 0.9901      | 1.000       | 1.0000      | 0.9901      | 1.0100      | 0.0000      |
| 2   | 1.020       | 0.9803      | 2.010       | 0.4975      | 1.9704      | 0.5075      | 0.4976      |
| 3   | 1.030       | 0.9706      | 3.030       | 0.3300      | 2.9410      | 0.3400      | 0.9934      |
| 4   | 1.041       | 0.9610      | 4.060       | 0.2463      | 3.9020      | 0.2563      | 1.4873      |
| 5   | 1.051       | 0.9515      | 5.101       | 0.1960      | 4.8534      | 0.2060      | 1.9801      |
| 6   | 1.062       | 0.9420      | 6.152       | 0.1625      | 5.7955      | 0.1725      | 2.4710      |
| 7   | 1.072       | 0.9327      | 7.214       | 0.1386      | 6.7282      | 0.1486      | 2.9601      |
| 8   | 1.083       | 0.9235      | 8.286       | 0.1207      | 7.6517      | 0.1307      | 3.4477      |
| 9   | 1.094       | 0.9143      | 9.369       | 0.1067      | 8.5660      | 0.1167      | 3.9335      |
| 10  | 1.105       | 0.9053      | 10.462      | 0.0956      | 9.4713      | 0.1056      | 4.4179      |
| 11  | 1.116       | 0.8963      | 11.567      | 0.0865      | 10.3676     | 0.0965      | 4.9005      |
| 12  | 1.127       | 0.8874      | 12.682      | 0.0788      | 11.2551     | 0.0888      | 5.3814      |
| 13  | 1.138       | 0.8787      | 13.809      | 0.0724      | 12.1337     | 0.0824      | 5.8607      |
| 14  | 1.149       | 0.8700      | 14.947      | 0.0669      | 13.0037     | 0.0769      | 6.3383      |
| 15  | 1.161       | 0.8613      | 16.097      | 0.0621      | 13.8650     | 0.0721      | 6.8143      |
| 16  | 1.173       | 0.8528      | 17.258      | 0.0579      | 14.7179     | 0.0679      | 7.2886      |
| 17  | 1.184       | 0.8444      | 18.430      | 0.0543      | 15.5622     | 0.0643      | 7.7613      |
| 18  | 1.196       | 0.8360      | 19.615      | 0.0510      | 16.3983     | 0.0610      | 8.2323      |
| 19  | 1.208       | 0.8277      | 20.811      | 0.0481      | 17.2260     | 0.0581      | 8.7016      |
| 20  | 1.220       | 0.8195      | 22.019      | 0.0454      | 18.0455     | 0.0554      | 9.1694      |
| 21  | 1.232       | 0.8114      | 23.239      | 0.0430      | 18.8570     | 0.0530      | 9.6354      |
| 22  | 1.245       | 0.8034      | 24.472      | 0.0409      | 19.6604     | 0.0509      | 10.0997     |
| 23  | 1.257       | 0.7954      | 25.716      | 0.0389      | 20.4558     | 0.0489      | 10.5625     |
| 24  | 1.270       | 0.7876      | 26.973      | 0.0371      | 21.2434     | 0.0471      | 11.0236     |
| 25  | 1.282       | 0.7798      | 28.243      | 0.0354      | 22.0231     | 0.0454      | 11.4830     |
| 26  | 1.295       | 0.7720      | 29.526      | 0.0339      | 22.7952     | 0.0439      | 11.9409     |
| 27  | 1.308       | 0.7644      | 30.821      | 0.0324      | 23.5596     | 0.0424      | 12.3970     |
| 28  | 1.321       | 0.7568      | 32.129      | 0.0311      | 24.3164     | 0.0411      | 12.8516     |
| 29  | 1.335       | 0.7493      | 33.450      | 0.0299      | 25.0658     | 0.0399      | 13.3044     |
| 30  | 1.348       | 0.7419      | 34.785      | 0.0287      | 25.8077     | 0.0387      | 13.7556     |
| 31  | 1.361       | 0.7346      | 36.133      | 0.0277      | 26.5423     | 0.0377      | 14.2052     |
| 32  | 1.375       | 0.7273      | 37.494      | 0.0267      | 27.2696     | 0.0367      | 14.6531     |
| 33  | 1.389       | 0.7201      | 38.869      | 0.0257      | 27.9897     | 0.0357      | 15.0994     |
| 34  | 1.403       | 0.7130      | 40.258      | 0.0248      | 28.7027     | 0.0348      | 15.5441     |
| 35  | 1.417       | 0.7059      | 41.660      | 0.0240      | 29.4086     | 0.0340      | 15.9871     |
| 36  | 1.431       | 0.6989      | 43.077      | 0.0232      | 30.1075     | 0.0332      | 16.4285     |
| 37  | 1.445       | 0.6920      | 44.508      | 0.0225      | 30.7995     | 0.0325      | 16.8682     |
| 38  | 1.460       | 0.6852      | 45.953      | 0.0218      | 31.4847     | 0.0318      | 17.3063     |
| 39  | 1.474       | 0.6784      | 47.412      | 0.0211      | 32.1630     | 0.0311      | 17.7427     |
| 40  | 1.489       | 0.6717      | 48.886      | 0.0205      | 32.8347     | 0.0305      | 18.1776     |
| 41  | 1.504       | 0.6650      | 50.375      | 0.0199      | 33.4997     | 0.0299      | 18.6108     |
| 42  | 1.519       | 0.6584      | 51.879      | 0.0193      | 34.1581     | 0.0293      | 19.0424     |
| 43  | 1.534       | 0.6519      | 53.398      | 0.0187      | 34.8100     | 0.0287      | 19.4723     |
| 44  | 1.549       | 0.6454      | 54.932      | 0.0182      | 35.4554     | 0.0282      | 19.9006     |
| 45  | 1.565       | 0.6391      | 56.481      | 0.0177      | 36.0945     | 0.0277      | 20.3273     |
| 46  | 1.580       | 0.6327      | 58.046      | 0.0172      | 36.7272     | 0.0272      | 20.7523     |
| 47  | 1.596       | 0.6265      | 59.626      | 0.0168      | 37.3537     | 0.0268      | 21.1757     |
| 48  | 1.612       | 0.6203      | 61.223      | 0.0163      | 37.9740     | 0.0263      | 21.5975     |
| 49  | 1.628       | 0.6141      | 62.835      | 0.0159      | 38.5881     | 0.0259      | 22.0177     |
| 50  | 1.645       | 0.6080      | 64.463      | 0.0155      | 39.1961     | 0.0255      | 22.4363     |

Interest Table for Annual Compounding with  $i = 1\%$  (Cont.)

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 51  | 1.661       | 0.6020      | 66.108      | 0.0151      | 39.7981     | 0.0251      | 22.8533     |
| 52  | 1.678       | 0.5961      | 67.769      | 0.0148      | 40.3942     | 0.0248      | 23.2686     |
| 53  | 1.694       | 0.5902      | 69.447      | 0.0144      | 40.9843     | 0.0244      | 23.6823     |
| 54  | 1.711       | 0.5843      | 71.141      | 0.0141      | 41.5687     | 0.0241      | 24.0944     |
| 55  | 1.729       | 0.5785      | 72.852      | 0.0137      | 42.1472     | 0.0237      | 24.5049     |
| 56  | 1.746       | 0.5728      | 74.581      | 0.0134      | 42.7200     | 0.0234      | 24.9138     |
| 57  | 1.763       | 0.5671      | 76.327      | 0.0131      | 43.2871     | 0.0231      | 25.3211     |
| 58  | 1.781       | 0.5615      | 78.090      | 0.0128      | 43.8486     | 0.0228      | 25.7268     |
| 59  | 1.799       | 0.5560      | 79.871      | 0.0125      | 44.4046     | 0.0225      | 26.1308     |
| 60  | 1.817       | 0.5504      | 81.670      | 0.0122      | 44.9550     | 0.0222      | 26.5333     |
| 61  | 1.835       | 0.5450      | 83.486      | 0.0120      | 45.5000     | 0.0220      | 26.9341     |
| 62  | 1.853       | 0.5396      | 85.321      | 0.0117      | 46.0396     | 0.0217      | 27.3334     |
| 63  | 1.872       | 0.5343      | 87.174      | 0.0115      | 46.5739     | 0.0215      | 27.7311     |
| 64  | 1.890       | 0.5290      | 89.046      | 0.0112      | 47.1029     | 0.0212      | 28.1272     |
| 65  | 1.909       | 0.5237      | 90.937      | 0.0110      | 47.6266     | 0.0210      | 28.5216     |
| 66  | 1.928       | 0.5185      | 92.846      | 0.0108      | 48.1451     | 0.0208      | 28.9145     |
| 67  | 1.948       | 0.5134      | 94.774      | 0.0106      | 48.6586     | 0.0206      | 29.3058     |
| 68  | 1.967       | 0.5083      | 96.722      | 0.0103      | 49.1669     | 0.0203      | 29.6955     |
| 69  | 1.987       | 0.5033      | 98.689      | 0.0101      | 49.6702     | 0.0201      | 30.0837     |
| 70  | 2.007       | 0.4983      | 100.676     | 0.0099      | 50.1685     | 0.0199      | 30.4702     |
| 71  | 2.027       | 0.4934      | 102.683     | 0.0097      | 50.6619     | 0.0197      | 30.8552     |
| 72  | 2.047       | 0.4885      | 104.710     | 0.0096      | 51.1504     | 0.0196      | 31.2386     |
| 73  | 2.068       | 0.4837      | 106.757     | 0.0094      | 51.6340     | 0.0194      | 31.6204     |
| 74  | 2.088       | 0.4789      | 108.825     | 0.0092      | 52.1129     | 0.0192      | 32.0006     |
| 75  | 2.109       | 0.4741      | 110.913     | 0.0090      | 52.5870     | 0.0190      | 32.3793     |
| 76  | 2.130       | 0.4694      | 113.022     | 0.0088      | 53.0565     | 0.0188      | 32.7564     |
| 77  | 2.152       | 0.4648      | 115.152     | 0.0087      | 53.5213     | 0.0187      | 33.1320     |
| 78  | 2.173       | 0.4602      | 117.304     | 0.0085      | 53.9815     | 0.0185      | 33.5059     |
| 79  | 2.195       | 0.4556      | 119.477     | 0.0084      | 54.4371     | 0.0184      | 33.8783     |
| 80  | 2.217       | 0.4511      | 121.671     | 0.0082      | 54.8882     | 0.0182      | 34.2492     |
| 81  | 2.239       | 0.4467      | 123.888     | 0.0081      | 55.3348     | 0.0181      | 34.6185     |
| 82  | 2.261       | 0.4422      | 126.127     | 0.0079      | 55.7771     | 0.0179      | 34.9862     |
| 83  | 2.284       | 0.4379      | 128.388     | 0.0078      | 56.2149     | 0.0178      | 35.3524     |
| 84  | 2.307       | 0.4335      | 130.672     | 0.0077      | 56.6484     | 0.0177      | 35.7170     |
| 85  | 2.330       | 0.4292      | 132.979     | 0.0075      | 57.0777     | 0.0175      | 36.0801     |
| 86  | 2.353       | 0.4250      | 135.309     | 0.0074      | 57.5026     | 0.0174      | 36.4416     |
| 87  | 2.377       | 0.4208      | 137.662     | 0.0073      | 57.9234     | 0.0173      | 36.8016     |
| 88  | 2.400       | 0.4166      | 140.038     | 0.0071      | 58.3400     | 0.0171      | 37.1601     |
| 89  | 2.424       | 0.4125      | 142.439     | 0.0070      | 58.7525     | 0.0170      | 37.5170     |
| 90  | 2.449       | 0.4084      | 144.863     | 0.0069      | 59.1609     | 0.0169      | 37.8724     |
| 91  | 2.473       | 0.4043      | 147.312     | 0.0068      | 59.5652     | 0.0168      | 38.2263     |
| 92  | 2.498       | 0.4003      | 149.785     | 0.0067      | 59.9656     | 0.0167      | 38.5786     |
| 93  | 2.523       | 0.3964      | 152.283     | 0.0066      | 60.3619     | 0.0166      | 38.9294     |
| 94  | 2.548       | 0.3925      | 154.806     | 0.0065      | 60.7544     | 0.0165      | 39.2787     |
| 95  | 2.574       | 0.3886      | 157.354     | 0.0064      | 61.1430     | 0.0164      | 39.6265     |
| 96  | 2.599       | 0.3847      | 159.927     | 0.0063      | 61.5277     | 0.0163      | 39.9727     |
| 97  | 2.625       | 0.3809      | 162.526     | 0.0062      | 61.9086     | 0.0162      | 40.3174     |
| 98  | 2.652       | 0.3771      | 165.152     | 0.0061      | 62.2858     | 0.0161      | 40.6606     |
| 99  | 2.678       | 0.3734      | 167.803     | 0.0060      | 62.6592     | 0.0160      | 41.0023     |
| 100 | 2.705       | 0.3697      | 170.481     | 0.0059      | 63.0289     | 0.0159      | 41.3425     |

Interest Table for Annual Compounding with  $i = 1.25\%$ 

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1   | 1.013       | 0.9877      | 1.000       | 1.0000      | 0.9877      | 1.0125      | 0.0000      |
| 2   | 1.025       | 0.9755      | 2.013       | 0.4969      | 1.9631      | 0.5094      | 0.4974      |
| 3   | 1.038       | 0.9634      | 3.038       | 0.3292      | 2.9265      | 0.3417      | 0.9921      |
| 4   | 1.051       | 0.9515      | 4.076       | 0.2454      | 3.8781      | 0.2579      | 1.4848      |
| 5   | 1.064       | 0.9398      | 5.127       | 0.1951      | 4.8179      | 0.2076      | 1.9756      |
| 6   | 1.077       | 0.9282      | 6.191       | 0.1615      | 5.7460      | 0.1740      | 2.4642      |
| 7   | 1.091       | 0.9167      | 7.268       | 0.1376      | 6.6628      | 0.1501      | 2.9507      |
| 8   | 1.104       | 0.9054      | 8.359       | 0.1196      | 7.5682      | 0.1321      | 3.4352      |
| 9   | 1.118       | 0.8942      | 9.463       | 0.1057      | 8.4624      | 0.1182      | 3.9176      |
| 10  | 1.132       | 0.8832      | 10.582      | 0.0945      | 9.3456      | 0.1070      | 4.3979      |
| 11  | 1.146       | 0.8723      | 11.714      | 0.0854      | 10.2179     | 0.0979      | 4.8762      |
| 12  | 1.161       | 0.8615      | 12.860      | 0.0778      | 11.0794     | 0.0903      | 5.3525      |
| 13  | 1.175       | 0.8509      | 14.021      | 0.0713      | 11.9302     | 0.0838      | 5.8266      |
| 14  | 1.190       | 0.8404      | 15.196      | 0.0658      | 12.7706     | 0.0783      | 6.2987      |
| 15  | 1.205       | 0.8300      | 16.386      | 0.0610      | 13.6006     | 0.0735      | 6.7687      |
| 16  | 1.220       | 0.8197      | 17.591      | 0.0568      | 14.4204     | 0.0693      | 7.2366      |
| 17  | 1.235       | 0.8096      | 18.811      | 0.0532      | 15.2300     | 0.0657      | 7.7025      |
| 18  | 1.251       | 0.7996      | 20.046      | 0.0499      | 16.0296     | 0.0624      | 8.1663      |
| 19  | 1.266       | 0.7898      | 21.297      | 0.0470      | 16.8194     | 0.0595      | 8.6281      |
| 20  | 1.282       | 0.7800      | 22.563      | 0.0443      | 17.5994     | 0.0568      | 9.0878      |
| 21  | 1.298       | 0.7704      | 23.845      | 0.0419      | 18.3698     | 0.0544      | 9.5454      |
| 22  | 1.314       | 0.7609      | 25.143      | 0.0398      | 19.1307     | 0.0523      | 10.0010     |
| 23  | 1.331       | 0.7515      | 26.458      | 0.0378      | 19.8821     | 0.0503      | 10.4546     |
| 24  | 1.347       | 0.7422      | 27.788      | 0.0360      | 20.6243     | 0.0485      | 10.9061     |
| 25  | 1.364       | 0.7330      | 29.136      | 0.0343      | 21.3574     | 0.0468      | 11.3555     |
| 26  | 1.381       | 0.7240      | 30.500      | 0.0328      | 22.0813     | 0.0453      | 11.8029     |
| 27  | 1.399       | 0.7150      | 31.881      | 0.0314      | 22.7964     | 0.0439      | 12.2482     |
| 28  | 1.416       | 0.7062      | 33.280      | 0.0300      | 23.5026     | 0.0425      | 12.6915     |
| 29  | 1.434       | 0.6975      | 34.696      | 0.0288      | 24.2001     | 0.0413      | 13.1327     |
| 30  | 1.452       | 0.6889      | 36.129      | 0.0277      | 24.8890     | 0.0402      | 13.5719     |
| 31  | 1.470       | 0.6804      | 37.581      | 0.0266      | 25.5694     | 0.0391      | 14.0091     |
| 32  | 1.488       | 0.6720      | 39.051      | 0.0256      | 26.2414     | 0.0381      | 14.4442     |
| 33  | 1.507       | 0.6637      | 40.539      | 0.0247      | 26.9051     | 0.0372      | 14.8773     |
| 34  | 1.526       | 0.6555      | 42.046      | 0.0238      | 27.5606     | 0.0363      | 15.3083     |
| 35  | 1.545       | 0.6474      | 43.571      | 0.0230      | 28.2080     | 0.0355      | 15.7373     |
| 36  | 1.564       | 0.6394      | 45.116      | 0.0222      | 28.8474     | 0.0347      | 16.1643     |
| 37  | 1.583       | 0.6315      | 46.680      | 0.0214      | 29.4789     | 0.0339      | 16.5892     |
| 38  | 1.603       | 0.6237      | 48.263      | 0.0207      | 30.1026     | 0.0332      | 17.0121     |
| 39  | 1.623       | 0.6160      | 49.867      | 0.0201      | 30.7186     | 0.0326      | 17.4330     |
| 40  | 1.644       | 0.6084      | 51.490      | 0.0194      | 31.3271     | 0.0319      | 17.8519     |
| 41  | 1.664       | 0.6009      | 53.134      | 0.0188      | 31.9280     | 0.0313      | 18.2688     |
| 42  | 1.685       | 0.5935      | 54.798      | 0.0182      | 32.5214     | 0.0307      | 18.6836     |
| 43  | 1.706       | 0.5862      | 56.483      | 0.0177      | 33.1076     | 0.0302      | 19.0964     |
| 44  | 1.727       | 0.5789      | 58.189      | 0.0172      | 33.6865     | 0.0297      | 19.5072     |
| 45  | 1.749       | 0.5718      | 59.916      | 0.0167      | 34.2583     | 0.0292      | 19.9160     |
| 46  | 1.771       | 0.5647      | 61.665      | 0.0162      | 34.8230     | 0.0287      | 20.3228     |
| 47  | 1.793       | 0.5577      | 63.436      | 0.0158      | 35.3808     | 0.0283      | 20.7276     |
| 48  | 1.815       | 0.5509      | 65.229      | 0.0153      | 35.9316     | 0.0278      | 21.1303     |
| 49  | 1.838       | 0.5441      | 67.044      | 0.0149      | 36.4757     | 0.0274      | 21.5311     |
| 50  | 1.861       | 0.5373      | 68.882      | 0.0145      | 37.0130     | 0.0270      | 21.9299     |

Interest Table for Annual Compounding with  $i = 1.25\%$  (Cont.)

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 51  | 1.884       | 0.5307      | 70.743      | 0.0141      | 37.5437     | 0.0266      | 22.3267     |
| 52  | 1.908       | 0.5242      | 72.628      | 0.0138      | 38.0679     | 0.0263      | 22.7215     |
| 53  | 1.932       | 0.5177      | 74.535      | 0.0134      | 38.5856     | 0.0259      | 23.1143     |
| 54  | 1.956       | 0.5113      | 76.467      | 0.0131      | 39.0969     | 0.0256      | 23.5052     |
| 55  | 1.980       | 0.5050      | 78.423      | 0.0128      | 39.6018     | 0.0253      | 23.8940     |
| 56  | 2.005       | 0.4987      | 80.403      | 0.0124      | 40.1006     | 0.0249      | 24.2809     |
| 57  | 2.030       | 0.4926      | 82.408      | 0.0121      | 40.5932     | 0.0246      | 24.6658     |
| 58  | 2.055       | 0.4865      | 84.438      | 0.0118      | 41.0797     | 0.0243      | 25.0488     |
| 59  | 2.081       | 0.4805      | 86.494      | 0.0116      | 41.5602     | 0.0241      | 25.4297     |
| 60  | 2.107       | 0.4746      | 88.575      | 0.0113      | 42.0347     | 0.0238      | 25.8088     |
| 61  | 2.134       | 0.4687      | 90.682      | 0.0110      | 42.5035     | 0.0235      | 26.1858     |
| 62  | 2.160       | 0.4629      | 92.816      | 0.0108      | 42.9664     | 0.0233      | 26.5609     |
| 63  | 2.187       | 0.4572      | 94.976      | 0.0105      | 43.4236     | 0.0230      | 26.9340     |
| 64  | 2.215       | 0.4516      | 97.163      | 0.0103      | 43.8752     | 0.0228      | 27.3052     |
| 65  | 2.242       | 0.4460      | 99.378      | 0.0101      | 44.3211     | 0.0226      | 27.6745     |
| 66  | 2.270       | 0.4405      | 101.620     | 0.0098      | 44.7616     | 0.0223      | 28.0418     |
| 67  | 2.299       | 0.4350      | 103.890     | 0.0096      | 45.1967     | 0.0221      | 28.4072     |
| 68  | 2.327       | 0.4297      | 106.189     | 0.0094      | 45.6263     | 0.0219      | 28.7706     |
| 69  | 2.356       | 0.4244      | 108.516     | 0.0092      | 46.0507     | 0.0217      | 29.1321     |
| 70  | 2.386       | 0.4191      | 110.873     | 0.0090      | 46.4698     | 0.0215      | 29.4917     |
| 71  | 2.416       | 0.4140      | 113.259     | 0.0088      | 46.8838     | 0.0213      | 29.8494     |
| 72  | 2.446       | 0.4088      | 115.675     | 0.0086      | 47.2926     | 0.0211      | 30.2051     |
| 73  | 2.477       | 0.4038      | 118.121     | 0.0085      | 47.6964     | 0.0210      | 30.5590     |
| 74  | 2.507       | 0.3988      | 120.597     | 0.0083      | 48.0952     | 0.0208      | 30.9109     |
| 75  | 2.539       | 0.3939      | 123.104     | 0.0081      | 48.4891     | 0.0206      | 31.2609     |
| 76  | 2.571       | 0.3890      | 125.643     | 0.0080      | 48.8782     | 0.0205      | 31.6091     |
| 77  | 2.603       | 0.3842      | 128.214     | 0.0078      | 49.2624     | 0.0203      | 31.9553     |
| 78  | 2.635       | 0.3795      | 130.817     | 0.0076      | 49.6419     | 0.0201      | 32.2996     |
| 79  | 2.668       | 0.3748      | 133.452     | 0.0075      | 50.0166     | 0.0200      | 32.6421     |
| 80  | 2.701       | 0.3702      | 136.120     | 0.0073      | 50.3868     | 0.0198      | 32.9826     |
| 81  | 2.735       | 0.3656      | 138.821     | 0.0072      | 50.7524     | 0.0197      | 33.3213     |
| 82  | 2.769       | 0.3611      | 141.557     | 0.0071      | 51.1135     | 0.0196      | 33.6582     |
| 83  | 2.804       | 0.3566      | 144.326     | 0.0069      | 51.4701     | 0.0194      | 33.9931     |
| 84  | 2.839       | 0.3522      | 147.130     | 0.0068      | 51.8223     | 0.0193      | 34.3262     |
| 85  | 2.875       | 0.3479      | 149.969     | 0.0067      | 52.1702     | 0.0192      | 34.6574     |
| 86  | 2.911       | 0.3436      | 152.844     | 0.0065      | 52.5138     | 0.0190      | 34.9868     |
| 87  | 2.947       | 0.3393      | 155.755     | 0.0064      | 52.8531     | 0.0189      | 35.3143     |
| 88  | 2.984       | 0.3351      | 158.702     | 0.0063      | 53.1883     | 0.0188      | 35.6400     |
| 89  | 3.021       | 0.3310      | 161.685     | 0.0062      | 53.5193     | 0.0187      | 35.9639     |
| 90  | 3.059       | 0.3269      | 164.706     | 0.0061      | 53.8462     | 0.0186      | 36.2859     |
| 91  | 3.097       | 0.3229      | 167.765     | 0.0060      | 54.1691     | 0.0185      | 36.6060     |
| 92  | 3.136       | 0.3189      | 170.862     | 0.0059      | 54.4880     | 0.0184      | 36.9244     |
| 93  | 3.175       | 0.3150      | 173.998     | 0.0057      | 54.8030     | 0.0182      | 37.2409     |
| 94  | 3.215       | 0.3111      | 177.173     | 0.0056      | 55.1141     | 0.0181      | 37.5557     |
| 95  | 3.255       | 0.3072      | 180.388     | 0.0055      | 55.4213     | 0.0180      | 37.8686     |
| 96  | 3.296       | 0.3034      | 183.643     | 0.0054      | 55.7247     | 0.0179      | 38.1797     |
| 97  | 3.337       | 0.2997      | 186.938     | 0.0053      | 56.0244     | 0.0178      | 38.4890     |
| 98  | 3.378       | 0.2960      | 190.275     | 0.0053      | 56.3204     | 0.0178      | 38.7965     |
| 99  | 3.421       | 0.2923      | 193.654     | 0.0052      | 56.6128     | 0.0177      | 39.1022     |
| 100 | 3.463       | 0.2887      | 197.074     | 0.0051      | 56.9015     | 0.0176      | 39.4061     |

Interest Table for Annual Compounding with  $i = 1.5\%$ 

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1   | 1.015       | 0.9852      | 1.000       | 1.0000      | 0.9852      | 1.0150      | 0.0000      |
| 2   | 1.030       | 0.9707      | 2.015       | 0.4963      | 1.9559      | 0.5113      | 0.4962      |
| 3   | 1.046       | 0.9563      | 3.045       | 0.3284      | 2.9122      | 0.3434      | 0.9900      |
| 4   | 1.061       | 0.9422      | 4.091       | 0.2444      | 3.8544      | 0.2594      | 1.4811      |
| 5   | 1.077       | 0.9283      | 5.152       | 0.1941      | 4.7826      | 0.2091      | 1.9700      |
| 6   | 1.093       | 0.9145      | 6.230       | 0.1605      | 5.6972      | 0.1755      | 2.4563      |
| 7   | 1.110       | 0.9010      | 7.323       | 0.1366      | 6.5982      | 0.1516      | 2.9402      |
| 8   | 1.126       | 0.8877      | 8.433       | 0.1186      | 7.4859      | 0.1336      | 3.4216      |
| 9   | 1.143       | 0.8746      | 9.559       | 0.1046      | 8.3605      | 0.1196      | 3.9006      |
| 10  | 1.161       | 0.8617      | 10.703      | 0.0934      | 9.2222      | 0.1084      | 4.3770      |
| 11  | 1.178       | 0.8489      | 11.863      | 0.0843      | 10.0711     | 0.0993      | 4.8510      |
| 12  | 1.196       | 0.8364      | 13.041      | 0.0767      | 10.9075     | 0.0917      | 5.3225      |
| 13  | 1.214       | 0.8240      | 14.237      | 0.0702      | 11.7315     | 0.0852      | 5.7914      |
| 14  | 1.232       | 0.8118      | 15.450      | 0.0647      | 12.5433     | 0.0797      | 6.2580      |
| 15  | 1.250       | 0.7999      | 16.682      | 0.0599      | 13.3432     | 0.0749      | 6.7221      |
| 16  | 1.269       | 0.7880      | 17.932      | 0.0558      | 14.1312     | 0.0708      | 7.1837      |
| 17  | 1.288       | 0.7764      | 19.201      | 0.0521      | 14.9076     | 0.0671      | 7.6428      |
| 18  | 1.307       | 0.7649      | 20.489      | 0.0488      | 15.6725     | 0.0638      | 8.0995      |
| 19  | 1.327       | 0.7536      | 21.797      | 0.0459      | 16.4261     | 0.0609      | 8.5537      |
| 20  | 1.347       | 0.7425      | 23.124      | 0.0432      | 17.1686     | 0.0582      | 9.0055      |
| 21  | 1.367       | 0.7315      | 24.470      | 0.0409      | 17.9001     | 0.0559      | 9.4547      |
| 22  | 1.388       | 0.7207      | 25.837      | 0.0387      | 18.6208     | 0.0537      | 9.9016      |
| 23  | 1.408       | 0.7100      | 27.225      | 0.0367      | 19.3308     | 0.0517      | 10.3460     |
| 24  | 1.430       | 0.6995      | 28.633      | 0.0349      | 20.0304     | 0.0499      | 10.7879     |
| 25  | 1.451       | 0.6892      | 30.063      | 0.0333      | 20.7196     | 0.0483      | 11.2274     |
| 26  | 1.473       | 0.6790      | 31.514      | 0.0317      | 21.3986     | 0.0467      | 11.6644     |
| 27  | 1.495       | 0.6690      | 32.987      | 0.0303      | 22.0676     | 0.0453      | 12.0990     |
| 28  | 1.517       | 0.6591      | 34.481      | 0.0290      | 22.7267     | 0.0440      | 12.5311     |
| 29  | 1.540       | 0.6494      | 35.999      | 0.0278      | 23.3760     | 0.0428      | 12.9608     |
| 30  | 1.563       | 0.6398      | 37.539      | 0.0266      | 24.0158     | 0.0416      | 13.3881     |
| 31  | 1.587       | 0.6303      | 39.102      | 0.0256      | 24.6461     | 0.0406      | 13.8129     |
| 32  | 1.610       | 0.6210      | 40.688      | 0.0246      | 25.2671     | 0.0396      | 14.2353     |
| 33  | 1.634       | 0.6118      | 42.298      | 0.0236      | 25.8789     | 0.0386      | 14.6553     |
| 34  | 1.659       | 0.6028      | 43.933      | 0.0228      | 26.4817     | 0.0378      | 15.0728     |
| 35  | 1.684       | 0.5939      | 45.592      | 0.0219      | 27.0755     | 0.0369      | 15.4880     |
| 36  | 1.709       | 0.5851      | 47.276      | 0.0212      | 27.6606     | 0.0362      | 15.9007     |
| 37  | 1.735       | 0.5764      | 48.985      | 0.0204      | 28.2371     | 0.0354      | 16.3110     |
| 38  | 1.761       | 0.5679      | 50.720      | 0.0197      | 28.8050     | 0.0347      | 16.7189     |
| 39  | 1.787       | 0.5595      | 52.480      | 0.0191      | 29.3645     | 0.0341      | 17.1244     |
| 40  | 1.814       | 0.5513      | 54.268      | 0.0184      | 29.9158     | 0.0334      | 17.5275     |
| 41  | 1.841       | 0.5431      | 56.082      | 0.0178      | 30.4589     | 0.0328      | 17.9282     |
| 42  | 1.869       | 0.5351      | 57.923      | 0.0173      | 30.9940     | 0.0323      | 18.3265     |
| 43  | 1.897       | 0.5272      | 59.792      | 0.0167      | 31.5212     | 0.0317      | 18.7225     |
| 44  | 1.925       | 0.5194      | 61.689      | 0.0162      | 32.0405     | 0.0312      | 19.1160     |
| 45  | 1.954       | 0.5117      | 63.614      | 0.0157      | 32.5523     | 0.0307      | 19.5072     |
| 46  | 1.984       | 0.5042      | 65.568      | 0.0153      | 33.0564     | 0.0303      | 19.8960     |
| 47  | 2.013       | 0.4967      | 67.552      | 0.0148      | 33.5531     | 0.0298      | 20.2824     |
| 48  | 2.043       | 0.4894      | 69.565      | 0.0144      | 34.0425     | 0.0294      | 20.6665     |
| 49  | 2.074       | 0.4821      | 71.608      | 0.0140      | 34.5246     | 0.0290      | 21.0482     |
| 50  | 2.105       | 0.4750      | 73.682      | 0.0136      | 34.9996     | 0.0286      | 21.4275     |

Interest Table for Annual Compounding with  $i = 1.5\%$  (Cont.)

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 51  | 2.137       | 0.4680      | 75.788      | 0.0132      | 35.4676     | 0.0282      | 21.8045     |
| 52  | 2.169       | 0.4611      | 77.925      | 0.0128      | 35.9287     | 0.0278      | 22.1792     |
| 53  | 2.201       | 0.4543      | 80.093      | 0.0125      | 36.3829     | 0.0275      | 22.5515     |
| 54  | 2.234       | 0.4475      | 82.295      | 0.0122      | 36.8305     | 0.0272      | 22.9215     |
| 55  | 2.268       | 0.4409      | 84.529      | 0.0118      | 37.2714     | 0.0268      | 23.2891     |
| 56  | 2.302       | 0.4344      | 86.797      | 0.0115      | 37.7058     | 0.0265      | 23.6545     |
| 57  | 2.336       | 0.4280      | 89.099      | 0.0112      | 38.1338     | 0.0262      | 24.0175     |
| 58  | 2.372       | 0.4217      | 91.436      | 0.0109      | 38.5555     | 0.0259      | 24.3782     |
| 59  | 2.407       | 0.4154      | 93.807      | 0.0107      | 38.9709     | 0.0257      | 24.7366     |
| 60  | 2.443       | 0.4093      | 96.214      | 0.0104      | 39.3802     | 0.0254      | 25.0927     |
| 61  | 2.480       | 0.4032      | 98.657      | 0.0101      | 39.7834     | 0.0251      | 25.4466     |
| 62  | 2.517       | 0.3973      | 101.137     | 0.0099      | 40.1807     | 0.0249      | 25.7981     |
| 63  | 2.555       | 0.3914      | 103.654     | 0.0096      | 40.5721     | 0.0246      | 26.1474     |
| 64  | 2.593       | 0.3856      | 106.209     | 0.0094      | 40.9578     | 0.0244      | 26.4943     |
| 65  | 2.632       | 0.3799      | 108.802     | 0.0092      | 41.3377     | 0.0242      | 26.8390     |
| 66  | 2.672       | 0.3743      | 111.434     | 0.0090      | 41.7120     | 0.0240      | 27.1815     |
| 67  | 2.712       | 0.3688      | 114.106     | 0.0088      | 42.0808     | 0.0238      | 27.5217     |
| 68  | 2.752       | 0.3633      | 116.817     | 0.0086      | 42.4441     | 0.0236      | 27.8596     |
| 69  | 2.794       | 0.3580      | 119.570     | 0.0084      | 42.8021     | 0.0234      | 28.1953     |
| 70  | 2.835       | 0.3527      | 122.363     | 0.0082      | 43.1548     | 0.0232      | 28.5288     |
| 71  | 2.878       | 0.3475      | 125.199     | 0.0080      | 43.5023     | 0.0230      | 28.8600     |
| 72  | 2.921       | 0.3423      | 128.076     | 0.0078      | 43.8446     | 0.0228      | 29.1891     |
| 73  | 2.965       | 0.3373      | 130.998     | 0.0076      | 44.1819     | 0.0226      | 29.5159     |
| 74  | 3.009       | 0.3323      | 133.963     | 0.0075      | 44.5141     | 0.0225      | 29.8405     |
| 75  | 3.055       | 0.3274      | 136.972     | 0.0073      | 44.8415     | 0.0223      | 30.1629     |
| 76  | 3.100       | 0.3225      | 140.027     | 0.0071      | 45.1641     | 0.0221      | 30.4831     |
| 77  | 3.147       | 0.3178      | 143.127     | 0.0070      | 45.4818     | 0.0220      | 30.8011     |
| 78  | 3.194       | 0.3131      | 146.274     | 0.0068      | 45.7949     | 0.0218      | 31.1169     |
| 79  | 3.242       | 0.3084      | 149.468     | 0.0067      | 46.1034     | 0.0217      | 31.4306     |
| 80  | 3.291       | 0.3039      | 152.710     | 0.0065      | 46.4072     | 0.0215      | 31.7421     |
| 81  | 3.340       | 0.2994      | 156.001     | 0.0064      | 46.7066     | 0.0214      | 32.0514     |
| 82  | 3.390       | 0.2950      | 159.341     | 0.0063      | 47.0016     | 0.0213      | 32.3586     |
| 83  | 3.441       | 0.2906      | 162.731     | 0.0061      | 47.2922     | 0.0211      | 32.6637     |
| 84  | 3.493       | 0.2863      | 166.172     | 0.0060      | 47.5786     | 0.0210      | 32.9666     |
| 85  | 3.545       | 0.2821      | 169.664     | 0.0059      | 47.8606     | 0.0209      | 33.2674     |
| 86  | 3.598       | 0.2779      | 173.209     | 0.0058      | 48.1386     | 0.0208      | 33.5660     |
| 87  | 3.652       | 0.2738      | 176.807     | 0.0057      | 48.4124     | 0.0207      | 33.8626     |
| 88  | 3.707       | 0.2698      | 180.459     | 0.0055      | 48.6822     | 0.0205      | 34.1571     |
| 89  | 3.762       | 0.2658      | 184.166     | 0.0054      | 48.9479     | 0.0204      | 34.4494     |
| 90  | 3.819       | 0.2619      | 187.929     | 0.0053      | 49.2098     | 0.0203      | 34.7397     |
| 91  | 3.876       | 0.2580      | 191.748     | 0.0052      | 49.4678     | 0.0202      | 35.0279     |
| 92  | 3.934       | 0.2542      | 195.624     | 0.0051      | 49.7219     | 0.0201      | 35.3140     |
| 93  | 3.993       | 0.2504      | 199.558     | 0.0050      | 49.9723     | 0.0200      | 35.5980     |
| 94  | 4.053       | 0.2467      | 203.552     | 0.0049      | 50.2191     | 0.0199      | 35.8800     |
| 95  | 4.114       | 0.2431      | 207.605     | 0.0048      | 50.4621     | 0.0198      | 36.1600     |
| 96  | 4.176       | 0.2395      | 211.719     | 0.0047      | 50.7016     | 0.0197      | 36.4379     |
| 97  | 4.238       | 0.2359      | 215.895     | 0.0046      | 50.9375     | 0.0196      | 36.7138     |
| 98  | 4.302       | 0.2325      | 220.133     | 0.0045      | 51.1700     | 0.0195      | 36.9877     |
| 99  | 4.367       | 0.2290      | 224.435     | 0.0045      | 51.3990     | 0.0195      | 37.2595     |
| 100 | 4.432       | 0.2256      | 228.802     | 0.0044      | 51.6246     | 0.0194      | 37.5293     |

Interest Table for Annual Compounding with  $i = 1.75\%$ 

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1   | 1.018       | 0.9828      | 1.000       | 1.0000      | 0.9828      | 1.0175      | 0.0000      |
| 2   | 1.035       | 0.9659      | 2.018       | 0.4957      | 1.9487      | 0.5132      | 0.4959      |
| 3   | 1.053       | 0.9493      | 3.053       | 0.3276      | 2.8980      | 0.3451      | 0.9886      |
| 4   | 1.072       | 0.9330      | 4.106       | 0.2435      | 3.8310      | 0.2610      | 1.4784      |
| 5   | 1.091       | 0.9169      | 5.178       | 0.1931      | 4.7479      | 0.2106      | 1.9655      |
| 6   | 1.110       | 0.9011      | 6.269       | 0.1595      | 5.6490      | 0.1770      | 2.4495      |
| 7   | 1.129       | 0.8856      | 7.378       | 0.1355      | 6.5347      | 0.1530      | 2.9308      |
| 8   | 1.149       | 0.8704      | 8.508       | 0.1175      | 7.4051      | 0.1350      | 3.4091      |
| 9   | 1.169       | 0.8554      | 9.656       | 0.1036      | 8.2605      | 0.1211      | 3.8845      |
| 10  | 1.189       | 0.8407      | 10.825      | 0.0924      | 9.1012      | 0.1099      | 4.3571      |
| 11  | 1.210       | 0.8263      | 12.015      | 0.0832      | 9.9275      | 0.1007      | 4.8268      |
| 12  | 1.231       | 0.8121      | 13.225      | 0.0756      | 10.7396     | 0.0931      | 5.2936      |
| 13  | 1.253       | 0.7981      | 14.457      | 0.0692      | 11.5377     | 0.0867      | 5.7575      |
| 14  | 1.275       | 0.7844      | 15.710      | 0.0637      | 12.3220     | 0.0812      | 6.2185      |
| 15  | 1.297       | 0.7709      | 16.985      | 0.0589      | 13.0929     | 0.0764      | 6.6767      |
| 16  | 1.320       | 0.7576      | 18.282      | 0.0547      | 13.8505     | 0.0722      | 7.1320      |
| 17  | 1.343       | 0.7446      | 19.602      | 0.0510      | 14.5951     | 0.0685      | 7.5844      |
| 18  | 1.367       | 0.7318      | 20.945      | 0.0477      | 15.3269     | 0.0652      | 8.0339      |
| 19  | 1.390       | 0.7192      | 22.311      | 0.0448      | 16.0461     | 0.0623      | 8.4806      |
| 20  | 1.415       | 0.7068      | 23.702      | 0.0422      | 16.7529     | 0.0597      | 8.9245      |
| 21  | 1.440       | 0.6947      | 25.116      | 0.0398      | 17.4476     | 0.0573      | 9.3655      |
| 22  | 1.465       | 0.6827      | 26.556      | 0.0377      | 18.1303     | 0.0552      | 9.8036      |
| 23  | 1.490       | 0.6710      | 28.021      | 0.0357      | 18.8013     | 0.0532      | 10.2388     |
| 24  | 1.516       | 0.6594      | 29.511      | 0.0339      | 19.4607     | 0.0514      | 10.6713     |
| 25  | 1.543       | 0.6481      | 31.028      | 0.0322      | 20.1088     | 0.0497      | 11.1009     |
| 26  | 1.570       | 0.6369      | 32.571      | 0.0307      | 20.7458     | 0.0482      | 11.5276     |
| 27  | 1.597       | 0.6260      | 34.141      | 0.0293      | 21.3718     | 0.0468      | 11.9515     |
| 28  | 1.625       | 0.6152      | 35.738      | 0.0280      | 21.9870     | 0.0455      | 12.3726     |
| 29  | 1.654       | 0.6046      | 37.363      | 0.0268      | 22.5917     | 0.0443      | 12.7909     |
| 30  | 1.683       | 0.5942      | 39.017      | 0.0256      | 23.1859     | 0.0431      | 13.2063     |
| 31  | 1.712       | 0.5840      | 40.700      | 0.0246      | 23.7699     | 0.0421      | 13.6189     |
| 32  | 1.742       | 0.5740      | 42.412      | 0.0236      | 24.3439     | 0.0411      | 14.0287     |
| 33  | 1.773       | 0.5641      | 44.155      | 0.0226      | 24.9080     | 0.0401      | 14.4358     |
| 34  | 1.804       | 0.5544      | 45.927      | 0.0218      | 25.4624     | 0.0393      | 14.8400     |
| 35  | 1.835       | 0.5449      | 47.731      | 0.0210      | 26.0073     | 0.0385      | 15.2414     |
| 36  | 1.867       | 0.5355      | 49.566      | 0.0202      | 26.5428     | 0.0377      | 15.6400     |
| 37  | 1.900       | 0.5263      | 51.434      | 0.0194      | 27.0691     | 0.0369      | 16.0359     |
| 38  | 1.933       | 0.5172      | 53.334      | 0.0187      | 27.5863     | 0.0362      | 16.4290     |
| 39  | 1.967       | 0.5083      | 55.267      | 0.0181      | 28.0947     | 0.0356      | 16.8193     |
| 40  | 2.002       | 0.4996      | 57.234      | 0.0175      | 28.5943     | 0.0350      | 17.2068     |
| 41  | 2.037       | 0.4910      | 59.236      | 0.0169      | 29.0853     | 0.0344      | 17.5916     |
| 42  | 2.072       | 0.4826      | 61.273      | 0.0163      | 29.5679     | 0.0338      | 17.9736     |
| 43  | 2.109       | 0.4743      | 63.345      | 0.0158      | 30.0421     | 0.0333      | 18.3529     |
| 44  | 2.145       | 0.4661      | 65.453      | 0.0153      | 30.5082     | 0.0328      | 18.7295     |
| 45  | 2.183       | 0.4581      | 67.599      | 0.0148      | 30.9663     | 0.0323      | 19.1033     |
| 46  | 2.221       | 0.4502      | 69.782      | 0.0143      | 31.4165     | 0.0318      | 19.4744     |
| 47  | 2.260       | 0.4425      | 72.003      | 0.0139      | 31.8590     | 0.0314      | 19.8428     |
| 48  | 2.300       | 0.4349      | 74.263      | 0.0135      | 32.2939     | 0.0310      | 20.2085     |
| 49  | 2.340       | 0.4274      | 76.563      | 0.0131      | 32.7212     | 0.0306      | 20.5715     |
| 50  | 2.381       | 0.4200      | 78.903      | 0.0127      | 33.1413     | 0.0302      | 20.9318     |

Interest Table for Annual Compounding with  $i = 1.75\%$  (Cont.)

| $n$ | $F/p, i, n$ | $P/F, i, n$ | $F/A, i, n$ | $A/F, i, n$ | $P/A, i, n$ | $A/P, i, n$ | $A/G, i, n$ |
|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 51  | 2.422       | 0.4128      | 81.283      | 0.0123      | 33.5541     | 0.0298      | 21.2895     |
| 52  | 2.465       | 0.4057      | 83.706      | 0.0119      | 33.9598     | 0.0294      | 21.6444     |
| 53  | 2.508       | 0.3987      | 86.171      | 0.0116      | 34.3585     | 0.0291      | 21.9967     |
| 54  | 2.552       | 0.3919      | 88.679      | 0.0113      | 34.7504     | 0.0288      | 22.3463     |
| 55  | 2.597       | 0.3851      | 91.231      | 0.0110      | 35.1355     | 0.0285      | 22.6933     |
| 56  | 2.642       | 0.3785      | 93.827      | 0.0107      | 35.5140     | 0.0282      | 23.0376     |
| 57  | 2.688       | 0.3720      | 96.469      | 0.0104      | 35.8860     | 0.0279      | 23.3793     |
| 58  | 2.735       | 0.3656      | 99.157      | 0.0101      | 36.2516     | 0.0276      | 23.7183     |
| 59  | 2.783       | 0.3593      | 101.893     | 0.0098      | 36.6109     | 0.0273      | 24.0548     |
| 60  | 2.832       | 0.3531      | 104.676     | 0.0096      | 36.9640     | 0.0271      | 24.3886     |
| 61  | 2.881       | 0.3471      | 107.508     | 0.0093      | 37.3111     | 0.0268      | 24.7199     |
| 62  | 2.932       | 0.3411      | 110.389     | 0.0091      | 37.6522     | 0.0266      | 25.0485     |
| 63  | 2.983       | 0.3352      | 113.321     | 0.0088      | 37.9874     | 0.0263      | 25.3746     |
| 64  | 3.035       | 0.3295      | 116.304     | 0.0086      | 38.3169     | 0.0261      | 25.6981     |
| 65  | 3.088       | 0.3238      | 119.339     | 0.0084      | 38.6407     | 0.0259      | 26.0191     |
| 66  | 3.142       | 0.3182      | 122.428     | 0.0082      | 38.9589     | 0.0257      | 26.3375     |
| 67  | 3.197       | 0.3127      | 125.570     | 0.0080      | 39.2716     | 0.0255      | 26.6534     |
| 68  | 3.253       | 0.3074      | 128.768     | 0.0078      | 39.5790     | 0.0253      | 26.9667     |
| 69  | 3.310       | 0.3021      | 132.021     | 0.0076      | 39.8811     | 0.0251      | 27.2775     |
| 70  | 3.368       | 0.2969      | 135.331     | 0.0074      | 40.1780     | 0.0249      | 27.5858     |
| 71  | 3.427       | 0.2918      | 138.700     | 0.0072      | 40.4697     | 0.0247      | 27.8916     |
| 72  | 3.487       | 0.2868      | 142.127     | 0.0070      | 40.7565     | 0.0245      | 28.1949     |
| 73  | 3.548       | 0.2818      | 145.614     | 0.0069      | 41.0383     | 0.0244      | 28.4957     |
| 74  | 3.610       | 0.2770      | 149.162     | 0.0067      | 41.3153     | 0.0242      | 28.7941     |
| 75  | 3.674       | 0.2722      | 152.773     | 0.0065      | 41.5875     | 0.0240      | 29.0900     |
| 76  | 3.738       | 0.2675      | 156.446     | 0.0064      | 41.8551     | 0.0239      | 29.3835     |
| 77  | 3.803       | 0.2629      | 160.184     | 0.0062      | 42.1180     | 0.0237      | 29.6745     |
| 78  | 3.870       | 0.2584      | 163.987     | 0.0061      | 42.3764     | 0.0236      | 29.9631     |
| 79  | 3.938       | 0.2540      | 167.857     | 0.0060      | 42.6304     | 0.0235      | 30.2493     |
| 80  | 4.006       | 0.2496      | 171.795     | 0.0058      | 42.8800     | 0.0233      | 30.5330     |
| 81  | 4.077       | 0.2453      | 175.801     | 0.0057      | 43.1253     | 0.0232      | 30.8144     |
| 82  | 4.148       | 0.2411      | 179.878     | 0.0056      | 43.3664     | 0.0231      | 31.0934     |
| 83  | 4.220       | 0.2369      | 184.026     | 0.0054      | 43.6033     | 0.0229      | 31.3700     |
| 84  | 4.294       | 0.2329      | 188.246     | 0.0053      | 43.8362     | 0.0228      | 31.6443     |
| 85  | 4.369       | 0.2289      | 192.540     | 0.0052      | 44.0651     | 0.0227      | 31.9162     |
| 86  | 4.446       | 0.2249      | 196.910     | 0.0051      | 44.2900     | 0.0226      | 32.1858     |
| 87  | 4.524       | 0.2211      | 201.356     | 0.0050      | 44.5110     | 0.0225      | 32.4531     |
| 88  | 4.603       | 0.2173      | 205.880     | 0.0049      | 44.7283     | 0.0224      | 32.7180     |
| 89  | 4.683       | 0.2135      | 210.482     | 0.0048      | 44.9418     | 0.0223      | 32.9807     |
| 90  | 4.765       | 0.2098      | 215.166     | 0.0046      | 45.1517     | 0.0221      | 33.2410     |
| 91  | 4.849       | 0.2062      | 219.931     | 0.0045      | 45.3579     | 0.0220      | 33.4991     |
| 92  | 4.934       | 0.2027      | 224.780     | 0.0044      | 45.5606     | 0.0219      | 33.7549     |
| 93  | 5.020       | 0.1992      | 229.714     | 0.0044      | 45.7598     | 0.0219      | 34.0085     |
| 94  | 5.108       | 0.1958      | 234.734     | 0.0043      | 45.9556     | 0.0218      | 34.2598     |
| 95  | 5.197       | 0.1924      | 239.842     | 0.0042      | 46.1480     | 0.0217      | 34.5089     |
| 96  | 5.288       | 0.1891      | 245.039     | 0.0041      | 46.3371     | 0.0216      | 34.7557     |
| 97  | 5.381       | 0.1858      | 250.327     | 0.0040      | 46.5229     | 0.0215      | 35.0004     |
| 98  | 5.475       | 0.1827      | 255.708     | 0.0039      | 46.7056     | 0.0214      | 35.2429     |
| 99  | 5.571       | 0.1795      | 261.183     | 0.0038      | 46.8851     | 0.0213      | 35.4831     |
| 100 | 5.668       | 0.1764      | 266.753     | 0.0037      | 47.0615     | 0.0212      | 35.7213     |

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# Engineering Economics

R. Panneerselvam

Designed as a text book for undergraduate students in various engineering disciplines — mechanical, civil and industrial engineering — and for postgraduate students in industrial engineering and water resource management, this comprehensive and well-organized book shows how complex economic decisions can be made from a number of given alternatives. It provides the managers not only a sound basis but also a clear-cut approach to decision making. These decisions will ultimately result in minimizing costs and/or maximizing benefits to their organizations. What is more, the book adequately illustrates these approaches with numerical problems and *Indian cases*.

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